FLAT COUNTING DISTRIBUTION FOR TRIANGULARLY-MODULATED POISSON PROCESS

M. C. TEICH and P. DIAMENT
Department of Electrical Engineering, Columbia University, New York, 10027, *USA*

Received 8 August 1969

Photocounting distributions for an amplitude-stabilized radiation source with triangularly-modulated intensity should exhibit the unusual property of extreme flatness over a tunable region of counts. The result applies to any Poisson process whose mean can be linearly swept.

We report on the photocounting distribution [1-4] obtained when the intensity of an amplitude-stabilized radiation source is modulated to an arbitrary depth m with a *triangular* waveform. Experimental counting distributions associated with small-depth square-wave and sinusoidal modulation of such a source have been investigated by Fray et al. [5]. More recently, Pearl and Troup [6] have investigated the sinusoidally-modulated amplitude-stabilized source for 100% modulation. We present the theory for the triangular case because of the unexpected character of the resultant counting distribution.

It is well known that an amplitude-stabilized laser with constant intensity gives rise to a Poisson photocounting distribution [1-4]. So, in fact will a source of arbitrary statistics, provided that its coherence time is considerably shorter than any other time associated with the experiment. We now sweep the mean of this source linearly between two levels, and perform photocounting measurements in a time interval T_0 much smaller than the modulation period T_1. If there are enough samples [5, 6] so that each infinitesimal time interval of the triangular waveform is sampled with equal weight, the resulting distribution will be an average of Poisson distributions. Each of these has a different mean count ranging from the lowest value $N(1-m)$ to the higest value $N(1+m)$, where N is the overall count. The counting distribution may therefore be written as

$$p(n, m, T_0) = \frac{1}{2mN} \int_{N(1-m)}^{N(1+m)} \frac{N^n \exp - N}{n!} \, \mathrm{d}N \,. \quad (1)$$

Evaluating the integral, we obtain the photocounting distribution

$$p(n, m, T_0) = \frac{\exp[-N(1-m)]}{2mN} \sum_{k=0}^{n} \frac{N(1-m)]^k}{k!} + \quad (2)$$

$$- \frac{\exp[-N(1+m)]}{2mN} \sum_{k=0}^{n} \frac{[N(1+m)]^k}{k!} .$$

For the particular case of 100% modulation (m=1), this becomes

$$p(n, 1, T_0) = \frac{1}{2N} \left[1 - \exp(-2N) \sum_{k=0}^{n} \frac{(2N)^k}{k!} \right] . \quad (3)$$

It is apparent from (3) that for $m = 1$ and large N, the distribution is flat at a value $1/2N$ for a range of n up to nearly $2N$. Fig. 1 presents the

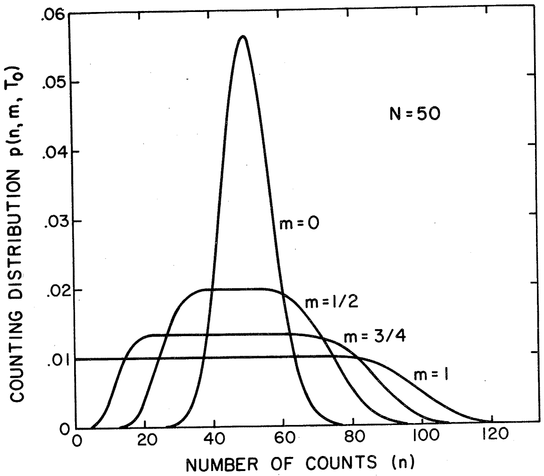

Fig. 1. Counting distribution $p(n, m, T_0)$ for triangularly-modulated Poisson process with the modulation depth m as a parameter. The mean count is $N = 50$ for all distributions shown.

photocounting distribution for a large mean count, $N = 50$, with the modulation depth m as a parameter. Quantitatively, for $m = 1$, the curve is flat to about 1 part in 10^{18} on the interval $0 \leqslant n \leqslant 25$; to 1 part in 10^8 on $0 \leqslant n \leqslant 50$; and to 1% on $0 \leqslant n \leqslant 77$. We observe that by choosing N sufficiently large, we can achieve an arbitrary degree of flatness over some "bandpass" in counts. The fractional deviation from flatness at any n within the bandpass is readily shown to be of the order of the counting probability for the pure Poisson process of mean $2N$, i.e. $[(2N)^n \exp(-2N)/n!]$. As m is decreased, it is seen that the distribution narrows and increases in height, achieving a trapezoid-like shape for intermediate values of $m(0.25 \leqslant m \leqslant 0.90)$, and finally reverting to the usual Poisson distribution for $m = 0$. Note that the central portion of the curve is still quite flat for intermediate values of m. For example, with $m = 0.75$, the curve is flat to 1 part in 10^3 on the interval $25 \leqslant n \leqslant 59$. The trailing edge of the curve is somewhat broader that the leading edge. The curve is, of course, always broader that the constituent pure Poisson process.

The distribution function possesses what may be termed a flat lowpass (for $m = 1$) or bandpass (for $0 < m < 1$) behavior in the count-number domain. The width of the bandpass may be tuned by varying m. It is expected that this property may find use in low-photon-number communications systems. It should be pointed out that the unusual characteristics of the distributions discussed in this paper arise from the particular combination of the Poisson process with triangular modulation. A more complete study of square-wave, sinusoidal, and triangular modulation of Gaussian, amplitude-stabilized, and Risken sources [4], to be published shortly, indicates that the triangularly-modulated Poisson case is unique in this respect. We note that the effect of finite sample time T_0 (while maintaining $T_0/T_1 \ll 1$) is to reduce the modulation depth by an amount of order T_0/T_1, thus not materially altering the flatness over some bandpass of counts.

Finally, we note that the results predicted here depend only on the existence of a Poisson process whose mean is triangularly modulated, and therefore by no means restricted to photocounting. Since many discrete random processes are Poisson distributed (e.g. radioactive decay, shot noise) the results given here are of general validity and are expected to find wide application. In particular, we note that the flat nature of the distribution may be useful for improving the signal-to-noise ratio in an optical communication system and also in simulating, in a very simple fashion, arbitrary random processes for direct input to an analog computer. It may also serve to provide a test for the Poisson nature of a random process.

It is a preasure to thank Professor O. Wing for informative discussions.

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