*Work supported in part by Atomic Energy Commission Contract No. AEC AT(11-1) 34 P107A.

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[2]S. L. Glashow and R. H. Socolow, Phys. Rev. Letters 15, 329 (1965). We would like to observe that the mass formulas (1) do not unambiguously require the nine observed 2^+ mesons to belong to a nonet. Equally well suited is a 36-dimensional reducible representation of SU(3) corresponding to a tensor $T_{cd}{}^{ab}$ which is symmetric in ab and cd but not traceless in any pair of indices. Such a 36-dimensional tensor would reduce into the irreducible SU(3) representations $\underline{27}+\underline{8}+\underline{1}$. Assumptions analogous to those used in deriving (1) from the nonet assignment lead to the same equal-spacing rule in the 36-plet. Of course, in this case 27 more mesons are expected. Thus 2^+ mesons with $I=2$, $Y=0$, $M\simeq 1300$ MeV; $I=1$, $Y=0$, $M\simeq 1520$ MeV; $I=0$, $Y=0$, $M\simeq 1800$ MeV; $I=\frac{1}{2}$, $Y=\pm1$, $M\simeq 1650$ MeV; $I=\frac{3}{2}$, $Y=\pm1$, $M\simeq 1450$ MeV; $I=1$, $Y=\pm2$, $M\simeq 1520$ MeV would be expected to accompany the already discovered nine mesons. Selection rules of the type $f'\not\to 2\pi$ can be formulated also for these mesons, e.g., the $I=\frac{1}{2}$, $Y=\pm1$, meson at 1650 MeV should predominantly decay into $K\eta$ (the mode K being forbidden to the same extent as $f'\not\to 2\pi$). The mass values given above should be handled with caution since the mass formulas used in deriving them are no better than (1). Such a 36-plet of 2^+ mesons is characteristic of the 405-dimensional representation of SU(6). Of course in the context of SU(6) the 2^+ mesons are to be accompanied by 0^+ and 1^+ mesons and in relativistic SU(6) theory a $\underline{405}$ is accompanied by a $\underline{35}^+$ and $\underline{1}^+$

$$[\underline{5940}_{M(12)}\rightarrow(\underline{21},\underline{21}^*)[U(6)\otimes U(6)]_M\rightarrow(\underline{405}\oplus\underline{35}\oplus\underline{1})]\;SU(6).$$

In terms of Regge analysis the $\underline{5940}$ representation is strongly favored over $\underline{4212}$ $(\underline{189}\oplus\underline{35}\oplus\underline{1})$ which does not couple to baryons. S. L. Glashow (private communication) has independently made similar considerations.

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MULTIPLE-PHOTON PROCESSES AND HIGHER ORDER CORRELATION FUNCTIONS*

M. C. Teich† and G. J. Wolga

Cornell University, Ithaca, New York
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In a previous letter, we have reported the observation of double-quantum photoelectric emission from sodium metal.[1,2] The two-quantum photoeffect is of particular interest because the nonlinear process occurs in the detector itself. This communication discusses the dependence of the two-quantum photoeffect, and of certain other nonlinear processes, upon the higher order correlation functions[3] of the radiation field. As a special case of the results described, it is shown that the absolute magnitude of the effect induced in a two-quantum detector[4] by a thermal source is expected to be twice as large as that induced by a single-mode ideal laser source[5] of the same intensity.[6] Physically, the effect occurs because of correlations in the photon arrival times at the absorbing atom, and is closely related to the Hanbury Brown–Twiss effect. In the following treatment, it is assumed that all sources possess precise first-order coherence.

The development is presented in terms of quantum mechanical coherence theory since, as pointed out by Titulaer and Glauber,[7] the higher order coherence properties of the field furnish a natural basis for describing the results of measurements of nonlinear functions of the intensity. Because the two-photon photoemission process represents the annihilation of two photons, the average counting rate at the space-time point $x_0=\vec{r}_0, t_0$ for the ideal[3] two-photon detector may be written as the second-order correlation function[3]:

$$G^{(2)}(x_0x_0x_0x_0) = \mathrm{tr}\{\rho E^-(x_0)E^-(x_0)E^+(x_0)E^+(x_0)\}. \quad (1)$$

Here, ρ is the density operator for the field, and E^- and E^+ are the negative- and positive-frequency portions of the electric field operator $\vec{E}$, respectively.

Measurements from two types of double-quantum experiments can yield information about the radiation field: (a) absolute measurements in which this information is provided by an accurate determination of the two-quantum photocurrent, and (b) relative measurements, in which two superimposed beams illuminate the double-quantum detector. In the latter case, the higher order correlation function is reflected in the relative magnitude of the two-quantum dc photomixing term resulting from the presence of both beams. Both of these methods will be considered in the following discussion.

Correlated photon annihilations must be discussed relative to the radiation source. The output of the double-quantum detector, when illuminated by a single beam, is proportional to $G^{(2)}(x_0x_0x_0x_0)$, where $x_0=\vec{r}_0, t_0$. When illuminated by a time-delayed form of the <u>same beam</u>, the output of the detector at time t_0 may be written as $G^{(2)}(x_1x_1x_1x_1)$, where $x_1=\vec{r}_1, t_1$ and $t_1>t_0$. That is, time delay of one beam relative to the other is equivalent to time displacement at the detector. Therefore, the x's above refer to space-time points relative to the radiation source.

The total two-quantum counting rate (or average two-quantum photocurrent $i_T^{(2)}$), for the case where two superimposed beams arising from the same source illuminate the detector, is therefore taken to be a sum of second-order correlation functions corresponding to the following contributions: (a) the absorption of two photons from the same space-time point (both from the same beam)—there are two of these terms (one for each beam); and (b) the absorption of two photons from different space-time points (each from a different beam)—there are also two of these terms (one for each permutation of photon absorption). The average counting rate for the two-photon detector illuminated by two beams is then seen to be proportional to the following sum:

$$i_T^{(2)} \propto G^{(2)}(x_0x_0x_0x_0)+G^{(2)}(x_1x_1x_1x_1) + G^{(2)}(x_0x_1x_1x_0)+G^{(2)}(x_1x_0x_0x_1). \quad (2)$$

Titulaer and Glauber[7] have shown that the second-order correlation function (for a source having first-order coherence) obeys the following relation:

$$G^{(2)}(x_ix_jx_jx_i) = g_2[G^{(1)}(x_ix_i)][G^{(1)}(x_jx_j)]. \quad (3)$$

Because the correlation function $G^{(1)}(x_jx_j)$ is proportional to the average single-quantum counting rate (and therefore to the intensity of the radiation) at the space-time point x_j, the double-quantum current becomes

$$i_T^{(2)} = g_2c(I_0^2+2I_0I_1+I_1^2), \quad \tau_\delta<\tau_c, \quad (4)$$

provided that the delay between the beams (τ_δ) is short compared to the coherence time of the radiation (τ_c). Here I_j represents the intensity of the jth beam, and c is a constant. Thus, the two-quantum current is proportional to g_2, a coherence parameter. In fact, the current is just g_2 times as large as that obtained when the excitation source is an ideal single-mode laser (where $g_2=1$), reflecting the effect of the coherence properties of the radiation on the absolute magnitude of the two-quantum photocurrent. This increase in current arises from the correlation in the photon arrival times from the thermal source radiation: The probability for the simultaneous arrival of two photons is greater than if no correlation were present (as in the case of the ideal single-mode laser source). This increase occurs whether one or two beams are present.

When the time delay between the beams is greater than the coherence time of the radiation, however, there is no correlation in the arrival times of a photon from one beam and a photon from the other beam. The counting rate for such a process, which is given by terms of the form $G^{(2)}(x_0x_1x_1x_0)$, then factors into the product

$$G^{(2)}(x_0x_1x_1x_0) = [G^{(1)}(x_0x_0)][G^{(1)}(x_1x_1)] \quad (5)$$

indicating that no excess coincidences occur. At a given x, correlations between photons from the same beam still remain, of course. Therefore, for large path-length differences between the beams (long time delays), $i_T^{(2)}$ is given by

$$i_T^{(2)} \propto g_2[G^{(1)}(x_0x_0)]^2+g_2[G^{(1)}(x_1x_1)]^2 + 2[G^{(1)}(x_0x_0)][G^{(1)}(x_1x_1)]. \quad (6)$$

Thus, the double-quantum current arising from the absorption of one photon from each beam is the same for this case as for the laser: It is equal to $c(2I_0I_1)$.

For the special case of chaotic fields (including thermal radiation),[8] $g_m = m!$, and the two-quantum photocounting rate $i_T^{(2)}$ is given by

$$i_T^{(2)} = 2c(I_0^2 + I_0I_1 + I_1^2), \quad \tau_\delta > \tau_c. \qquad (7)$$

Again, this current is higher than that for a single-mode laser, where $g_2 = 1$. Here, however, it is seen that the cross term no longer has a coefficient of two (relative to the coefficients of the terms I_0^2 and I_1^2). Physically, this corresponds to the enhancement of the single-beam counting rates $[G^{(2)}(x_0x_0x_0x_0)$ and $G^{(2)}(x_1x_1x_1x_1)]$, arising from the tendency of these photons to arrive in correlated pairs. Since the beam delay is greater than the coherence time of the radiation, however, there is no correlation between the arrival time of a photon from one beam and the arrival time of a photon from the other.

The (relative) accidental coincidence rate is obtained from the cross-term coefficient in Eq. (7) [for $\tau_\delta > \tau_c$]; it is unity. The (relative) observed coincidence rate for delay times short compared with the coherence time is obtained from the cross-term coefficient in Eq. (4) [for $\tau_\delta < \tau_c$], and is equal to 2. A measure of the difference between the observed and accidental coincidence rates is therefore $(2-1)/1 = 1 = g_2 - 1$. This factor, $g_2 - 1$, was also obtained by Titulaer and Glauber[7] for the Hanbury Brown–Twiss experiment with small detector separation compared to the coherence length, and small delay time compared to the coherence time. For the limiting case of very small resolving time of the coincidence counter, Hanbury Brown and Twiss also found this result.[9]

The double-quantum detector, illuminated by two beams which are superimposed but have a time delay between them, is therefore seen to behave like a self-integrating Hanbury Brown–Twiss apparatus. For the two-beam double-quantum experiment, the relative magnitude of the cross term reflects the correlations. The relatively large excess coincidence counting rate occurs because of the extremely short resolving time of the two-photon detector ($\sim 10^{-15}$-10^{-14} sec which is the "lifetime" of an intermediate state in the two-photon absorption). In performing such an experiment, it should be remarked that the use of unfocused radiation would facilitate superposing the beams. Time-delay type photon-correlation experiments could not be performed satisfactorily with an experimental setup in our laboratory because focused beams (which were required in order to give an observable double-quantum current[1,2]) precluded proper beam superposition. Experiments with spatially displaced beams, although possible, were not attempted.

In the same way, the correlation in photon arrival times will be reflected in any process consisting of the annihilation of two or more photons. For example, consider a Hanbury Brown–Twiss experiment using two double-quantum detectors rather than the conventional single-quantum detectors. Assuming that the electronic resolving time is less than the coherence time of the radiation, the excess coincidence counting rate for such an experiment is calculated to be $\{[g_4/(g_2)^2]-1\}$ (for a thermal source, this quantity is equal to 5). This is in analogy with the case for the ordinary Hanbury Brown–Twiss experiment where the excess coincidence counting rate is given by $g_2 - 1$ (which is equal to unity for a thermal source). In the experiment using double-quantum detectors, the excess coincidence rate reflects the fourth-order coherence properties of the field. This experiment would, however, be difficult to perform with currently available two-photon detectors.

It is therefore seen that multiple-photon processes can provide information about the higher order correlation functions of the incident radiation field, and conversely, that full information about a nonlinear process requires a knowledge of the nature of the radiation field inducing the process. In particular, for sources possessing precise first-order coherence, the single-beam m-photon detector output is enhanced over the classically (semiclassically) calculated value by the factor g_m, which in general differs from unity.

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†James Clerk Maxwell Fellow at Cornell University. Present address: Massachusetts Institute of Technology Lincoln Laboratory, Lexington, Massachusetts.

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[2]For new experimental and theoretical results concerning the two-quantum photoelectric effect in sodium, see M. C. Teich, thesis, Cornell University, 1966 (unpublished). This thesis is available as a report from the Materials Science Center at Cornell University, Ithaca, New York. A detailed publication is in preparation.

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TWO-FLUID MODEL OF THE SOLAR WIND*

P. A. Sturrock† and R. E. Hartle‡

Space Sciences Division, Ames Research Center, National Aeronautics and Space Administration, Moffett Field, California
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The original analysis by Parker[1] of the dynamical expansion of the solar corona, to produce what is now known as the solar wind, was based on fluid-dynamical equations. The importance of heat flow was recognized by Parker[2,3] and has also been discussed by Noble and Scarf[4] and by Whang and Chang,[5] but in the context of a single-fluid model.

Consideration of the collision frequency ν_E for energy exchange between protons and electrons shows that there is no simple mechanism for maintaining thermal equilibrium between these species, so that one should expect the proton and electron temperatures to be quite different in the solar wind. If, in the formula[6]

$$\nu_E \approx 8.5\times10^{-2} n T^{-3/2}, \tag{1}$$

we use the values of density and temperature computed by Noble and Scarf[4] for the vicinity of the earth, $n = 7\ \mathrm{cm}^{-3}$, $T = 2.8\times10^5\ {}^\circ\mathrm{K}$, we obtain $\nu_E = 4\times10^{-9}\ \mathrm{sec}^{-1}$ which is smaller, by a factor of about 10^{-3}, than the expansion rate $-(v/n)dn/dr \approx 2v/r$ corresponding to a velocity, v, of 350 km sec^{-1} at the radial distance, r, of 1 A.U. from the center of the sun. It follows that one should consider at least a two-fluid model of the solar wind, not only to study the temperatures of the individual species, but also to obtain a correct formulation of the dynamics.

We have made such an investigation of the solar wind, making the conventional assumptions of stationary spherically symmetric flow with no rotation or magnetic fields and without viscous stresses. In this case the equation of continuity leads to

$$nvr^2 = J, \tag{2}$$

where J is a constant and the ions (only protons are considered) and electrons have the same number density n. The dynamical equation is, to good approximation,

$$nm_p v\frac{dv}{dr} = -\frac{d}{dr}(nkT_p + nkT_e) - \frac{GM_\odot n m_p}{r^2}, \tag{3}$$

where m_p, m_e, T_p, T_e are the proton and electron masses and temperatures, k is Boltzmann's constant, G is the gravitational constant, and $M_\odot$ is the solar mass. On combining the energy equation,[7] the continuity equation, Eq. (2), and the equation of motion, Eq. (3), we arrive at the following "heat equation" for protons:

$$\frac{3}{2}\frac{1}{T_p}\frac{dT_p}{dr} - \frac{1}{n}\frac{dn}{dr} = \frac{1}{JkT_p}\frac{d}{dr}\left(r^2\kappa_p\frac{dT_p}{dr}\right) + \frac{3}{2}\frac{\nu_E}{v}\frac{T_e - T_p}{T_p} \tag{4}$$

and a similar equation for electrons. The thermal conductivities of protons and electrons,