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Performance of a lightwave system incorporating a cascade of erbium-doped fiber amplifiers

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The photon-number distribution (PND) at the output of a *cascade* of erbium-doped fiber amplifiers (EDFAs), with coherent light at the input, turns out to be the noncentral-negative-binomial (NNB) distribution, even in the presence of intervening loss, provided that the normalized bandwidth is the same for all amplifiers. The probability of error (PE) for a cascade of several high-gain EDFAs is essentially the same as that for a single EDFA. The performance of a sequence of postamplifiers is superior to that of a sequence of preamplifiers.

1. Introduction

We previously showed that the evolution of the photon statistic in an EDFA can be characterized by the probability generating function (PGF) of a birth-death-immigration (BDI) process, and that the photon-number distribution at the output of an EDFA with *coherent light at the input* is the NNB distribution [1]. Indeed, these results are generally applicable to all traveling-wave amplifiers (TWAs). In this paper we show that the same results obtain for a *cascade* of EDFAs (TWAs), even in the presence of loss, provided that all the amplifiers have the same normalized bandwidth (characterized by the mode parameter M). Furthermore, the probability of error (PE) of an on/off keying (OOK) direct-detection lightwave communication system incorporating a cascade of high-gain (~ 100) EDFAs is essentially the same as that obtained using a single EDFA. The performance of a sequence of postamplifiers turns out to be superior to that of a sequence of preamplifiers. This is also true for an OOK system using a sequence of EDFAs as repeaters with unity gain.

It has previously been shown that the photon-number PGF at the output of an optical amplifier with a *single photon at its input* is [2]

$$\begin{aligned} G_{\mathrm{BDI}}(s) &= G_{\mathrm{BD}}(s)\, G_{\mathrm{I}}(s) \\ &= \frac{1+(g-\langle n_{\mathrm{th}}\rangle)(s-1)}{1-\langle n_{\mathrm{th}}\rangle(s-1)} \\ &\quad \times [1-\langle n_{\mathrm{th}}\rangle(s-1)]^{-M}, \end{aligned} \tag{1}$$

where g and $\langle n_{\mathrm{th}}\rangle$ are the gain and amplification-noise parameter of the amplifier, respectively. The relationship between the output and input photon-number PGFs of an EDFA is [1]

$$G_{\mathrm{out}}(s) = [G_{\mathrm{in}}(G_{\mathrm{BD}}(s))]\, G_{\mathrm{I}}(s)\,, \tag{2}$$

where

$$G_{\mathrm{BD}}(s) = \frac{1+(g-\langle n_{\mathrm{th}}\rangle)(s-1)}{1-\langle n_{\mathrm{th}}\rangle(s-1)}\,, \tag{3}$$

and

$$G_{\mathrm{I}}(s) = [1-\langle n_{\mathrm{th}}\rangle(s-1)]^{-M}\,. \tag{4}$$

With a *Poisson number of photons at the input* characterizing coherent light, we have

$$G_{\mathrm{in}}(s) = \exp[\langle N\rangle(s-1)]\,, \tag{5}$$

where $\langle N\rangle$ is the mean input photon number, so that $G_{\mathrm{out}}^{\mathrm{Poisson}}(s)$ is [1]

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$$G_{\text{out}}^{\text{Poisson}}(s)=\exp\left[\frac{g\langle N\rangle(s-1)}{1-\langle n_{\text{th}}\rangle(s-1)}\right]$$
$$\times[1-\langle n_{\text{th}}\rangle(s-1)]^{-M}, \quad (6)$$

corresponding to the NNB distribution [1]

$$P(n)=\frac{1}{n!}\left[\frac{\partial^n G_{\text{out}}(s)}{\partial s^n}\right]_{s=0}$$
$$=\frac{\langle n_{\text{th}}\rangle^n}{(1+\langle n_{\text{th}}\rangle)^{n+M}}\exp\left(-\frac{g\langle N\rangle}{1+\langle n_{\text{th}}\rangle}\right)$$
$$\times L_n^{M-1}\left(-\frac{g\langle N\rangle}{\langle n_{\text{th}}\rangle(1+\langle n_{\text{th}}\rangle)}\right), \quad (7)$$

where L_n^{M-1} is the generalized Laguerre polynomial

$$L_n^{M-1}(-x)=\sum_{k=0}^{n} x^k\frac{(n+M-1)!}{(k+M-1)!(n-k)!k!}. \quad (8)$$

2. Preamplifiers and postamplifiers

We now consider a lightwave system incorporating a single EDFA as a *preamplifier* or *postamplifier*, with loss introduced by a transmission fiber *before* or *after* the EDFA, as shown in figs. 1a and 1b respectively. Preamplification refers to amplification before the detector whereas postamplification signifies amplification following the source. These definitions are standard in lightwave communication systems.

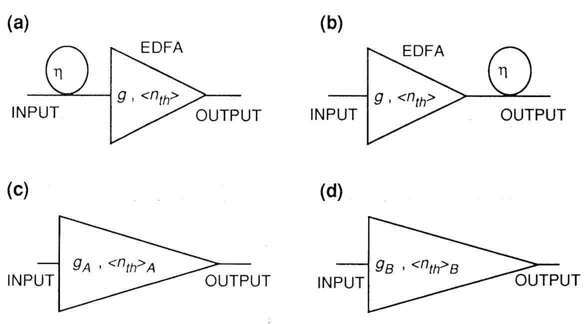

Fig. 1. (a) Preamplifier configuration (attenuation precedes amplification). (b) Postamplifier configuration (attenuation follows amplification). (c) Equivalent amplifier of the configuration shown in (a). (d) Equivalent amplifier of the configuration shown in (b).

The modification of the photon statistics arising from loss is introduced by Bernoulli random deletion [3]. This is characterized by the PGF

$$G_{\text{BERN}}(s)=1+\eta(s-1), \quad (9)$$

where η ($0\leqslant\eta\leqslant1$) is the transmittance. For a single photon at the input to the *preamplifier* configuration shown in fig. 1a, then, we use eq. (2) together with

$$G_{\text{in}}(s)=G_{\text{BERN}}(s) \quad (10)$$

to obtain the PGF at the output

$$G_{\text{outA}}(s)=[G_{\text{BERN}}(G_{\text{BD}}(s))]\,G_{\text{I}}(s)$$
$$=\frac{1+(g_{\text{A}}-\langle n_{\text{th}}\rangle_{\text{A}})(s-1)}{1-\langle n_{\text{th}}\rangle_{\text{A}}(s-1)} \quad (11)$$
$$\times[1-\langle n_{\text{th}}\rangle_{\text{A}}(s-1)]^{-M}, \quad (11)$$

where

$$g_{\text{A}}=\eta g, \quad (12)$$
$$\langle n_{\text{th}}\rangle_{\text{A}}=\langle n_{\text{th}}\rangle. \quad (13)$$

For the *postamplifier* case, with a single photon at the input to the configuration shown in fig. 1b, we have instead

$$G_{\text{outB}}(s)=G_{\text{BDI}}(G_{\text{BERN}}(s))$$
$$=\frac{1+(g_{\text{B}}-\langle n_{\text{th}}\rangle_{\text{B}})(s-1)}{1-\langle n_{\text{th}}\rangle_{\text{B}}(s-1)}$$
$$\times[1-\langle n_{\text{th}}\rangle_{\text{B}}(s-1)]^{-M}, \quad (14)$$

where now

$$g_{\text{B}}=\eta g, \quad (15)$$
$$\langle n_{\text{th}}\rangle_{\text{B}}=\eta\langle n_{\text{th}}\rangle. \quad (16)$$

Comparing eqs. (11) and (14) with eq. (1) demonstrates that loss does not alter the BDI nature of the amplification process. The preamplifier and postamplifier configurations with loss can therefore be replaced by equivalent amplifiers, as shown in figs. 1c and 1d respectively. It is of interest to note that, from a noise point-of-view, the postamplifier is superior to the preamplifier since

$$g_{\text{B}}=g_{\text{A}} \quad (17)$$

but

$$\langle n_{\text{th}}\rangle_{\text{B}}\leqslant\langle n_{\text{th}}\rangle_{\text{A}}. \quad (18)$$

3. Cascade of amplifiers

We now proceed to examine the statistics for a k-stage cascade of EDFAs, as shown in fig. 2a, in which the gain and the amplification-noise parameter of the jth state are g_j and $\langle n_{\mathrm{th}} \rangle_j$, respectively, and all stages have the same value of M. With a single photon at the input to EDFA_1, according to eqs. (2) and (1), the PGF at the output of EDFA_2 is, since

$$G_{\mathrm{in}2}(s) = G_{\mathrm{out}1}(s) = G_{\mathrm{BDI}1}(s) ,$$

$$G_{\mathrm{out}2}(s) = [G_{\mathrm{BDI}1}(G_{\mathrm{BD}2}(s))]\, G_{\mathrm{I}2}(s) = \frac{1+(G_2-\langle N_{\mathrm{th}}\rangle_2)(s-1)}{1-\langle N_{\mathrm{th}}\rangle_2(s-1)} \times [1-\langle N_{\mathrm{th}}\rangle_2(s-1)]^{-M} , \tag{19}$$

with

$$G_2 = g_1 g_2 , \tag{20}$$

$$\langle N_{\mathrm{th}}\rangle_2 = g_2 \langle n_{\mathrm{th}}\rangle_1 + \langle n_{\mathrm{th}}\rangle_2 . \tag{21}$$

Equation (19) has the same form as eq. (1) so that the output of amplifier 2 remains a BDI process, with gain and amplification-noise parameter given by eqs. (2) and (21).

By induction, it is readily shown that the PGF at the output of a k-stage cascade of EDFAs is, with a single photon at the input,

$$G_{\mathrm{out}k}(s) = \frac{1+(G_k-\langle N_{\mathrm{th}}\rangle_k)(s-1)}{1-\langle N_{\mathrm{th}}\rangle_k(s-1)} \times [1-\langle N_{\mathrm{th}}\rangle_k(s-1)]^{-M} ,$$

with

$$G_k = \prod_{j=1}^{k} g_j , \tag{23}$$

$$\langle N_{\mathrm{th}}\rangle_k = \sum_{j=1}^{k} \left(\langle n_{\mathrm{th}}\rangle_j \prod_{l=j+1}^{k} g_l \right) , \tag{24}$$

which maintains the form of a BDI process regardless of the number of stages. In the special case where

$$g_j = g, \quad \langle n_{\mathrm{th}}\rangle_j = \langle n_{\mathrm{th}}\rangle, \quad j=1,2,...,k ; \tag{25}$$

we obtain

$$G_k = g^k , \tag{26}$$

$$\langle N_{\mathrm{th}}\rangle_k = \langle n_{\mathrm{th}}\rangle \frac{g^k-1}{g-1} . \tag{27}$$

We conclude that a cascade of k EDFAs, all with the same value of M, has behavior equivalent to that of a single EDFA, as illustrated in fig. 2b, with overall gain and amplification-noise parameter given in eqs. (23) and (24), or eqs. (26) and (27) for identical amplifiers.

Based on these results we construct a statistical model for a long-haul lightwave communication system using a sequence of k EDFAs as repeaters, either in preamplifier or postamplifier configurations. Preamplifier results are obtained by constructing a cascade in fig. 2a from EDFA elements of the form in fig. 1c; postamplifier results are obtained using elements of the form in fig. 1d. Using eqs. (12), (13), (23), and (24), the overall gain and amplification-noise parameters for the BDI process representing the preamplifier cascade are

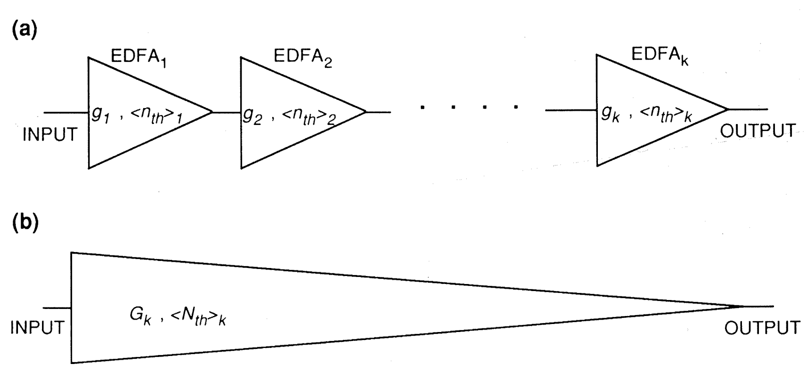

Fig. 2. (a) k-stage cascade of EDFAs. (b) Equivalent amplifier of the cascade shown in (a).

$$G_{kA}=\prod_{j=1}^{k}\eta_j g_j\,, \tag{28}$$

$$\langle N_{\rm th}\rangle_{kA}=\sum_{j=1}^{k}\left(\langle n_{\rm th}\rangle_j\prod_{l=j+1}^{k}\eta_l g_l\right). \tag{29}$$

In the special case of identical preamplifier segments, we obtain

$$G_{kA}=(\eta g)^k\,, \tag{30}$$

$$\langle N_{\rm th}\rangle_{kA}=\langle n_{\rm th}\rangle\,\frac{(\eta g)^k-1}{\eta g-1}\,, \tag{31}$$

in agreement with the expressions obtained by Yamamoto [4]. For the postamplifier cascade, eqs. (15), (16), (23), and (24) yield

$$G_{kB}=\prod_{j=1}^{k}\eta_j g_j\,, \tag{32}$$

$$\langle N_{\rm th}\rangle_{kB}=\sum_{j=1}^{k}\left(\eta_j\langle n_{\rm th}\rangle_j\prod_{l=j+1}^{k}\eta_l g_l\right), \tag{33}$$

which again has lower noise than the preamplifier cascade in eq. (29).

4. Error performance

With the help of eqs. (6), (7), (28), (29), (32), and (33) it is clear that, with coherent light at the input (with mean photon number $\langle N\rangle$), the PND at the output of a sequence of EDFAs, in either preamplifier or postamplifier configurations, is the NNB distribution.

It is of interest to compare the performance of a single EDFA with that of a cascade of k EDFAs. Figure 3 shows the PE for an OOK direct-detection system incorporating a single EDFA (solid curve) [1], along with that for a cascade of 10 EDFAs (dashed curve). The PEs are virtually identical; it is clear that cascading multiple high-gain (>100) EDFAs does not degrade the PE of the system. This behavior is not unlike that obtained using a photomultiplier tube, in which the number of stages does not appreciably affect the PE [5].

A sequence of k identical EDFAs may also be used as repeaters, in which case the gain g in each amplifier exactly compensates the transmission loss of

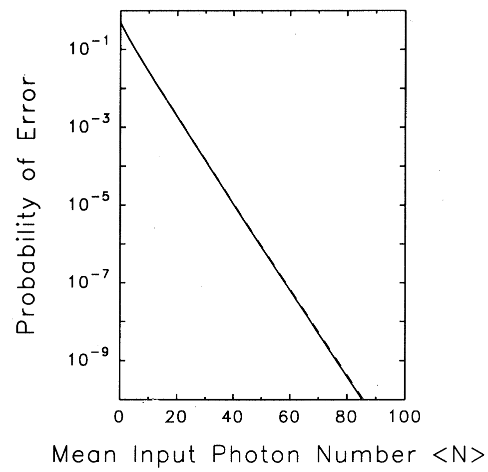

Fig. 3. Probability of error (PE) versus mean input photon number $\langle N\rangle$ for an OOK direct-detection system incorporating a single EDFA (solid curve) and a cascade of 10 identical EDFAs (dashed curve). For each EDFA, the gain $g=100$ (20 dB), the mode parameter $M=1$, and uniform pumping is assumed.

the adjacent fiber section. In that case $\eta g=1$ and the signal power at the receiver input is identical to the signal power at the transmitter output. For a given transmission distance, the question is how far apart the amplifiers should be placed, or how high the gain of each amplifier should be made to achieve optimal PE performance. We address this question for a long-haul optical-fiber communication system. We choose a transmission distance $L=1000$ km and a fiber-attenuation coefficient $\alpha=0.3$ dB/km for the purposes of our illustration, but the results obtained are generally valid. In this example, the total loss is [6] $\eta_k=\eta^k=10^{-\alpha L/10}=10^{-30}$, so that the total gain required to compensate this loss is $G_k=g^k=1/\eta_k=10^{30}$. We compare two different system configurations: (i) a high-gain configuration in which $k=10$ and $g=1000$, representing a relatively small number of high-gain amplifiers placed at large distances; and (ii) a low-gain configuration in which $k=30$ and $g=10$, representing a large number of smaller-gain amplifiers with closer spacings.

For the preamplifier sequence, eqs. (30) and (31) become

$$G_{kA}=1\,, \tag{34}$$

$$\langle N_{\rm th}\rangle_{kA}=k\langle n_{\rm th}\rangle\,, \tag{35}$$

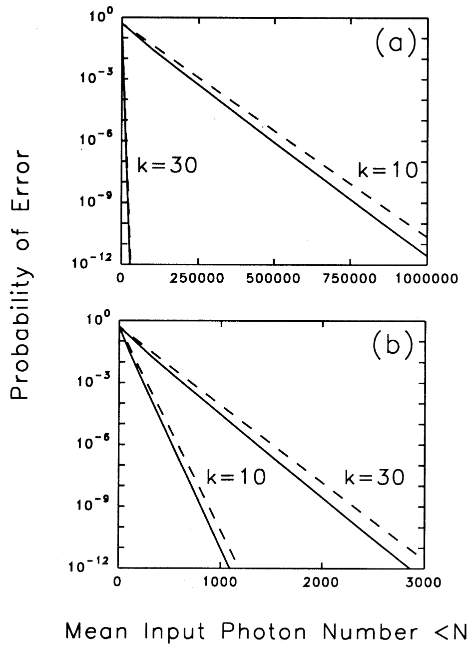

Fig. 4. Probability of error (PE) versus mean input photon number $\langle N\rangle$ for an OOK direct-detection optical fiber communication system using a sequence of identical EDFAs as repeaters. Results are calculated using both the exact NNB distributions (solid curves) and the gaussian approximation (dashed curves). The PE for both the high-gain system configuration ($k=10$ and $g=1000$) and the low-gain system configuration ($k=30$ and $g=10$) are calculated. For the purposes of this example, $\langle n_{\mathrm{th}}\rangle = g-1$, $M=1$, and $\eta g=1$. (a) PE for a sequence of preamplifiers. (b) PE for a sequence of postamplifiers.

whereas for the postamplifier sequence, eqs. (32) and (33) become

$$G_{k\mathrm{B}} = 1\,, \tag{36}$$

$$\langle N_{\mathrm{th}}\rangle_{k\mathrm{B}} = \eta k \langle n_{\mathrm{th}}\rangle\,. \tag{37}$$

Using the NNB distribution in eq. (7), together with eqs. (34), (35), (36) and (37), we calculate the PE for both system configurations and illustrate the results in fig. 4. In the case of the preamplifier sequence, fig. 4a shows that the high-gain system configuration ($k=10$) has *inferior* PE performance to that of the low-gain configuration ($k=30$). For the postamplifier sequence, in contrast, fig. 4b shows that the high-gain system configuration has *better* PE performance than the low-gain configuration. Noting the scales of the abscissas in figs. 4a and b, it is evident that for the same system configuration the postamplifier sequence is vastly superior to the preamplifier configuration. This is because both the signal and noise are attenuated in the postamplifier configuration whereas only the signal is attenuated in the preamplifier configuration. Finally, fig. 4 shows that, as in the case of the single EDFA, the PE obtained using the gaussian approximation is not too different from that obtained using the exact NNB distributions.

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