ANTIBUNCHING IN THE FRANCK–HERTZ EXPERIMENT

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We propose a method by which the sub-poissonian character of a space-charge-limited electron beam is transferred to photons. It is shown that a modified version of the Franck–Hertz experiment should generate a strong source of antibunched, sub-poissonian light at 253.7 nm. The relation between photon antibunching and sub-poissonian photon statistics is elucidated, and conditions are obtained when each implies the other.

1. Introduction

It is well known that under most circumstances, the statistical behavior of photon registrations [1] can be described in terms of the doubly stochastic Poisson point process (DSPP) [2]. For such light, the ratio of photon-count variance Var(n), to photon-count mean $\langle n\rangle$, designated as the Fano factor $F_n(T)$, is $\geqslant 1$ for all choices of the counting time interval $[0, T]$. For pure Poisson light, $F_n(T) = 1$, independent of T. DSPP light is sometimes also called super-poissonian or bunched, since the photons tend to be emitted in bunches rather than strictly at random [1].

It has long been recognized that under special circumstances, it is possible to generate so-called antibunched light, for which successive photon emissions are less likely than for poisson light [3,4]. When the condition $0 \leqslant F_n(T) < 1$ is obeyed, the light is called sub-poissonian. (In the course of our presentation, the relationship between antibunched and sub-poissonian light will be elucidated.) The measure $F_n(T)$ is useful because it succinctly describes the noisiness of a light source (in terms of noise-to-signal ratio) relative to that of an ideal amplitude-stabilizer laser (coherent source), for which $F_n(T) = 1$ for all T. Although ideal laser light has the useful property of exhibiting coherence to all orders [5], sub-poissonian light is less noisy in a photon-counting paradigm. Such quiet light may therefore find use in applications such as optical signal processing and optical communications. It could also serve as an improved source for investigating many physical and biological processes, such as the behavior of the human visual system at threshold [6].

In recent years, a number of optical processes have been proposed for generating antibunched light, including degenerate parametric amplification [4,7] and many other effects in which light interacts with a nonlinear medium [8]. Resonance fluorescence is, however, the only phenomenon in which the antibunching effect has actually been observed [9]. Most of the proposed processes operate by reducing the Poisson fluctuations of the exciting optical beam.

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This may be accomplished by removing pairs (or clusters) of photons [8], or by making use of an atomic emitter that exhibits a natural dead time [9]. Sub-poissonian photon statistics, though predicted [10, 11] have not yet been observed.

In this letter, we propose a uniquely different way of generating antibunched and sub-poissonian light that circumvents many of the difficulties inherent in other optical techniques. We consider a Franck–Hertz experiment [12], in which space charge [13] is used to regularize the spacing of the electrons before they enter the passive atomic medium. In the Franck–Hertz experiment, each electron is given a small (but specific) kinetic energy by acceleration in an electric field, so that it can undergo an inelastic resonant collision with an atom (e.g., Hg), losing all of its energy to the atomic excitation. This then results in the spontaneous emission of a single photon, after a random time delay characterized by the lifetime of the excited state τ_c. By suitable construction of the apparatus, and choice of the atomic species, the excitation, emission, and collection processes can each be made to occur with very high probability, so the overall quantum efficiency for the effect can be large.

The technique works by transferring the anticlustering properties of the electrons, resulting from Coulomb repulsion, to the photons. The direction of the transfer is the inverse of that encountered in the usual photodetection process, where the statistical character of the photons is imparted to the photoelectrons. Since the excitation itself can be substantially sub-poissonian, we avoid the struggle against the Poisson statistics of the exciting photons faced by techniques using optical excitation [14]. Furthermore, a large number of excitations and emissions can be produced per unit time, so that the antibunched light generated in this manner can be both cw and strong.

We proceed by developing the relationship between antibunching and sub-poissonian behavior. This is followed by a description of the expected photon statistics for our quiet light source. We then propose a specific implementation for such a special Franck–Hertz photon-counting experiment.

2. Relation between antibunching and sub-poissonian behavior

The relation between the antibunching properties of an optical field, and the sub-poissonian character of its corresponding photon-counting distribution, may be established by use of the equation [1,10,14]

$$\mathrm{Var}(n) = \langle n\rangle + (\langle n\rangle^2/T^2) \times \int_0^T \int_0^T [g^{(2)}(t_2 - t_1) - 1]\, \mathrm{d}t_1\, \mathrm{d}t_2 \,, \tag{1}$$

where n is the number of photons emitted in the time interval $[0, T]$, and $g^{(2)}(\tau) = \langle \hat{I}(t)\hat{I}(t+\tau)\rangle/\langle \hat{I}\rangle^2$ is the quantum-mechanical normalized (second-order) intensity correlation function (normally ordered and time ordered) [3,5,10]. The Fano factor $F_n(T) = \mathrm{Var}(n)/\langle n\rangle$ therefore satisfies the equation

$$[F_n(T) - 1] = (\langle n\rangle/M)[g^{(2)}(0) - 1] \,, \tag{2}$$

where M is the number of degrees of freedom (number of modes) [1], and is given by

$$M^{-1} = (2/T) \int_0^T (1 - \tau/T)\, \xi(\tau)\, \mathrm{d}\tau \,,$$

$$\xi(\tau) = [g^{(2)}(\tau) - 1]/[g^{(2)}(0) - 1] \,, \tag{3}$$

for stationary light. The quantity $[F_n - 1]$ is also normalized second-order factorial moment, and is sometimes denoted by Q [10,15]. The parameter $F_n(T)$ characterizes the nature of the photon-counting distribution (super-poissonian for $F_n(T) > 1$, sub-poissonian for $F_n(T) < 1$), whereas the parameter $g^{(2)}(0)$ characterizes the state of bunching of the optical field (bunched for $g^{(2)}(0) > 1$, antibunched for $g^{(2)}(0) < 1$).

It is apparent from eq. (2) that if $M > 0$, then

$$g^{(2)}(0) \gtrless 1 \leftrightarrow F_n \gtrless 1 \,, \tag{4}$$

i.e., bunching/antibunching implies super-/sub-poissonian photon-counting behavior, and vice versa. In general, M need not be positive. Nevertheless, there are a number of important conditions where it does turn out to be >0:

(1) When the counting time T is much smaller than the width of the function $[g^{(2)}(\tau) - 1]$, eq. (3) gives $M = 1 > 0$ and eq. (4) follows.

(2) When $g^{(2)}(\tau)$ is monotonic and $g^{(2)}(\infty) = 1$, it

can be shown from eq. (3) that $M > 0$. In this case, and when T is also much larger than the width of the function $[g^{(2)}(\tau) - 1]$, we may write $M = T/\tau_c > 0$, where $\tau_c = 2\int_0^\infty \xi(\tau)\,d\tau$ represents the width of $\xi(\tau)$ or $[g^{(2)}(\tau) - 1]$. Eq. (4) then follows.

(3) When $[g^{(2)}(\tau) - 1] \geqslant 0$, $\forall\tau$, then $M > 0$, and eq. (4) follows. The light is then both bunched and super-poissonian. This is the situation encountered for classical fields. When $[g^{(2)}(\tau) - 1] \leqslant 0$, $\forall\tau$, then M is also >0, and the light is both antibunched and sub-poissonian.

It is clear that the time dependence of the optical field plays an important role in the determination of both $F_n(T)$ and $g^{(2)}(\tau)$. The quantity $g^{(2)}(0)$, on the other hand, though it is the usual basis for the definition of antibunching, reflects only (instantaneous) photon coincidences. Mandel has shown that, for steady-state resonances fluorescence radiation, antibunching and sub-poissonian behavior accompany each other for all T when there is no detuning between the exciting field and atom [10]. When there is detuning [16], and in the transient regime, this need not be so, however, as recently demonstrated by Singh [15], Hildred and Hall [17] have pointed out that a related distinction can be made for the case of two-photon absorption. Finally, we mention that in addition to antibunching and sub-poissonian behavior, nonclassical fields are also characterized by their squeezedness. Indeed, squeezedness and sub-poissonian behavior need not accompany each other, though for resonance fluorescence generated under specified conditions, Mandel has shown that they do [18].

3. Photon statistics for Franck–Hertz light

Consider a beam of electrons, with an average flux of u electrons/sec, interacting with a collection of independent atoms (in their ground states) in a Franck–Hertz tube. Let η be the probability that a given electron excites an atom and results in the emission of a photon. Let n be the (random) number of photons emitted from the tube during the time interval $[0, T]$. If m is the (random) number of electrons in the active region of the tube in the same time interval, then according to the Burgess variance theorem [19] ‡

$$\langle n\rangle = \eta\langle m\rangle\,, \tag{5}$$

$$\mathrm{Var}(n) = \eta^2\mathrm{Var}(m) + \eta(1-\eta)\langle m\rangle \tag{6}$$

or

$$[F_n - 1] = \eta[F_m - 1]\,. \tag{7}$$

Here $[F_n - 1]$ and $[F_m - 1]$ refer to the photons and electrons, respectively. We have implicitly assumed that the energy transfer and emission processes are instantaneous (i.e., that the observation time T is much larger than the scattering time and the atomic lifetime τ_c).

By use of eqs. (2) and (7) we conclude that the emitted field must have a normalized intensity correlation function (at $\tau = 0$) given by

$$[g^{(2)}(0) - 1] = (\eta M/\langle n\rangle)[F_m - 1]\,, \tag{8}$$

where $M \approx T/\tau_c$ is taken to be positive. The implications of eqs. (7) and (8) are important: from eq. (7) it is readily seen that if the electron beam is super-/sub-poissonian, the emitted photons will also be super-/sub-poissonian; from eq. (8) it follows that the photons will also be bunched/antibunched.

The relation between the Fano factors of the emitted photons and of the exciting electrons is determined by the efficiency η in accordance with eq. (7). On the other hand, the relation between the parameter $g^{(2)}(0)$ of the emitted light and the Fano factor of the electrons [eq. (8)] is governed by the parameter

$$\eta M/\langle n\rangle = 1/u\tau_c = 1/\delta'\,, \tag{9}$$

i.e., by the inverse of the average number of electrons in an atomic lifetime (δ' is therefore a kind of degeneracy parameter). We note that the Fermi–Dirac statistics of the electrons could in principle also give rise to quieting [20], but these effects are expected to be negligible in our proposed experiment (typical electron spacing $\gg$ electron de Broglie wavelength).

The limiting case for sub-poissonian electrons is the deterministic beam, in which the electrons are perfectly regularly spaced at intervals of $1/u$ s. Then

‡ In this reference, the definition of antibunching was taken to be identical to that of sub-poissonian. The definition of antibunching used in the current work $[g^{(2)}(0) < 1]$ is in accord with the more usual usage of the term.

$F_m = 0$ and

$$F_n = 1 - \eta \,, \tag{10}$$

$$g^{(2)}(0) = 1 - 1/u\tau_c = 1 - 1/\delta' \,, \tag{11}$$

representing both sub-poissonian and antibunched behavior. The photon-counting distribution $p(n)$ is also known in this case. It is a Bernoulli-deleted deterministic number of events, i.e., the binomial distribution [19,21].

4. Proposed space-charge-limited Franck–Hertz photon-counting experiment

We consider a Franck–Hertz experiment consisting of a triode or tetrode vacuum tube, containing a small amount of Hg. It is heated in an oven to permit the vapor pressure of the Hg to be temperature controlled. By means of an external voltage, the cathode is heated to an appropriate temperature to produce thermionically emitted electrons and the formation of a space-charge cloud. The grid voltages are adjusted to provide the desired space-charge-limited electron current and electron energy (4.88 eV to collisionally excite the $6\,^3P_1$ level of atomic Hg). The reduction in the Poisson fluctuations of the emitted electrons due to space charge has been studied extensively, and compact expressions exist for the noise-reduction factor (Fano factor F_m). The quieting effect (relative to the temperature-limited shot-noise case) can be substantial; values of F_m smaller than 0.1 are typical and values of F_m as low as 0.01 are possible [13].

In the most common configuration of the experiment, a small retarding potential is provided between the (final) grid and the collector (plate), to prevent those electrons that have given their energy to Hg atoms, from contributing to the collector current. On advancing the accelerating potential between the grids, therefore, the collector current will dip in the vicinity of 4.9 V (after accounting for the contact potential associated with the cathode, which is ~2 V) [12]. A deep dip in the current can be obtained by proper adjustment of the electrode spacings and oven temperature (~170°C), which control the collision probability. A decrease in the current by about a factor of 10 in the vicinity of the dip is typical; this implies that electron excitation efficiencies of roughly 90% can be obtained comfortably. The $6\,^3P_1 \to 6\,^1S_0$ (ground state) transition of Hg is radiative, with a lifetime $\tau_c \approx 1.17 \times 10^{-7}$ s, and a wavelength of 253.7 nm, in the near ultraviolet [22]. For 4.88 eV electrons traversing a 1 cm path, the transit time $\tau_{tr} \sim 7 \times 10^{-9}$ s $\ll \tau_c$. For an oven temperature of 170°C ($kT \approx 0.038$ eV), and a photon energy $h\nu \approx$ 4.88 eV, $h\nu/kT \approx 128$; thus the thermally excited spontaneous emission will be very small.

The expected values for F_n and $g^{(2)}(0)$ for our proposed experiment may now be estimated. From eq. (7) it is obvious that sub-poissonian photon behavior is enhanced by minimizing F_m and maximizing η. Thus, it is important to (1) produce as much space-charge limiting of the electron current as possible; (2) maximize the number of electrons that transfer their energy to the atoms; and (3) collect as many of the electron-excited photons as possible (which may be accomplished with carefully designed collection optics). For an experiment designed with some (but not extraordinary) care, our previous discussion provides us with the rough values $F_m \approx 0.05$ and $\eta \approx 0.5$. Inserting these estimated values into eq. (7), we obtain $F_n = 0.525$, indicating that the emitted photon flux will be quieted by about a factor of 2 relative to that of a coherent course. Using eq. (9), with a typical beam current $i_e \approx 1$ nA and $\tau_c \approx 1.17 \times 10^{-7}$ s, leads to $\delta' \approx 730$. Thus, eq. (8) provides $g^{(2)}(0) = 1 + [F_m - 1]/\delta' \approx 1 - (1.3 \times 10^{-3})$, which is just a hair less than unity. If i_e is reduced to 10^{-11} A, however, $g^{(2)}(0) \approx 0.87$. (A comparison of these numbers with those calculated on the basis of eqs. (10) and (11) explicitly shows that the noisiness in the electron beam is small.) Although the light emitted by our space-charge-limited Franck–Hertz apparatus is almost imperceptably antibunched, it is nevertheless substantially sub-poissonian.

We have recently demonstrated that Bernoulli deletion and/or the addition of Poisson noise counts preserve the super- or sub-poissonian nature of a photon-counting distribution [19]. Thus, even in the presence of spontaneous emission, the process of photodetection should permit the sub-poissonian nature of the Franck–Hertz photons to be observed. It is important to note that the intensity of our quiet light source should be adjusted by varying i_e (while maintaining the space-charge limiting). Optical filters and other losses further reduce η, and thereby the sub-poissonian behavior.

A broad variety of higher atomic excited states, as well as other atomic and molecular species, could be used in a similar configuration. Thus, there is the possibility of producing radiation with antibunched and sub-poissonian photon statistics in various regions of the electromagnetic spectrum, including the X-ray. Other charged particles (e.g., protons) could also be used as the excitation. A solid-state implementation of this effect may also be possible, since space-charge-limited electron currents and photon emission are both well-known processes in semiconductor devices.

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