Network Centrality and Device Ecosystems *

Doh-Shin Jeon† Yassine Lefouili‡ Yaxin Li§ Timothy Simcoe¶

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Abstract

We develop a model of a device ecosystem to study how network structure affects demand, pricing, and output for the underlying products. Prices depend on each device's Katz-Bonacich centrality in a network defined by the demand-side externalities linking devices. We show how the relevant network differs for an ecosystem monopolist, a social planner, a regulator that sets Ramsey prices, or a "decentralized" ecosystem where devices sell at marginal cost. Finally, we extend our baseline model by adding complementary goods and allowing for imperfect competition among downstream device producers.

Keywords: Ecosystem, Multi-sided Market, Network, Centrality

JEL classification: D21, D42, D85, L12

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†Toulouse School of Economics, University of Toulouse Capitole, and CEPR, dohshin.jeon@tse-fr.eu

‡Toulouse School of Economics, University of Toulouse Capitole, yassine.lefouili@tse-fr.eu

§Renmin University of China, School of Applied Economics, liyaxin@ruc.edu.cn

¶Boston University Questrom School of Business and NBER, tsimcoe@bu.edu

1 Introduction

Several of the world's largest companies offer a group of inter-connected products and services that industry observers have called *ecosystems*. The Apple ecosystem, for example, consists of a set of devices (iPhone, iPad, Apple Watch, Macintosh computers) along with the proprietary software and services (iOS, App Store, Apple Music, Apple TV) that run on those devices. Google, Amazon, and Microsoft all have similar business models.

While the economic literature on multi-sided platforms (e.g., Caillaud and Jullien, 2003; Anderson and Coate, 2005; Rochet and Tirole, 2006; Armstrong, 2006) offers many insights into ecosystems, it has not considered the importance of a device's *position* within the overall network. Conversely, the literature on social networks (e.g., Ballester et al., 2006; Galeotti et al., 2010; Candogan et al., 2012; Bloch and Quérou, 2013) examines how network structure mediates spillovers and externalities, but has not focused on device ecosystems. This paper connects the two streams of literature by using a simple model of a device ecosystem to study how network structure influences demand, pricing, and output for the underlying products.

Our first set of results characterize optimal pricing by an ecosystem monopolist and show that the relevant measure of a device's position within an ecosystem is Katz-Bonacich (KB) centrality. For the monopolist, device prices reflect a well-known trade-off between internalizing externalities (subsidizing devices that generate larger positive externalities) and extracting value. These forces are captured by a weighted average of all externalities to/from all other devices, where the weights correspond to the Katz-Bonacich centrality of each device. In equilibrium, moreover, the output of each device is proportional to its KB-centrality.

The second part of our analysis, in Section 3, compares welfare under ecosystem monopoly to several benchmarks. We show how the matrix used to compute KB-centrality differs for a monopolist, a social planner, a regulator that sets Ramsey prices, or a "decentralized" ecosystem where devices sell at marginal cost. Finally, in Section 4 we extend our baseline model of a device ecosystem to account for either complementary goods or market power in downstream device manufacturing.

This paper contributes to several strands of literature. First, there is a large literature on pricing by platforms. In addition to the papers cited above, which mainly analyze pricing by two-sided platforms under monopoly or competition, Weyl (2010) studies a monopoly platform with many sides and highlights the role of a Spence distortion. More recently, Tan and Zhou (2021) analyze platform competition in a many-sided market, characterize the symmetric equilibrium prices, and perform comparative statics to find that an increase in the number of platforms can lead to an increase in the prices. Our main contribution to this literature is to show how, for a many-sided platform, the KB-centrality of each side/device plays a crucial role. By characterizing equilibrium pricing in terms of centrality measures, we find that KB-centrality is a natural concept to use in an ecosystem composed of multiple sides, because it captures both *direct* and *indirect* cross-side network effects.[1]

Second, this paper is related to the literature on pricing in networks in the presence of consumption and price externalities. Building on Ballester et al. (2006)'s approach to network games with strategic complementarities among players, Candogan et al. (2012) and Bloch and Quérou (2013) show that if network effects are symmetric and marginal production costs are constant, a monopolist's optimal prices do not depend on the network structure even if the monopolist is able to price-discriminate. Bloch and Quérou (2013), Chen et al. (2018), Zhang and Chen (2020) and Chen et al. (2022) show that this irrelevance result does not hold in a competitive setting. In that case, firms price-discriminate consumers based on their network positions in terms of KB-centrality.[2] While this literature considers network externalities among consumers, our paper focuses on externalities among products. In that sense, we respond to Caffarra et al. (2023)'s call to bridge network economics and industrial organization.

Finally, we contribute to a broad literature, with roots in both management (Adner and Kapoor, 2010) and economics (Rochet and Tirole, 2003; Rysman, 2009), that explores the relationship between platforms and ecosystems. Many authors use the term ecosystem to describe a set of complementary products whose interactions are orchestrated by a single firm (UK Competition and Markets Authority, 2020, p.57). For example, Heidhues et al. (2024) analyze a multi-product firm's ability to leverage market power in an "access point" by steering customers to its other offerings. Rhodes et al. (2025) develop a model where data-based scope economies provide a competitive advantage to an ecosystem in competition with single-product firms. Other authors, particularly within management, adopt a broader industry-level definition of an ecosystem that emphasizes buyer-supplier relationships and the governance of investments that produce complemen-

[1] When there are more than two sides, even if direct externalities between side i and side j are zero, there can be indirect externalities between the two through other sides.

[2] See also Fainmesser and Galeotti (2016) and Fainmesser and Galeotti (2020).

tarity (e.g., Gawer and Cusumano, 2014; Jacobides et al., 2018). The ecosystem in this paper is formally equivalent to a multi-sided platform, but our emphasis on network structure and KB-centrality justifies, we think, the change in label. And although we do not analyze competition between ecosystems, we offer a tractable framework that represents a first step in that direction, as called for by various competition authorities and commentators (Cremer et al., 2019; Furman et al., 2019; Scott Morton et al., 2019).

2 Ecosystem Monopoly

Consider a multi-product ecosystem with $n > 1$ devices managed by a monopoly platform. Let p_i denote the price charged for device i. This price can represent either a direct sale to end-users or an access fee paid by downstream producers in a perfectly competitive industry. For instance, most Apple devices are sold directly to users, but one could also interpret the monopolist as a licensing platform, such as a patent pool, that charges royalties to producers of various devices using the 5G communications protocol.[3] We normalize marginal costs to zero, so the final price of each device is p_i.

Connectivity among devices creates externalities in demand. Specifically, demand for device i is given by

$$q_i = \alpha_i - \beta_i p_i + \sum_{j \neq i} \gamma_{ij} q_j, \tag{1}$$

where (α_i, β_i) parameterize the standalone demand for device i, and γ_{ij} captures the strength of the externality exerted by device j's users on the users of device i. Appendix A provides a micro-foundation for this demand system.

The literature on two-sided markets emphasizes that externalities, γ_{ij}, emerge from opportunities for *interaction* among different types of agents, such as buyers and sellers on an exchange, or readers, publishers, and advertisers on a web site. Within a device ecosystem, externalities can also arise because users of one device generate *data* that improves the quality of other devices. For example, data from search engines might help improve the quality of maps and shopping sites, or vice versa. Externalities also reflect the idea that devices can access a shared pool of *complements*. Users of Android Phones and Pixel Watches, for example, both access apps in Google's Play Store and content on YouTube Music. So, if the number of Android users encourages greater quality or variety

[3] Avanci, for example, manages a 5G "Internet of Things" patent pool (www.avanci.com/iot/).

in app or music supply, that will also benefit Pixel users.[4]

Using matrices, the demand system (1) can be written as

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{p} + \mathbf{G}\mathbf{q} \tag{2}$$

where $\mathbf{q}$ is an $n \times 1$ vector of quantities q_i, $\mathbf{p}$ is an $n \times 1$ vector of prices p_i, $\mathbf{a}$ is an $n \times 1$ vector of intercepts α_i, $\mathbf{B}$ is an $n \times n$ matrix of slopes β_i with zero for all off-diagonal elements, and

$$\mathbf{G} = \begin{bmatrix} 0 & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & 0 & \cdots & \gamma_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & 0 \end{bmatrix}.$$

Hence, if $\mathbf{I} - \mathbf{G}$ is invertible, the demand system can be written as:

$$\mathbf{q} = (\mathbf{I} - \mathbf{G})^{-1}(\mathbf{a} - \mathbf{B}\mathbf{p}).$$

If $\lambda_{\mathbf{G}}$ represents the largest eigenvalue of $\mathbf{G}$, then a sufficient condition for existence and non-negativity of $(\mathbf{I} - \mathbf{G})^{-1}$ is that $\lambda_{\mathbf{G}} < 1$.[5] The eigenvalue $\lambda_{\mathbf{G}}$ reflects the overall strength of network effects in the ecosystem, and if they are too large then demand will "explode" given the recursive nature of (2). This suggests that an ecosystem monopolist will seek to increase $\lambda_{\mathbf{G}}$ by designing interoperability into its devices or leveraging data harvested from various applications. In this paper, however, we take $\mathbf{G}$ as fixed and focus on pricing decisions.

2.1 Monopoly Pricing and Device Centrality

To begin, suppose $\mathbf{B} = \mathbf{I}$. The monopolist maximizes $\Pi^M = \mathbf{p}'\mathbf{q}$, and its system of first-order conditions can be written as

$$(\mathbf{I} - \mathbf{G})^{-1}(\mathbf{a} - \mathbf{p}) - (\mathbf{I} - \mathbf{G}')^{-1}\mathbf{p} = 0. \tag{3}$$

[4]It is tempting to interpret γ_{ij} as also capturing "ordinary" complementarities rooted in consumer utility. However, both Nocke and Schutz (2017) and Amir et al. (2017) show that for multi-product demand systems without network effects, complementarity implies symmetry ($\gamma_{ij} = \gamma_{ji}$), which we do not impose.

[5]See Theorem III^* of Debreu and Herstein (1953)

where $\mathbf{G}'$ denotes the transpose of $\mathbf{G}$. Appendix B shows that the solution to (3), if one exists, is given by:

$$\mathbf{p}^M = \frac{1}{2}\mathbf{a} + \frac{1}{4}\left(\mathbf{G} - \mathbf{G}'\right)\left[\mathbf{I} - \frac{1}{2}\left(\mathbf{G} + \mathbf{G}'\right)\right]^{-1}\mathbf{a}. \tag{4}$$

Each element of the matrix $(\mathbf{G} - \mathbf{G}')$ measures the *net* externalities ("inbound" γ_{ij} less "outbound" γ_{ji}) associated with an ordered pair of devices. That matrix is post-multiplied by a vector of weights that is familiar from the literature on networks (e.g., Jackson, 2008, Ch. 2). We take from that literature

Definition 1 *The* $n \times 1$ *vector* $\left[\mathbf{I} - \frac{1}{2}\left(\mathbf{G} + \mathbf{G}'\right)\right]^{-1}\mathbf{a} \equiv \mathbf{c}^{KB}$ *measures Katz-Bonacich (KB) centrality in the directed network* $(\mathbf{G} + \mathbf{G}')$ *with decay parameter* $\frac{1}{2}$ *and device weights* $\mathbf{a}$*.*

Katz-Bonacich centrality is a well-known measure of the influence exerted by each node in a network. If $\overline{\mathbf{G}} = (\mathbf{G} + \mathbf{G}')$, then KB-centrality can be decomposed as $\mathbf{c}^{KB} = \mathbf{a} + \frac{1}{2}\overline{\mathbf{G}}\mathbf{a} + \sum_{t=2}^{\infty}(\frac{\mathbf{1}}{\mathbf{2}}\overline{\mathbf{G}})^{\mathbf{t}}\mathbf{a}$. The term $\frac{1}{2}\overline{\mathbf{G}}\mathbf{a}$ measures direct centrality: the total externality of all 1-step links to a device, discounted by the decay parameter $\frac{1}{2}$ (which we show below relates to monopoly pricing) and weighted by $\mathbf{a}$. The term $\sum_{t=2}^{\infty}(\frac{\mathbf{1}}{\mathbf{2}}\overline{\mathbf{G}})^{\mathbf{t}}\mathbf{a}$ measures indirect centrality: the total externalities from all t-step links to a device, discounted by 2^{-t} and weighted by $\mathbf{a}$. Indirect centrality is a geometric sequence that will converge if $\lambda_{\frac{1}{2}\overline{\mathbf{G}}} < 1$, and the same condition guarantees that demand is well-behaved.[6] Thus, we have

Proposition 1 *If* $\lambda_{\frac{1}{2}\overline{\mathbf{G}}} < 1$*, then there exists a unique vector of optimal monopoly prices*

$$\mathbf{p}^M = \frac{1}{2}\left[\mathbf{a} + \frac{1}{2}\left(\mathbf{G} - \mathbf{G}'\right)\mathbf{c}^{KB}\right]. \tag{5}$$

Candogan et al. (2012) derive equation (5) as the solution to a problem of price discrimination of buyers influenced by their social network $\mathbf{G}$. The individual prices can be written in scalar form as

$$p_i^M = \frac{\alpha_i}{2} + \frac{1}{4}\sum_{j \neq i}\left(\gamma_{ij} - \gamma_{ji}\right)c_j^{KB}, \tag{6}$$

and for intuition it is useful to separate these prices into three parts:

[6] See theorem 10.28 of Zhang (2011).

1. Baseline price: In the absence of demand externalities, the standard monopoly price is given by $p_i = \alpha_i/2$.

2. Value capture: The term $\gamma_{ij}c_j^{KB}$ reflects the inbound externality that (users of) device i receive from device j. Increasing this externality increases the value of device i, which leads the platform to raise p_i to capture that value.

3. Externality internalization (or value creation): The term $\gamma_{ji}c_j^{KB}$ captures the outbound externality that (users of) device i exert on device j. An increase in this outbound externality raises demand for device j and that leads the monopolist to lower p_i to internalize this positive externality.

Throughout the paper, we will say that device i is *subsidized* (respectively, *exploited*) if p_i is lower (respectively, higher) than its baseline price. When $\mathbf{G}$ is symmetric, equations (5) and (6) show that incentives for value extraction and externality internalization are in perfect balance, leading to a corollary that is known in the context of social networks and two-sided platforms (e.g, Candogan et al., 2012; Belleflamme and Peitz, 2018).

Corollary 1 *For symmetric demand externalities,* $\mathbf{G} = \mathbf{G}'$*, when* $\mathbf{B} = \mathbf{I}$ *a monopolist charges the baseline prices* $\mathbf{p}^M = \frac{1}{2}\mathbf{a}$*.*

Finally, to solve for output, we can substitute the monopoly prices from (5) into the demand system (2), which yields

$$(\mathbf{I} - \mathbf{G})\mathbf{q} = \mathbf{a} - \mathbf{p}^M = \frac{1}{2}\mathbf{a} - \frac{1}{4}\left(\mathbf{G} - \mathbf{G}'\right)\mathbf{c}^{KB}.$$

Adding $\frac{1}{2}\mathbf{G}\mathbf{c}^{KB}$ to both sides of the equation and using the definition of $\mathbf{c}^{KB}$, this equality simplifies to

$$(\mathbf{I} - \mathbf{G})\mathbf{q} + \frac{1}{2}\mathbf{G}\mathbf{c}^{KB} = \frac{1}{2}\mathbf{a} + \frac{1}{4}\left(\mathbf{G} + \mathbf{G}'\right)\mathbf{c}^{KB} = \frac{1}{2}\mathbf{c}^{KB}$$
$$\Rightarrow \mathbf{q} = \frac{1}{2}\mathbf{c}^{KB},$$

and we restate this result as

Corollary 2 *For linear demand with* $\mathbf{B} = \mathbf{I}$*, the quantity of each device supplied by an ecosystem monopolist is proportional to its KB-centrality.*

Corollary 2 says that a monopolist prices its ecosystem such that output is proportional to the centrality of each device in the network defined by $\overline{\mathbf{G}}$. This helps to explain the intuition that ecosystems often comprise a core product, such as a mobile operating system, with many other devices and services built around that core.[7] It may also explain why large platforms heavily subsidize products like Smart Home devices (e.g., Amazon Echo/Alexa or Google Nest/Assistant) that could form the core of a new ecosystem.[8]

2.1.1 Armstrong's Formulation

Our results can be linked to the two-sided model of Armstrong (2006), which uses a change of variable to express output in terms of utility for each device

$$u_i = \sum_{j \neq i} \gamma_{ij} q_j - p_i, \tag{7}$$

so that quantities are given by $q_i = \alpha_i + u_i$. The platform's profit is $\Pi = \sum p_i q_i$, and its first-order condition with respect to u_i (holding q_j for all $j \neq i$ constant) is therefore

$$\sum_{j \neq i} \gamma_{ij} q_j - u_i - q_i + \sum_{j \neq i} \gamma_{ji} q_j = 0. \tag{8}$$

Rearranging the first-order condition gives the generalized Armstrong pricing rule

$$\frac{p_i + \sum_{j \neq i} \gamma_{ji} q_j}{p_i} = \frac{1}{\varepsilon_i}$$

where $\varepsilon_i = -\frac{\partial q_i}{\partial p_i} / \frac{q_i}{p_i} = p_i / q_i$. The appearance of demand externalities where we would normally observe marginal costs in the Lerner markup rule highlights the marginal effect of reducing p_i on sales of other devices.

Substituting (7) into (8) yields a modified Armstrong pricing formula

$$p_i = \frac{\alpha_i}{2} + \frac{1}{2} \sum_{j \neq i} \left(\gamma_{ij} - \gamma_{ji}\right) q_j. \tag{9}$$

[7] See, for example, the European Commission discussion of ecosystems in its notice on market definition (European Commission, 2024, para. 140).

[8] The Wall Street Journal reports that Amazon's "strategy to set prices low for Echo speakers and other smart devices, expecting them to generate income elsewhere in the tech giant, hasn't paid off." (Alexa Is in Millions of Households—and Amazon Is Losing Billions, Dana Mattioli, WSJ, July 22, 2024).

Our characterization of monopoly prices in (6) takes the same shape, but expresses q_j in terms of centrality. Setting equations (6) and (9) equal reveals, again, that $q_i = c_i^{KB}/2$.

2.1.2 Demand Heterogeneity

Thus far we have assumed linear demand and equal slopes ($\mathbf{B} = \mathbf{I}$). Both assumptions can be relaxed as long as we retain the assumption that network externalities can only shift (and not rotate) the demand curve for each device. In particular, suppose demand is $q_i(p_i)$ with $\frac{\partial q_i}{\partial p_j} = 0$ for all $i \neq j$. Demand for device i can then be approximated using the first-order terms of a Taylor expansion: $\beta_i = q_i'(p_i)$ and $\alpha_i = q_i(p_i) - q_i'(p_i)p_i$. The monopolist's system of first-order conditions in a neighborhood of any profit maximizing price vector can therefore be written as

$$(\mathbf{I} - \mathbf{G})^{-\mathbf{1}}[\mathbf{a} - \mathbf{B}\mathbf{p}] - \mathbf{B}'(\mathbf{I} - \mathbf{G}')^{-\mathbf{1}}\mathbf{p} = 0$$

and Appendix B shows that the solution to this system is

$$\mathbf{p}^M = \frac{1}{2}\mathbf{B}^{-\mathbf{1}}\mathbf{a} + \frac{1}{4}\left(\mathbf{B}^{-\mathbf{1}}\mathbf{G}\mathbf{B} - \mathbf{G}'\right)\mathbf{c}^{KB(\mathbf{B})} \tag{10}$$
$$\text{for } \mathbf{c}^{KB(\mathbf{B})} \equiv \left[\mathbf{I} - \frac{1}{2}\left(\mathbf{B}^{-\mathbf{1}}\mathbf{G}\mathbf{B} + \mathbf{G}'\right)\right]^{-1}\mathbf{B}^{-\mathbf{1}}\mathbf{a}.$$

Equation (10) resembles (5), but with two changes. First, the intercepts $\mathbf{a}$ (and hence, the baseline prices) are scaled by $\mathbf{B}^{-1}$, reflecting the standard incentive to charge higher markups when demand is less elastic. Second, the value capture matrix $\mathbf{G}$ is replaced by $\mathbf{B}^{-\mathbf{1}}\mathbf{G}\mathbf{B}$ (with representative element $\gamma_{ij}\beta_j/\beta_i$). Intuitively, $\mathbf{G}\mathbf{B}$ are the marginal inbound externalities from a change in p_j (with $j \neq i$), and $\frac{1}{2}\mathbf{B}^{-\mathbf{1}}$ is the pass-through rate from inbound externalities (which are equivalent to a demand shift) to $\mathbf{p}^M$.

One implication of (10) is that Corollary 1 no longer holds. An ecosystem monopolist may deviate from baseline prices even if $\mathbf{G}$ is symmetric because externalities interact with demand elasticity. This can also be illustrated by using Armstrong's approach to calculate an element-wise version of (10)

$$p_i = \frac{\alpha_i}{2\beta_i} + \frac{\sum\limits_{j\neq i}\left(\frac{\gamma_{ij}}{\beta_i}\beta_j - \gamma_{ji}\right)\frac{q_j}{\beta_j}}{2} = \frac{\alpha_i}{2\beta_i} + \frac{\sum\limits_{j\neq i}\left(\frac{\gamma_{ij}}{\beta_i} - \frac{\gamma_{ji}}{\beta_j}\right)q_j}{2}.$$

The second part of this equality shows that the inbound externality from device j to device i is discounted by β_i whereas the outbound externality from device i to device j is discounted by β_j. So, even if all of the γ_{ij} are identical, devices with relatively large (small) β_i will be subsidized (exploited). Moreover, comparing the first part of the equality with (10) reveals that

$$\frac{c_j^{KB(\mathbf{B})}}{2} = \frac{q_j}{\beta_j}.$$

2.2 Examples

One contribution of this paper is to document the link between network centrality and ecosystem pricing. To illustrate, this sub-section provides two examples. For each example, the externality between any pair of devices takes one of three values, $\gamma_{ij} \in \{\mu, \eta, 0\}$. We set $\alpha_i = 1$ and $\beta_i = 1$ for all i, and define two parameters $c \equiv \mu + \eta$ and $d \equiv \mu - \eta$. Figure 1 provides a graphical depiction of each example.

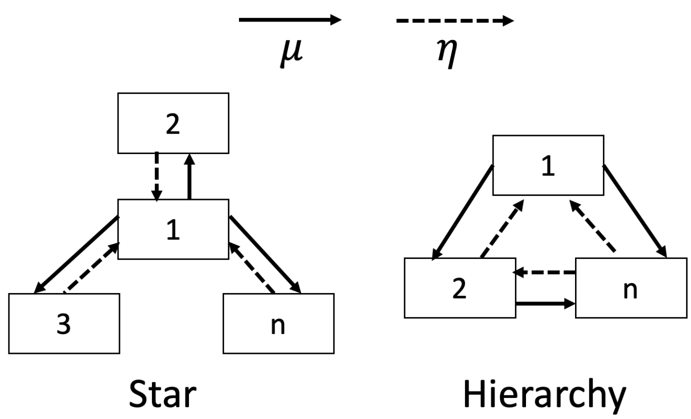


Figure 1: Star and Hierarchy Ecosystems

2.2.1 Star

A star network is defined by $\gamma_{1j} = \eta$ for all $j > 1$; $\gamma_{j1} = \mu$ for all $j > 1$; and $\gamma_{ij} = 0$ for all $i, j > 1$. For this demand system, all externalities either originate from or terminate at the "star" device ($i = 1$). One might think of the star as a smartphone that exhibits bilateral demand externalities with a series of peripheral devices, such as watches, cars, thermostats, etc. that do not interact with one another.

Using (5) and the fact that all peripheral devices ($j > 1$) are symmetric, we can write the monopoly prices as

$$p_1^M = \frac{1}{2} - \frac{1}{4}d(n-1)c_j^{KB}$$
$$p_j^M = \frac{1}{2} + \frac{1}{4}dc_1^{KB}.$$

Because the KB-centrality of each device, c^{KB}, is strictly positive, the star device will be subsidized if and only if $d > 0$ (i.e., when its *net* externalities to each peripheral are positive). Moreover, when the star device is subsidized, the peripherals are exploited, and vice versa. When $d > 0$, the amount of subsidy to the star device is proportionate to $(n-1)$ times the centrality of a peripheral device whereas the amount of exploitation of a peripheral device is proportionate to the centrality of the star.

2.2.2 Hierarchy

Next, consider a "hierarchical" ecosystem, where $\gamma_{ij} = \eta$ for all $i < j$, and $\gamma_{ij} = \mu$ for all $i > j$. When $\mu > \eta$, device 1 generates the most and receives the fewest externalities, device 2 generates the second-most and receives the second-least amount of externalities, and so on. This example might correspond to a set of integrated software tools or applications: all devices interact with each other, but some produce more externalities.

For this demand system, every non-diagonal element in the matrix $[\mathbf{I} - \overline{\mathbf{G}}]$ equals $-\frac{c}{2}$, and because its inverse exhibits the same symmetry, all devices have the same KB-centrality. Together with (5), this implies that monopoly prices for each device are

$$p_i^M = \frac{1}{2} - \frac{d}{4}\left(n+1-2i\right)c^{KB}.$$

When $d > 0$, a monopolist will subsidize devices that are "higher" in the hierarchy ($i < \frac{n+1}{2}$) and exploit the devices that are "lower" in the hierarchy. For devices near the middle of the hierarchy, which generate and receive similar amounts of externalities, prices will be close to the monopoly baseline. It is also worth emphasizing that in this example, all of the price distortions reflect the trade-off between value extraction and externality internalization, as captured by $[\mathbf{G} - \mathbf{G}']$, given that every device has the same KB-centrality.

3 Welfare Benchmarks

We now compare output under ecosystem monopoly with three natural benchmarks: first-best, marginal cost pricing, and Ramsey pricing. Because each case will depend on a different type of centrality, it is useful to define the matrix

$$\widehat{\mathbf{G}}(\delta, \kappa) = \delta \left[\mathbf{G} + \kappa \mathbf{G}'\right]. \tag{11}$$

We have already seen, for example, that $\delta = \frac{1}{2}$ and $\kappa = 1$ in the case of an ecosystem monopolist. The scalar δ is the KB-centrality decay parameter, which reflects the pass through rate from increased externalities to prices and output. With linear demand and monopoly pricing, this pass through rate is $\frac{1}{2\beta_i}$. The scalar κ measures the weight placed on outbound externalities, or value creation, relative to inbound externalities.

3.1 Social Planner

For the utility functions in Appendix A that rationalize our system of linear demand functions, we show in Appendix C that social welfare is equal to

$$W = \sum_i (\alpha_i q_i - \frac{q_i^2}{2}) + \sum_i \sum_{j \neq i} \gamma_{ij} q_i q_j. \tag{12}$$

Differentiating with respect to q_i implies that at the social optimum

$$\alpha_i - q_i + \sum_{j \neq i} (\gamma_{ij} + \gamma_{ji}) q_j = 0. \tag{13}$$

Substituting (1) for q_i in this expression, and putting the result in matrix form, we have the following relationship between welfare-maximizing prices and quantities

$$\mathbf{p}^W = -\mathbf{G}' \mathbf{q}^W \tag{14}$$

Finally, substituting equilibrium demand from (2) into this expression shows that

Proposition 2 *If $\lambda_{\widehat{\mathbf{G}}(1,1)} < 1$, welfare-maximizing prices are given by*

$$\mathbf{p}^W = -\mathbf{G}'[\mathbf{I} - (\mathbf{G} + \mathbf{G}')]^{-1} \mathbf{a} \tag{15}$$

At first-best prices, the output of each device is $\mathbf{q}^W = [\mathbf{I} - (\mathbf{G} + \mathbf{G}')]^{-1} \mathbf{a}$, which is the

KB-centrality vector in the network $(\mathbf{G}+\mathbf{G}') = \widehat{\mathbf{G}}(1,1)$. Compared to the monopolist, a social planner allows more externality pass-through ($\delta = 1 > \frac{1}{2}$) and places the same relative weights on value creation and externality internalization ($\kappa = 1$). In Appendix C we show that the different centrality measures reflect the fact that a social planner cares about the social marginal surplus from expanding output, whereas a monopoly platform cares about its marginal profit.

For intuition, it is helpful to compare the welfare-maximizing prices in (15) to the monopoly prices in (5). The social planner's "baseline prices" should equal marginal cost, which we have normalized to zero. When externalities are present, the monopolist faces a tradeoff between value capture and externality internalization, as reflected in the term $(\mathbf{G}-\mathbf{G}')\,\mathbf{c}^{KB}$. The social planner, on the other hand, cares only about externality internalization; extracting surplus is a pure transfer. Thus, only the externality internalization matrix $-\mathbf{G}'$ is multiplied by the centrality vector $(\mathbf{q}^W)$ in equation (14), which implies that a social planner subsidizes all devices.

Our characterization of the monopoly and first-best prices can also be compared to the four-part decomposition of monopoly price distortions in Tan and Wright (2021). Specifically, combining equations (6) and (14) shows that

$$\mathbf{p}^M - \mathbf{p}^W = \frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{G}+\mathbf{G}')\mathbf{q}^M + \mathbf{G}'(\mathbf{q}^W - \mathbf{q}^M). \tag{16}$$

In their terminology, the first two terms are a markup distortion, and the third is a scale distortion. There is no Spence or displacement distortion in our model, because there is no consumer-level heterogeneity in $\mathbf{G}$.

3.2 Marginal Cost Pricing

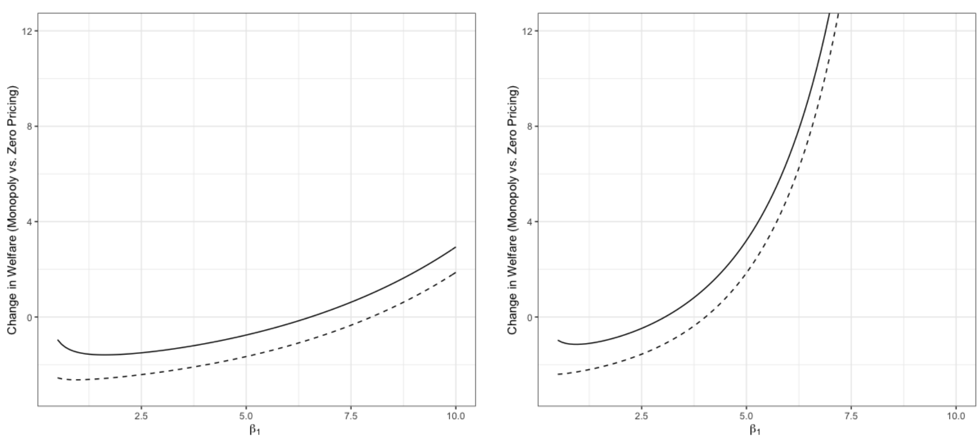
We have seen that a social planner sets all prices below marginal cost, whereas a monopolist may price some devices below marginal cost in order to capture value from others. This raises the question of whether ecosystem monopoly could be preferable to decentralized governance with all devices priced at marginal cost.

When all devices are priced at marginal cost, total output is $\mathbf{q}^{MC} = [\mathbf{I}-\mathbf{G}]^{-1}\mathbf{a}$, which is equivalent to centrality in the network $\widehat{\mathbf{G}}(1,0)$. Relative to a social planner, the decentralized ecosystem has the same externality pass-through rate $\delta = 1$, but places no weight on outbound externalities ($\kappa = 0 < 1$). Compared to a monopolist, marginal cost pricing allows for greater pass-through of externalities ($\delta = 1 > \frac{1}{2}$) but places less weight on internalization ($\kappa = 0 < 1$).

To show that welfare under monopoly can exceed welfare under marginal cost pricing, we use an example based on the star network. For this example, recall that η (μ) represents the inbound (outbound) externality to (from) the star device from (to) a peripheral and that $d = \mu - \eta$. As a first step, we can show that[9]

Lemma 1 *If $\alpha_i = 1$ for all i, and $\beta_j = 1$ for all peripherals (i.e., $j > 1$), then a star device (peripheral device) is subsidized (exploited) if and only if $\mu > \frac{\eta}{\beta_1}$.*

When $d = 0$ and $\beta_1 = 1$, the ecosystem monopolist will choose the same positive baseline price for every device, so welfare under monopoly must be lower than under zero pricing. If we increase β_1, however, Lemma 1 says that a monopolist will subsidize the star device. (A larger β_1 leads to a lower price on the star, and that in turn reduces the marginal benefit of inbound relative to outbound externalities.) To see whether there is a threshold level of β_1, beyond which an ecosystem monopolist generates more welfare than zero pricing, we fix d and use equation (12) to compute welfare at different values of β_1. These calculations are summarized in Figure 2.



(a) d=0 (b) d=0.3

Figure 2: Monopoly vs. Zero pricing

Welfare = Solid line, Consumer Surplus = Dashed Line

These calculations show that even when $d = 0$, so externalities are symmetric, monopoly pricing can dominate zero-pricing if β_1 is sufficiently large. When d increases from zero to

[9]The proof of this result appears in Appendix E.

0.3, outbound externalities become relatively larger and the threshold value of β_1 declines. We summarize these findings as

Proposition 3 *An ecosystem monopolist that internalizes downstream externalities may produce more welfare and consumer surplus than marginal cost pricing of each device.*

Proposition 3 suggests that when evaluating antitrust remedies for an ecosystem monopolist, it is important to consider incentives for internalizing externalities. This idea is familiar from the two-sided platform literature, and has already been adopted in that context by U.S. Courts.[10] Proposition 3 also has interesting implications if one interprets the ecosystem monopolist as a licensing platform. The standard argument for granting temporary monopoly power to a patent holder is based on the trade-off between ex ante innovation incentives and ex post market power. An implicit assumption behind this argument is that each patent is associated with a single product. Proposition 3 suggests that things could change dramatically when a patent (or a bundle of complementary patents) is associated with a multi-product ecosystem. In that case, it is possible that the usual dynamic trade-off no longer exists, because ecosystem monopoly generates higher welfare (and consumer surplus) than the zero-price equilibrium that occurs without any intellectual property.

3.3 Ramsey Pricing

Ramsey pricing provides an alternative second-best benchmark based on maximizing social welfare subject to a profitability constraint. The first-order condition to this constrained optimization problem combines the social planner's optimality condition (13) and the monopoly first-order condition (3). Specifically, the Ramsey first-order conditions are

$$\mathbf{a} - \mathbf{q} + (\mathbf{G} + \mathbf{G}')\,\mathbf{q} - \rho[\mathbf{a} - \mathbf{2q} + (\mathbf{G} + \mathbf{G}')\,\mathbf{q}] = \mathbf{0} \tag{17}$$

where $\rho < 0$ is the Lagrange multiplier.

In Appendix D we solve for Ramsey prices and output, and show that the latter is given by

$$\mathbf{q}^R = \left(\frac{1-\rho}{1-2\rho}\right)\left[\mathbf{I} - \left(\frac{1-\rho}{1-2\rho}\right)[\mathbf{G} + \mathbf{G}']\right]^{-1}\mathbf{a} \tag{18}$$

[10] In *Ohio vs. American Express* (138 S Ct 2274), the U.S. Supreme Court held that, "Evidence of a price increase on one side of a two-sided transaction platform cannot, by itself, demonstrate an anticompetitive exercise of market power."

Thus, centrality for Ramsey pricing is defined by the network $\widehat{\mathbf{G}}(\delta, 1)$, where $\frac{1}{2} < \delta = \frac{(1-\rho)}{1-2\rho} < 1$. Ramsey prices internalize externalities while reducing the monopoly prices to increase externality pass-through.

Table 1 summarizes how the centrality network, $\widehat{\mathbf{G}}(\delta, \kappa)$, differs across the four scenarios we have analyzed: first-best, Ramsey, monopoly and marginal cost.

	Externality Pass-Through (δ)	Externality Internalization (κ)
First Best	1	1
Ramsey	($\frac{1}{2}$,1)	1
Monopoly	$\frac{1}{2}$	1
Marginal Cost	1	0

Table 1: Centrality Network $\widehat{\mathbf{G}}(\delta, \kappa)$

4 Extensions

This section generalizes our model in two dimensions: adding complementary goods and allowing for imperfect competition among downstream device producers.

4.1 Complementary Devices

In practice, products not supplied by an ecosystem monopolist can still generate externalities that influence demand for monopolized devices. Google's Android operating system, for example, runs on a wide variety of smartphones. Smart Home devices, such as the Amazon Echo or Google Nest, connect to wireless routers produced by various manufacturers. To capture this idea, we now add third-party complements to the ecosystem and evaluate their impact on the "core" products sold by an ecosystem monopolist. We assume the third-party products are sold in competitive markets, and therefore their prices are zero.[11]

[11] In our model, it does not matter whether the monopolist supplies a particular complement. If the monopolist enters and subsidizes a device, then competitors will exit and that device becomes one of the k core ecosystem components. If the device is not subsidized, the monopolist earns zero profit because of Bertrand competition. In practice, platform entry into complementary markets is a strategic decision that has received considerable scholarly attention (e.g., Farrell and Katz, 2000; Gawer and Henderson, 2007).

Suppose the ecosystem sponsor monopolizes the first $k < n$ devices, and Bertrand competition drives prices to zero for the remaining $m = n - k$ devices. We can partition the demand system into

$$\mathbf{q} = \begin{bmatrix} \tilde{\mathbf{q}} \\ \overline{\mathbf{q}} \end{bmatrix}, \mathbf{p} = \begin{bmatrix} \tilde{\mathbf{p}} \\ \mathbf{0} \end{bmatrix}, \mathbf{a} = \begin{bmatrix} \tilde{\mathbf{a}} \\ \overline{\mathbf{a}} \end{bmatrix},$$

where $\tilde{\mathbf{q}}$ is the k-element vector corresponding to monopolized devices, and $\overline{\mathbf{q}}$ is the m-element vector for competitively supplied devices (and likewise for $\mathbf{p}$ and $\mathbf{a}$). It is also useful to decompose the externality matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{G_1} & \mathbf{G_2} \\ \mathbf{G_3} & \mathbf{G_4} \end{bmatrix}$$

so that $\mathbf{G_1}$ is a $k \times k$ matrix of externalities among monopolized devices, $\mathbf{G_2}$ is a $k \times m$ matrix of externalities from competitive to monopolized devices, and so on. We can then apply formulas for the inverse of partitioned matrices to show that

$$(\mathbf{I} - \mathbf{G})^{-\mathbf{1}} = \begin{bmatrix} \mathbf{M_1} & \mathbf{M_2} \\ \mathbf{M_3} & \mathbf{M_4} \end{bmatrix}$$

where $\mathbf{M_1} = [\mathbf{I} - \mathbf{G_1} - \mathbf{G_2}(\mathbf{I} - \mathbf{G_4})^{-\mathbf{1}}\mathbf{G_3}]^{-1}$ and $\mathbf{M_2} = \mathbf{M_1}\mathbf{G_2}(\mathbf{I} - \mathbf{G_4})^{-1}$.[12]

Equilibrium demand is given by equation (2), and the monopolist maximizes $\Pi = \tilde{\mathbf{p}}'\tilde{\mathbf{q}}$. The solution to its system of k first-order conditions can be written as

$$\tilde{\mathbf{p}}^{\mathbf{M}} = [\mathbf{M_1} + \mathbf{M_1'}]^{-\mathbf{1}}\mathbf{M_1}(\mathbf{a} + \mathbf{G_2}(\mathbf{I} - \mathbf{G_4})^{-\mathbf{1}}\overline{\mathbf{a}}) \tag{19}$$

To simplify, we define two new terms. First, $\tilde{\mathbf{G}} = \mathbf{G_1} + \mathbf{C}$, where $\mathbf{C} = \mathbf{G_2}(\mathbf{I} - \mathbf{G_4})^{-\mathbf{1}}\mathbf{G_3}$ represents externalities among the k core devices that are mediated by complementary products. Second, $\widehat{\mathbf{a}} = (\mathbf{a} + \mathbf{G_2}(\mathbf{I} - \mathbf{G_4})^{-\mathbf{1}}\overline{\mathbf{a}})$ are a set of adjusted demand intercepts for core products that reflect the impact of complements.

Substituting $\mathbf{M_1}$ for $(\mathbf{I} - \mathbf{G})^{-1}$ in (3), and using the definitions of $\tilde{\mathbf{G}}$ and $\widehat{\mathbf{a}}$, Proposition 1 implies that

$$\tilde{\mathbf{p}}^{\mathbf{M}} = \frac{1}{2}\widehat{\mathbf{a}} + \frac{1}{4}(\tilde{\mathbf{G}}' - \tilde{\mathbf{G}}) \left[\mathbf{I} - \frac{\tilde{\mathbf{G}}' + \tilde{\mathbf{G}}}{\mathbf{2}}\right]^{-1} \widehat{\mathbf{a}} \tag{20}$$

[12] For details, see Minka (2000, p.12).

and Corollary 2 says that core devices' equilibrium output will be half their KB-centrality. Thus, we have

Proposition 4 *In an ecosystem with competitively supplied complements, the optimal monopoly prices for core devices are given by* (20)*, and the relevant measure of centrality is* $\tilde{c}^{KB} \equiv \left[\mathbf{I} - \frac{\tilde{\mathbf{G}}' + \tilde{\mathbf{G}}}{\mathbf{2}}\right]^{-1} \widehat{\mathbf{a}}$.

With complementary devices, the prices charged by an ecosystem monopolist take the same general form as (5). However, centrality is defined on an adjusted network $\tilde{\mathbf{G}}$ that combines direct externalities among the core devices ($\mathbf{G_1}$) with externalities that are mediated by complements ($\mathbf{C}$). Moreover, the weights in the new KB-Centrality measure combine standalone demand for monopolized devices ($\mathbf{a}$) with the externality from complements ($\mathbf{G_2}(\mathbf{I} - \mathbf{G_4})^{-\mathbf{1}}\overline{\mathbf{a}}$).

4.2 Downstream Market Power

Returning to the monopoly case, suppose that for each device there are $l_i \geq 1$ symmetric downstream producers that compete à la Cournot. Each downstream firm sells a single device.[13] To distinguish the upstream input prices from the downstream device prices, let r_i be the price (royalty) charged by the platform to device i. We continue to use p_i to denote downstream prices.

Given r_i and the output of all other devices, $\mathbf{q}_{-i}$, each producer of device i selects its output. For instance, firm $i1$ chooses a quantity q_{i1} to maximize $(p_i - r_i)q_{i1}$, where

$$p_i = \frac{\alpha_i + \sum_{j \neq i} \gamma_{ij} q_j - (q_{i1} + \sum_{k \neq 1} q_{ik})}{\beta_i}.$$

From the first-order condition, and using symmetry, we find that each firm's equilibrium output $q_{i1} = ... = q_{il_i} = \tilde{q}_i$ is given by

$$\alpha_i + \sum_{j \neq i} \gamma_{ij} q_j - l_i \tilde{q}_i - \beta_i r_i - \tilde{q}_i = 0.$$

[13] If downstream firms produce multiple devices, they can also engage in platform pricing to internalize externalities among devices. This is an interesting topic for future research.

This implies that

$$q_i = l_i \tilde{q}_i = L_i \left[\alpha_i - \beta_i r_i + \sum_{j \neq i} \gamma_{ij} q_j \right] \tag{21}$$

where $L_i \equiv \frac{l_i}{l_i+1}$. Note that as l_i goes to infinity for all i, the demand system (21) converges to (1), the input demand under perfect downstream competition. We can therefore state

Proposition 5 *If each device i is produced by $l_i \geq 1$ symmetric downstream firms that compete à la Cournot, then the unique vector of optimal prices for an ecosystem monopolist are given by (10) after replacing $\left(\alpha_i, \beta_i, \gamma_{ij}\right)$ with $\left(L_i \alpha_i, L_i \beta_i, L_i \gamma_{ij}\right)$.*

5 Conclusions

We develop a tractable model of a device ecosystem and use it to study a number of questions. Our first set of results generalizes Armstrong's two-sided platform to show how a monopolist prices its ecosystem of inter-related products. We show that prices and output are a function of Katz-Bonacich centrality, and use examples to show how ecosystem pricing responds to the structure of demand externalities. Next, we show how the relevant network (and hence, centrality measure) changes if prices are set by a social planner, at marginal cost, or by a regulator that sets Ramsey prices. When externalities are present, a monopolist that internalizes those network effects may outperform marginal cost (zero) prices: a result that has interesting implications for patent licensing of platform technologies.

Our theoretical framework might be extended in several directions. A key simplifying assumption throughout the analysis is linearity of both demand and network externalities. We show how the former assumption can be relaxed, but have not considered a more general (nonlinear) specification of the network effects. One promising extension is to analyze ecosystem competition. In particular, future research might characterize the link between pricing and network centrality measures when two or more ecosystems compete in the market for one or more devices.

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Appendices

A Micro-foundations for demand

Consider a unit-mass of heterogeneous consumers indexed by $\theta \in [0,1]$. Denote p_i the price of device i and N_i the mass of consumers buying device i. We assume that the utility of consumer $\theta \in [0,1]$ is given by

$$u^\theta = \sum_i u_i^\theta$$

where

$$u_i^\theta = a_i^\theta - p_i + \sum_{j \neq i} \gamma_{ij} N_j$$

is the utility obtained by the consumer from using device i. The parameter $\gamma_{ij} \geq 0$ captures the network externality exerted by the users of device j on the users of device i.

For the sake of simplicity, we assume that $a_1^\theta, a_2^\theta, ..., a_n^\theta$ are not correlated for any $\theta \in [0,1]$ and that a_i^θ is uniformly distributed over an interval $[\underline{a}_i, \overline{a}_i]$ where $\underline{a}_i < \overline{a}_i$. The assumption that $a_1^\theta, a_2^\theta, ..., a_n^\theta$ are not correlated for any $\theta \in [0,1]$ implies that there are no complementarities between the devices at the individual level. In other words, network externalities are the only source of complementarities.

For given expectations N_j, $j \neq i$, the demand for device i is

$$\begin{aligned} q_i &= \Pr[u_i^\theta \geq 0] \\ &= \Pr[a_i^\theta \geq p_i - \sum_{j \neq i} \gamma_{ij} N_j] \\ &= \frac{\overline{a}_i - p_i + \sum_{j \neq i} \gamma_{ij} N_j}{\overline{a}_i - \underline{a}_i} \end{aligned}$$

over the range of prices for which this expression is between 0 and 1.

It is sufficient to define $\alpha_i \equiv \frac{\overline{a}}{\overline{a}_i - \underline{a}_i}$ and $\beta_i \equiv \frac{1}{\overline{a}_i - \underline{a}_i}$, so that we obtain

$$q_i = \alpha_i - \beta_i p_i + \sum_{j \neq i} \gamma_{ij} N_j$$

which in a fulfilled expectation equilibrium, where $q_j = N_j$, is identical to the the demand system in equation (1).

Importantly, the above microfoundation can be extended to the case in which each consumer may only be interested in a subset of devices. This follows easily from our assumption that $a_1^\theta, a_2^\theta, ..., a_n^\theta$ are not correlated for any $\theta \in [0,1]$.

B Optimal Monopoly Pricing

Let $\mathbf{V} = \mathbf{I} - \mathbf{G}$. The monopolist's profit is given by

$$\mathbf{\Pi} = \mathbf{p}'\mathbf{V}^{-1}(\mathbf{a} - \mathbf{B}\mathbf{p})$$

and the first-order condition associated with the maximization of $\mathbf{\Pi}$ with respect to $\mathbf{p}$ is

$$\mathbf{V}^{-1}(\mathbf{a} - \mathbf{B}\mathbf{p}) - \mathbf{B}'\left(\mathbf{V}^{-1}\right)'\mathbf{p} = \mathbf{0}.$$

which (after some matrix manipulation) leads to

$$\begin{aligned} \mathbf{p}^* &= \left[\mathbf{V}^{-1}\mathbf{B} + \mathbf{B}'(\mathbf{V}^{-1})'\right]^{-1}\mathbf{V}^{-1}\mathbf{a} \\ &= \left[\mathbf{B} + \mathbf{V}\mathbf{B}'(\mathbf{V}^{-1})'\right]^{-1}\mathbf{a} \end{aligned}$$

Pre-multiplying each side of this expression by $\mathbf{B}$ and rearranging yields

$$\begin{aligned} \mathbf{B}\mathbf{p}^* &= \left[\left(\mathbf{B} + \mathbf{V}\mathbf{B}'(\mathbf{V}^{-1})'\right)\mathbf{B}^{-1}\right]^{-1}\mathbf{a} \\ &= \left[\mathbf{I} + \mathbf{V}\mathbf{B}'\left(\mathbf{B}\mathbf{V}'\right)^{-1}\right]^{-1}\mathbf{a} \\ &= \left[2\mathbf{I} + \left(\mathbf{V}\mathbf{B}' - \mathbf{B}\mathbf{V}'\right)\left(\mathbf{B}\mathbf{V}'\right)^{-1}\right]^{-1}\mathbf{a} \end{aligned}$$

Applying the formula $(\mathbf{X} + \mathbf{Y})^{-1} = \mathbf{X}^{-1} - \mathbf{X}^{-1}\left(\mathbf{X}^{-1} + \mathbf{Y}^{-1}\right)^{-1}\mathbf{X}^{-1}$, this becomes

$$\begin{aligned} \mathbf{B}\mathbf{p}^* &= \left[\frac{1}{2}\mathbf{I} - \frac{1}{2}\left(\mathbf{I} + 2\mathbf{B}\mathbf{V}'\left[\mathbf{V}\mathbf{B}' - \mathbf{B}\mathbf{V}'\right]^{-1}\right)^{-1}\right]\mathbf{a} \\ &= \frac{1}{2}\mathbf{a} - \frac{1}{2}\left(\left[\mathbf{V}\mathbf{B}' - \mathbf{B}\mathbf{V}' + 2\mathbf{B}\mathbf{V}'\right]\left[\mathbf{V}\mathbf{B}' - \mathbf{B}\mathbf{V}'\right]^{-1}\right)^{-1}\mathbf{a} \\ &= \frac{1}{2}\mathbf{a} - \frac{1}{4}\left[\mathbf{V}\mathbf{B}' - \mathbf{B}\mathbf{V}'\right]\left(\frac{\mathbf{V}\mathbf{B}' + \mathbf{V}\mathbf{B}'}{2}\right)^{-1}\mathbf{a} \end{aligned}$$

Finally, by substituting $\mathbf{V} = \mathbf{I} - \mathbf{G}$, we can solve for the vector of prices

$$\begin{aligned} \mathbf{B}\mathbf{p}^* &= \frac{1}{2}\mathbf{a} + \frac{1}{4}\left(\mathbf{G}\mathbf{B} - \mathbf{B}'\mathbf{G}'\right)\left(\mathbf{B} - \frac{\mathbf{G}\mathbf{B}' + \mathbf{B}\mathbf{G}'}{2}\right)^{-1}\mathbf{a} \\ \mathbf{p}^* &= \frac{1}{2}\mathbf{B^{-1}a} + \frac{1}{2}\left(\mathbf{B^{-1}GB} - \mathbf{G}'\right)\left[\mathbf{I} - \frac{\mathbf{B^{-1}GB} + \mathbf{G}'}{2}\right]^{-1}\mathbf{B^{-1}a} \end{aligned} \tag{B.1}$$

When $\mathbf{B} = \mathbf{I}$, equation (B.1) simplifies to the monopoly pricing formula in Proposition 1. For a monopoly with demands having different elasticity (i.e., $\mathbf{B} \neq \mathbf{I}$), equation (B.1) is equivalent to (10), given the definition of $\mathbf{c}^{KB(\mathbf{B})}$.

C Social welfare

Let q_i denote the demand for device i and denote $\tilde{p}_i = p_i - \sum_{j\neq i}\gamma_{ij}q_j = \sum_{k=1}^m p_i^k - \sum_{j\neq i}\gamma_{ij}q_j$ the "externality-adjusted" price of device i. Recall that $q_i = \alpha_i - \tilde{p}_i$.

Aggregate consumer surplus is given by

$$\begin{aligned} CS &= \sum_i \int_{\tilde{p}_i}^{\alpha_i} (\alpha_i^\theta - \tilde{p}_i) d\alpha_i^\theta \\ &= \sum_i \left(\int_{\tilde{p}_i}^{\alpha_i} \alpha_i^\theta d\alpha_i^\theta - q_i\tilde{p}_i \right). \end{aligned}$$

Since

$$\int_{\tilde{p}_i}^{\alpha_i} \alpha_i^\theta d\alpha_i^\theta = \frac{1}{2}(\alpha_i^2 - \tilde{p}_i^2) = \frac{1}{2}(\alpha_i - \tilde{p}_i)(\alpha_i + \tilde{p}_i) = \frac{q_i}{2}(2\alpha_i - q_i)$$

we get

$$CS = \sum_i (\alpha_i q_i - \frac{q_i^2}{2} - q_i\tilde{p}_i) = \sum_i (\alpha_i q_i - \frac{q_i^2}{2} - q_i p_i) + \sum_i \sum_{j\neq i} \gamma_{ij} q_i q_j.$$

Therefore, social welfare is given by

$$W = \sum_i (\alpha_i N_i - \frac{q_i^2}{2}) + \sum_i \sum_{j\neq i} \gamma_{ij} q_i q_j$$

which is equivalent to equation (12) in the paper.

The welfare maximizing prices, as shown in the paper, are

$$\mathbf{p^W} = -\,\mathbf{G}'[\mathbf{I} -\, (\mathbf{G} + \mathbf{G})']^{-1}\mathbf{a}$$

The matrix to compute the centrality measure used by the social planner is different from the one used by a monopoly platform. While the social planner cares about the social marginal surplus, a monopoly platform cares about its marginal profit. The social marginal surplus can be expressed by rewriting (13) in a matrix form as

$$\underbrace{\mathbf{G} - [\mathbf{I} - (\mathbf{G} + \mathbf{G}')]\mathbf{Q}}_{\text{social marginal surplus}} = \mathbf{0},$$

while the marginal profit is obtained from the first-order condition of the monopolist's profit, $\mathbf{\Pi}^M = [\mathbf{G} - (\mathbf{I} - \mathbf{G})\mathbf{Q}]'\mathbf{Q}$, with respect to $\mathbf{Q}$:

$$\underbrace{\mathbf{G} - 2\left[\mathbf{I} - \frac{(\mathbf{G} + \mathbf{G}')}{2}\right]\mathbf{Q}}_{\text{marginal profit}} = 0$$

Comparing the social marginal surplus and the marginal profit shows why the matrix to compute the centrality measure used by the social planner is different from the one of the monopolist.

D Prices and Output Under Ramsey Pricing

The Ramsey pricing first-order condition is

$$\mathbf{a} - \mathbf{q} + (\mathbf{G} + \mathbf{G}')\,\mathbf{q} - \rho[\mathbf{a} - \mathbf{2q} + (\mathbf{G} + \mathbf{G}')\,\mathbf{q}] = \mathbf{0} \tag{D.1}$$

where $\rho < 0$ is the Lagrange multiplier. Rearranging terms yields

$$[(1-2\rho)\mathbf{I} - (1-\rho)[\mathbf{G} + \mathbf{G}']]\,\mathbf{q} = (1-\rho)\mathbf{a}$$

Pre-multiplying yields the following expression for output at Ramsey prices:

$$\mathbf{q} \equiv \mathbf{q}^R = \left(\frac{1-\rho}{1-2\rho}\right)\left[\mathbf{I} - \left(\frac{1-\rho}{1-2\rho}\right)[\mathbf{G} + \mathbf{G}']\right]^{-1}\mathbf{a}$$

which is equation (17) in the paper. And finally, substituting into (2), we have

$$\begin{aligned}\mathbf{p}^R &= \mathbf{a} - (\mathbf{I} - \mathbf{G})\mathbf{q}^R \\ &= \mathbf{a} - (\mathbf{I} - \mathbf{G})[(1-2\rho)\mathbf{I} - (1-\rho)(\mathbf{G} + \mathbf{G}')]^{-1}\mathbf{a}(1-\rho)\end{aligned}$$

E Proof of Lemma 1

Recall that η (μ) represents the inbound (outbound) externality to the star device. Let $\alpha_i = \beta_i = 1$ for all peripheral devices ($i > 1$), while maintaining the general notation (α_1, β_1) for the star device. Denote $\lambda = \mu - \frac{\eta}{\beta_1}$ and $\sigma = \mu + \frac{\eta}{\beta_1}$. Let k be a generic indicator for a peripheral device. Calculations show that the monopoly prices are given by

$$p_1 = \frac{1}{2}\frac{\alpha_1}{\beta_1} - \frac{1}{4}n\lambda c_k \tag{E.1}$$

$$p_k = \frac{1}{2} + \frac{1}{4}\beta_1\lambda c_1. \tag{E.2}$$

where c_1 (c_k) is the centrality of the star (peripheral) device, which is strictly positive. It follows immediately that the star is subsidized if and only if $\lambda > 0 \Leftrightarrow \mu > \frac{\eta}{\beta_1}$.