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ROLE OF ENTANGLEMENT IN QUANTUM HOLOGRAPHY

BAHAA E. A. SALEH, AYMAN F. ABOURADDY,
ALEXANDER V. SERGIENKO, AND MALVIN C. TEICH

*Quantum Imaging Laboratory, Departments of Electrical & Computer Engineering,
Boston University, 8 Saint Mary's Street, Boston, MA 02215, USA*
E-mail: besaleh@bu.edu

Two optical beams in a two-photon state may be used for distributed imaging. Coherent imaging and holography truly require spatial entanglement in the source and cannot be achieved using a classical source with correlations but without entanglement.

1 Introduction

Light in a single-photon pure quantum state may exhibit interference, e.g., in a Young's double-slit configuration, and may in principle be used for coherent imaging and holography. In a multipath holographic configuration, the hologram is the spatial distribution of the probability density function $p(\mathbf{x})$ for detecting the photon at the position $\mathbf{x}$ in the detection plane. Such function is of course measured by use of an ensemble of single-photon experiments. No advantage is gained, however, by use of such quantum state, as opposed to conventional light in the coherent state.

Similarly, light in a two-beam two-photon state exhibits fourth-order (or coincidence) interference that may also be utilized for incoherent imaging[3,4] and coherent imaging and holography.[2,5,6] The hologram is a spatial distribution obtained from the joint probability density function $p(\mathbf{x}_1, \mathbf{x}_2)$ for detecting the photons at positions $\mathbf{x}_1$ and $\mathbf{x}_2$. This paper addresses the role of two-particle spatial entanglement in such imaging systems and whether the same information extracted from an entangled-based system can be retrieved

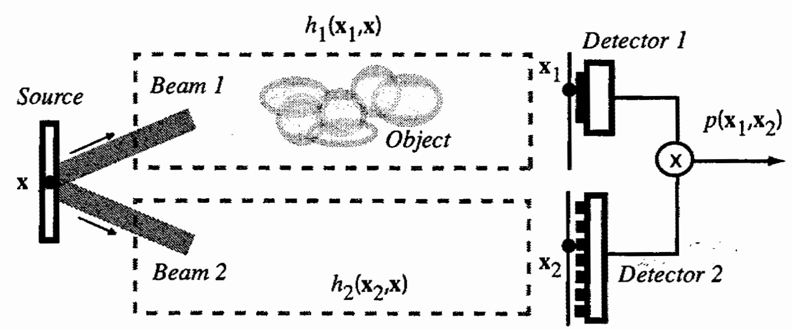

Figure 1. Two-beam distributed imaging system

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by classical means.

2 Pure-State Two-Photon Imaging

Consider a two-beam light source in a two-photon arbitrary quantum state. As illustrated in Fig. 1, the object is placed in the path of one of the beams, say beam 1, and two photodetectors are used to record the arrival of the two photons in the two beams. Detector 1 is a single point detector of finite area A placed in beam 1 after the object, and detector 2 is a bank of point detectors (or a single point detector whose position is scanned) used to detect the photon in beam 2 with high spatial resolution. The hologram is a record of the spatial distribution

$$I(\mathbf{x}_2) = \int_A p(\mathbf{x}_1, \mathbf{x}_2) d\mathbf{x}_1, \qquad (1)$$

which is the probability density for detecting a photon in beam 1 anywhere within the detector area A, and a photon in beam 2 at position $\mathbf{x}_2$. In this configuration, the beam that interacts with the object is observed with no spatial resolution (since the detector is a single-pixel camera), whereas the other beam is measured with high spatial resolution using the many-pixel detector.

Is coherent imaging possible under these conditions? The answer is obviously dependent on the quantum state of the two-photon light source. However, in the limit in which the state is separable, the readings of the two detectors are independent and therefore the multi-pixel detector in beam 2 provides no information about the object in beam 1. Since beam 1 is measured with no spatial resolution, no imaging is possible.

For an arbitrary pure two-photon quantum state

$$|\Psi\rangle = \iint d\mathbf{x} d\mathbf{x}' \psi(\mathbf{x}, \mathbf{x}') |1_{\mathbf{x}}\rangle |1_{\mathbf{x}'}\rangle, \qquad (2)$$

the photon coincidence rate is[7]

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto \left| \iint d\mathbf{x} d\mathbf{x}' h_2(\mathbf{x}_2, \mathbf{x}') \psi(\mathbf{x}, \mathbf{x}') h_1(\mathbf{x}_1, \mathbf{x}) \right|^2, \qquad (3)$$

where $h_1(\mathbf{x}_1, \mathbf{x})$ and $h_2(\mathbf{x}_2, \mathbf{x})$ are the impulse response functions of the linear systems that represent propagation of light from the source plane to the detector planes in the paths of beams 1 and 2, respectively. Since the object is placed in beam 1, the function h_1 depends on the object. It is proportional to the optical field generated at point $\mathbf{x}_1$ in the detector plane when the object is illuminated by a point source at position $\mathbf{x}$ in the source plane. The function h_2 is similarly defined and is independent of the object.

Based on Eq. (3), the measurement $p(\mathbf{x}_1, \mathbf{x}_2)$ may be regarded as the coherent image created when a wave emitted by a point source at $\mathbf{x}_1$ travels through a cascade of three linear systems of impulse response functions $h_1(\mathbf{x}, \mathbf{x}_1)$, $\psi(\mathbf{x}', \mathbf{x})$, and $h_2(\mathbf{x}_2, \mathbf{x}')$, respectively. The state function $\psi(\mathbf{x}', \mathbf{x})$ therefore serves as the impulse response function of a coherent system that couples the two beams into one single *coherent* optical system. This description is consistent with Klyshko's advanced wave interpretation.[1] In the maximally-entangled case, $\psi(\mathbf{x}', \mathbf{x}) = \psi_o(\mathbf{x})\delta(\mathbf{x} - \mathbf{x}')$, so that the intermediate system is equivalent to modulation by a complex pupil function $\psi_o(\mathbf{x})$. On the other hand, if the state function is separable, $p(\mathbf{x}_1, \mathbf{x}_2)$ is also separable so that the two beams are decoupled, and the measurement of the photon in beam 2 provides no information about the object in beam 1.

A coherent imaging system may be used for holography if the wave scattered from the object is mixed with a reference wave reaching the detector via a separate path. In the two-beam two-photon setup depicted in Fig. 1, one cannot regard beam 2 as the reference wave. After all, beams 1 and 2 together constitute cascaded serial, and not parallel, parts of a single optical system. To record a hologram of the object, beam 1 must be split into two parallel paths, one scattered from the object and another serving as a reference. For a weakly scattering object, the unscattered wave can serve as this path, as in Gabor holography. Mathematically, this is represented by writing the impulse response function h_1 as a sum $h_1 = h_{1o} + h_{1r}$ of systems representing these two paths. By substitution into Eq. (3) and expanding the square of the sum we obtain the usual four components of conventional holography.[6]

When detector 1 is of finite area A, the apparatus measures the image $I(\mathbf{x}_2)$ given by Eq.(1). In view of Eq.(3), the overall imaging process becomes equivalent to a partially-coherent system since the contributions of points within the area A to the image $I(\mathbf{x}_2)$ are added incoherently. In the limit in which A covers the entire plane, $I(\mathbf{x}_2)$ equals the marginal probability density function for measuring a photon in beam 2 at position $\mathbf{x}_2$ given that a photon has been measured anywhere in beam 1. In this case, if the object is a pure phase object, i.e., does not absorb photons, the imaging system is not capable of extracting any information about the object. Note, however, that for an absorptive object, the two-beam two-photon imaging system is capable of recovering information about the object, even if the single-pixel detector 1 extends over the entire plane. For example, if detector 1 takes the form of an integrating sphere with an opening through which beam 1 enters and scatters from the object therein, the high-resolution measurement on the external beam 2 does yield information about the object, even though the position of the photon scattered from the object is not recorded. This is not applicable to pure phase objects.

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3 Mixed-State Maximally-Correlated Two-Photon Imaging

Can similar results be obtained by employing a two-photon source that exhibits classical statistical correlations but not entanglement? That is, can distributed quantum-imaging be achieved without entanglement? To answer this question we take a mixed state that exhibits the strongest possible classical correlations, i.e., one for which the density operator is

$$\hat{\rho} = \int d\mathbf{x}\, \gamma(\mathbf{x}) |1_{\mathbf{x}}, 1_{\mathbf{x}}\rangle\langle 1_{\mathbf{x}}, 1_{\mathbf{x}}|. \tag{4}$$

This state represents a superposition of photon-pair emission probabilities from various locations within the source. In this case,[5]

$$p(\mathbf{x}_1, \mathbf{x}_2) = \int d\mathbf{x} |h_2(\mathbf{x}_2, \mathbf{x}')|^2 \gamma(\mathbf{x}) |h_1(\mathbf{x}_1, \mathbf{x})|^2. \tag{5}$$

This function is generated when the object is illuminated by a point source at $\mathbf{x}_1$ (through system h_1), and the resultant coherent image is detected in the source plane, modulated by the function $\gamma(\mathbf{x})$, and imaged incoherently through the optical system h_2. The overall system is incoherent.

4 Can Distributed Imaging be Implemented Classically?

Can the holographic information extracted by a two-beam two-photon source in an entangled state be extracted by use of a classical source? The answer depends on the conditions placed on the measurement apparatus. In the absence of any restrictions, the function $p(\mathbf{x}_1, \mathbf{x}_2)$ in Eq.(3), may be measured by placing a classical point source at $\mathbf{x}_1$ and detecting the light with the multipixel detector in beam 2 after it has traveled backward through the object. This is consistent with the advanced-wave interpretation.[1] Other classical implementations are also possible.[8]

Consider, however, a restricted measurement apparatus. Specifically, assume that the object and the single-pixel photodetector are placed within a closed chamber that has an opening (aperture) through which beam 1 enters and scatters from the object before reaching the detector, as illustrated in Fig. 2. The first restriction is that beam 1 may travel through the aperture into the chamber but it cannot travel out of the aperture in the opposite direction. This condition implies a specific space-time configuration for the measurement apparatus. The second restriction is that the spatial distribution of the optical wave crossing the aperture is prespecified and fixed within the aperture and cannot be changed during the course of the experiment. If either condition were not met, one could mimic the information extracted by an entangled-photon source by use of a classical source.

The function $p(\mathbf{x}_1, \mathbf{x}_2)$ in Eq.(3) may be measured classically in two ways, each of which violates one of the above constraints. (i) One could replace the detector with a source and measure the back-propagated wave using detector

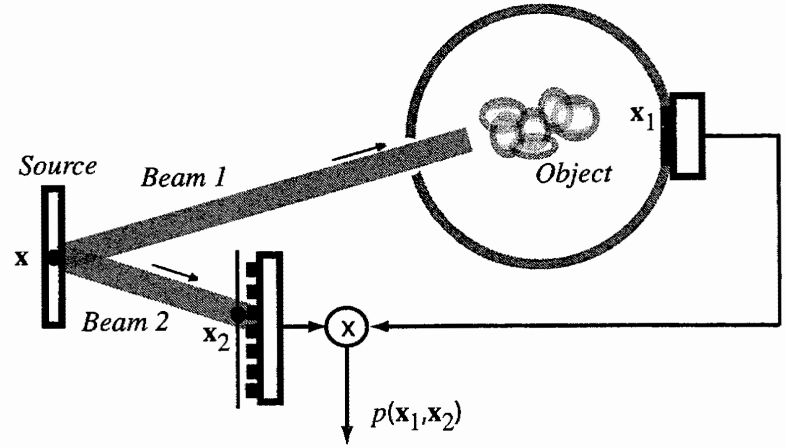

Figure 2. Configuration for coherent imaging with two beams traveling in prescribed directions

2; this of course violates the first constraint. (ii) Alternatively, one may dispense with the system h_2 altogether and employ a bank of classical fields using point sources at $\mathbf{x}_2$ scanned in such a way that full information is recovered from the readings of the single-pixel detector at $\mathbf{x}_1$ inside the chamber. Here, the second constraint is violated since the aperture field is modified.

5 Conclusion

A source of light in the form of two beams in a two-photon quantum state may be used for distributed coherent imaging and holography. One beam probes the object and is measured with a single-pixel detector while the other is measured with high spatial resolution. The photon coincidence rate, measured as a function of position in the second beam, forms a *coherent* image of the object only when the two beams are entangled. Two classically-correlated beams cannot be used for coherent imaging. If the single-pixel detector has an extended area, then the imaging system becomes partially coherent, even in the maximally-entangled case. When the probe distribution is prespecified and the light must travel in a prescribed direction, this distributed imaging system cannot be mimicked by a classical imaging system.

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