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Measurement of photon statistics of wiggler radiation from an electron storage ring

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The number of visible photons emitted by an electron bunch moving through a wiggler in the Brookhaven Synchrotron Light Source storage ring was repetitively measured using an analog photon counting technique, and the photon counting distribution, which is the probability of finding n photons versus n, was obtained. The photoelectron-counting distribution of detected spontaneous light from the wiggler obeys a negative-binomial distribution consistent with a multi-electron, multimode description of the light generation process. In the absence of the wiggler, the bending-magnet light emerging from the pyrex exit port obeys the Neyman type-A distribution.

There have been a number of theoretical investigations of the photon-number statistics of the radiation emitted from an electron beam as it traverses a wiggler, viz., the spontaneous radiation from a free electron laser (FEL) [1–6]. A single electron gives rise to photons characterized by a Poisson number distribution [1], $P(n)$, which has variance given by $\mathrm{Var}(n)=\langle n\rangle$, with $\langle n\rangle$ being the photon number mean. However, the radiation is better described by a multi-electron theory; several researchers [2–6] have shown that in this case

$$\mathrm{Var}(n)=\langle n\rangle+\langle n\rangle^2(1-1/N_e), \tag{1}$$

with $\langle n\rangle=N_e\langle a\rangle$, where N_e is the fixed number of electrons in the storage ring bunch and $\langle a\rangle$ is the mean number of photons spontaneously emitted by a single electron during a pass through the wiggler. This result can be viewed [7] as arising from the superposition of a fixed number N_e of independent, statistically identical coherent emissions, each of which contains a Poisson number of photons of mean $\langle a\rangle$. When $N_e \gg 1$, eq. (1) reduces to the variance of the Bose–Einstein photon number distribution associated with single-mode thermal light [7–9], i.e.,

$$\mathrm{Var}(n)=\langle n\rangle+\langle n\rangle^2. \tag{2}$$

These results do not apply to light radiated from the electron beam, because of the finite counting time and area, quantum efficiency of the photodetector, and imperfect polarization of light measured in an experiment. A multimode description of thermal light is required [10], in which the detected photoelectrons are described by the negative binomial distribution [7,9–11], rather than the Bose–Einstein, with variance

$$\mathrm{Var}(n)=\langle m\rangle+\langle m\rangle^2/M. \tag{3}$$

Here, $\langle m\rangle=\eta\langle n\rangle$, where η is the quantum efficiency of the detector, and M is the number of modes (degrees of freedom). Indeed, this description is appropriate for describing the light from many types of lasers operated below threshold. For $\eta=M=1$, we recover the result of eq. (2).

We have carried out a series of experiments on the photon statistics of the light emitted from the vacuum ultraviolet (VUV) electron storage ring at the National Synchrotron Light Source at Brookhaven National Laboratory, and have verified eq. (3). The statistical properties of light emitted from the storage ring and from FELs are of interest inasmuch as these sources are finding an increasingly broad range of applications.

A single pulse of electrons circulates in the ring; it produces a pulse of light 480 ps in duration every 170.2 ns at the exit window. Wiggler radiation as well as synchrotron radiation from the bending magnets can be examined. The light is emitted from the same ensemble of electrons since the current decay time (~ 100 min) is $\gg$ than the time of the experiment. The operating parameters of the ring and wiggler are provided in table 1.

The experimental arrangement for measuring the statistical properties of the light, using an analog photon-counting [12] technique, is illustrated schematically

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Table 1
Wiggler (BNL designation U13-TOK) and storage-ring parameters used in our experiments. The emittance values and the source size are for wiggler light. SQ represents the skew quadrupole parameter

Wiggler period, λ_0	10 cm
Number of wiggler periods, N	22.5
Peak wiggler magnetic field, B_0	0.61 T
Wiggler strength parameter, $K = eB_0\lambda_0/2\pi\sqrt{2}\,mc^2$	4.0
Operating energy	$\approx$ 650 MeV (wiggler light), $\approx$ 745 MeV (bending magnet light)
Operating current	$\approx$ 50 mA (wiggler light), $\approx$ 150 mA (bending magnet light)
Horizontal damped emittance, ϵ_x	8.0×10^{-8} m rad (SQ = 300), 7.6×10^{-8} m rad (SQ = 0) (wiggler light), 1.5×10^{-7} m rad (bending magnet light)
Vertical damped emittance, ϵ_y	2.0×10^{-8} m rad (SQ = 300), 2.6×10^{-8} m rad (SQ = 0) (wiggler light), $> 2.8\times10^{-10}$ m rad (bending magnet light)
Source size σ_h, σ_v	1.0 mm, 0.32 mm (SQ = 300); 0.96 mm, 0.36 mm (SQ = 0) (wiggler light); 0.5 mm, > 0.06 mm (bending magnet light)

in fig. 1. The radiation has a center wavelength of 532 nm (the fundamental) and a FWHM of 20 nm. The light emerges form a pyrex exit port and is directed upon an optical interference filter at 532 nm with FWHM of 3.2 nm. The operating energy of the ring is adjusted so the wiggler radiates at the maximum transmission wavelength of the filter. The light may be focussed by a 50 mm focal length glass lens onto a Si pin photodiode detector with quantum efficiency of 0.78.

The photodiode output is amplified by two low-noise video amplifiers, bandwidth 500 MHz and overall gain of 100; this broadens the light pulses to 5 ns from 480 ps. The pulses are then fed into a gated integrator and boxcar averager; this device high-pass filters the pulses to eliminate noise below 10 kHz and also provides an electronic gate with a 15 ns width. A set of divide-by-ten counters causes an output to appear for every thousandth pulse. Therefore, the time between each selected pulse of light is 170 μs, sufficiently slow for the electronics to register them. The selected pulses are then integrated and amplified to produce a sequence of voltage pulses that are proportional to the integrated current, i.e., the charge. A 10 μs width sample of each of these voltages is obtained using a digital delay/pulse generator synchronized to the pulses, in conjunction with an integrated circuit switch. After suitable normalization, this sequence of voltage pulses represents the number of photons per pulse (see fig. 1); they are fed into a multichannel analyzer which sorts them into a histogram that represents the photoelectron counting distribution. The typical counting time is ~ 5 s, during which ~ 30000 samples are collected.

The noise in the system was determined by feeding a sequence of identical deterministic electrical pulses in place of the photodiode output into the amplifier, and then measuring the variance at the output of the multichannel analyzer as a function of the pulse level. The resultant noise count variance was found to be approximately constant at $\sim 3\times10^8$ for count means

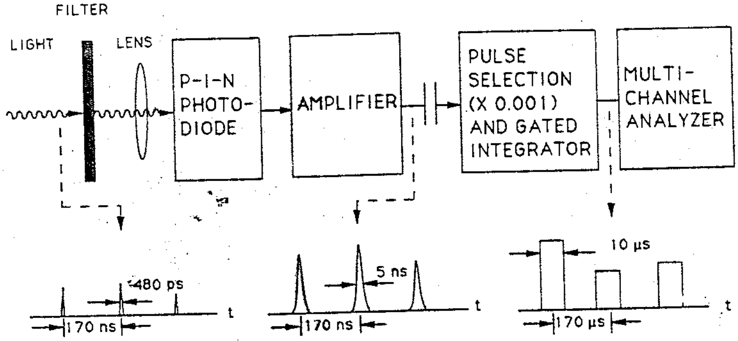

Fig. 1. Block diagram of the experiment used for the analog photon counting measurement.

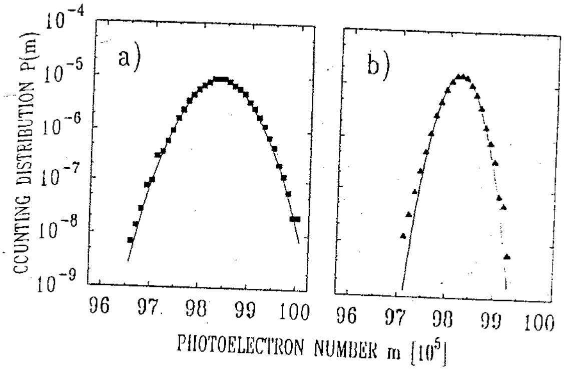

Fig. 2. (a) Squares represent the experimental photoelectron counting distribution $P(m)$ versus photoelectron number m for wiggler light from a tightly focussed electron beam. (b) Triangles represent data for light in absence of wiggler. Solid line (a) is a negative binomial distribution convolved with Gaussian noise and solid line (b) is fit by a Neyman type-A distribution convolved with Gaussian noise.

above 5×10^6; however, it decreased with decreasing count mean below this value. Measurements of noise arising from the individual components of the photocounting system gave a variance that was consistent with this value; the largest source of noise was found to be in the first amplifier.

The experimental photoelectron counting distribution $P(m)$ is shown in fig. 2a for wiggler light. The mean photoelectron number was adjusted to 9.8×10^6 by means of an adjustable neutral density filter placed in front of the lens. The resulting distribution for the wiggler light is well fitted by the negative binomial theoretical distribution, after convolution with a zero-mean Gaussian distribution (with variance $= 3 \times 10^8$) which represents the system noise. The number of modes experimentally determined is $M = 54000$. In another set of experiments, the wiggler was "removed" (by increasing the gap between its magnets) and the photoelectron counting distribution of the bending magnet light was observed. The apparatus of fig. 1 was modified by removal of the interference filter, and the operating energy of the ring was set at its normal value (see table 1) since the radiation is now broadband; the ring current was increased and the electron beam emittance and source size are somewhat different than in the wiggler case. The experimental photoelectron counting distribution is shown for the bending magnet light in fig. 2b, adjusting the mean number again at 9.8×10^6. It is clear that this distribution is substantially narrower than that associated with the wiggler light.

The bending magnet photoelectron counting distribution is well fit by a Neyman type-A (NTA) theoretical distribution [13–17] convolved with the zero-mean Gaussian noise distribution used in conjunction with the theoretical curve in fig. 2a. The NTA distribution, has variance [7,11,18]

$$\mathrm{Var}(m) = (1 + \eta\langle\alpha\rangle)\langle m\rangle. \tag{4}$$

As an example, this distribution will describe the statistics of photoluminescence light with an arbitrary spectrum [17]. Again, $\langle m\rangle = \eta\langle n\rangle$, and $\langle\alpha\rangle$ is the mean number of secondary photons per primary photon. As is evident in eq. (4), finite efficiency reduces both the mean number of secondary photons per primary photon $\langle\alpha\rangle$ as well as the overall mean $\langle n\rangle$. This is in contrast to the negative binomial distribution in which only $\langle n\rangle$ is altered. The experimentally determined value for $\eta\langle\alpha\rangle$, from fig. 2b is 40. We note that the synchrotron light contains energetic photons (into the X-ray region) as well as visible photons, and that the high energy photons, when passing into an optical medium such as the exit window, could give rise to many visible wavelength photons, as in a scintillation crystal. If the photon statistics of a pulse of synchrotron light is Poisson, and the number of visible photons created per energetic synchrotron photon is also Poisson – as is plausible – the resulting photon statistics will be described by the NTA distribution. The light emitted by a mechanism such as this would be diffuse, in accord with our observations.

We have experimentally measured $\mathrm{Var}(m)$ versus $\langle m\rangle$ for both wiggler light and bending-magnet light (fig. 3). Wiggler light for a tightly focussed electron beam (squares, SQ = 300) and a loosely focussed beam (dots, SQ = 0) is shown for comparison. The mean

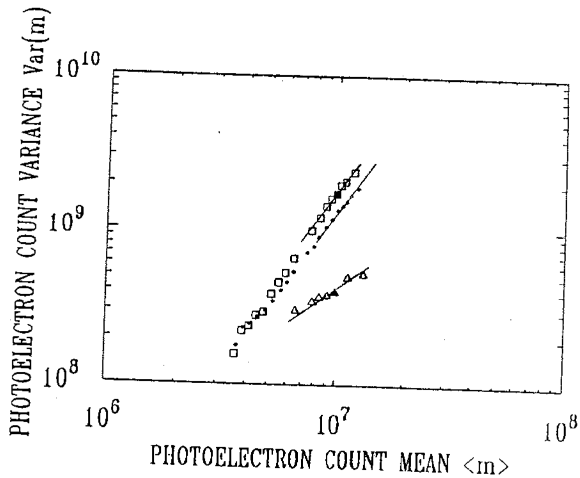

Fig. 3. Photoelectron count variance versus mean for wiggler light from a tightly focussed beam (squares), a loosely focussed beam (dots), and for light in the absence of the wiggler (triangles). The noise variance has been subtracted from this data. The filled square and triangle represent data obtained from figs. 2a and 2b, respectively. Residual variance increases as the square of the mean for wiggler light, and directly with the mean for the synchrotron light (straight solid lines).

count was controlled by use of a continuously variable neutral density filter. The variance of the wiggler light scales as $\langle m \rangle^2$ as expected from eq. (3), whereas the variance of the synchrotron light varies as $\langle m \rangle$ in accordance with eq. (4).

The number of modes, M, was determined from fig. 3 with the help of eq. (3), to be ~54000 and 78000 for the tightly and loosely focussed beam, respectively. M was estimated as the product of three factors (associated with time, area, and polarization) applicable for cross-spectrally pure light [9]. We take $M = M_T M_A M_P = (T/\tau_c)(A/A_c)[2/(1+P)]$, which is appropriate when the first two factors are somewhat greater than unity, with T the counting time, τ_c the coherence time, A the photodiode active area, A_c the coherence area, and P the degree of polarization. These parameters we estimate as: $M_T \sim 2450$ using $\tau_c = 0.2$ ps as determined by the Gaussian passband of the interference filter; $M_A \sim 27.2$ for SQ = 300 and 30.5 for SQ = 0, assuming that this term, which depends on the emittance (see table 1), can be reasonably represented as the computed ratio of the total photon flux to the coherent photon flux [19]; and the measured value of $P = 0.87$. The theoretically expected values of M are therefore 71000 for SQ = 300 and 80000 for SQ = 0, which are close to the experimental result. Other experiments were done, demonstrating that longer pulsewidths, unfocussed wiggler light, or rectangular slits (in the horizontal or vertical directions) would alter the measured M in the expected way.

The result reported in eq. (2) is also obtained from amplified spontaneous emission resulting from an optical wave moving along an electron beam inside a wiggler [6]. When the device is configured as an FEL which operates far above threshold, it is predicted the radiation is not in a coherent state, but suffers larger fluctuations which are related to the spread of electron momentum introduced by the FEL interaction [20].

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References

[1] R. Bonifacio, Opt. Commun. 32 (1980) 440.
[2] G. Dattoli and A. Renieri, in: Laser Handbook, eds. M.L. Stitch and M. Bass (North-Holland, Amsterdam, 1988) pp. 1–141.
[3] W. Becker and J.K. McIver, Phys. Rev. A27 (1983) 1030.
[4] W. Becker and J.K. McIver, Phys. Rev. A28 (1983) 1838.
[5] A.T. Georges, in: Free Electron Generators of Coherent Radiation, eds. C.A. Brau, S.F. Jacobs and M. Scully, SPIE 453 (1985) 297.
[6] J. Gea-Banacloche, Phys. Rev. A31 (1985) 1607.
[7] M.C. Teich and B.E.A. Saleh, in: Progress in Optics, vol. 26, ed. E. Wolf (North Holland, Amsterdam, 1988) p. 1–104.
[8] S. Benson and J.M.J. Madey, Nucl. Instr. and Meth. A237 (1985) 55.
[9] B. Saleh, Photoelectron Statistics (Springer, New York, 1978).
[10] G. Bedard, J.C. Chang and L. Mandel, Phys. Rev. 160 (1967) 1496.
[11] M.C. Teich and B.E.A. Saleh, Opt. Lett. 7 (1982) 365.
[12] P.L. Liu et al., IEEE J. Quantum Electron. QE-19 (1983) 1348.
[13] M.C. Teich, Appl. Opt. 20 (1981) 2457.
[14] B.E.A. Saleh, J.T. Tavolacci and M.C. Teich, IEEE J. Quantum Electron. QE-17 (1981) 2341.
[15] B.E.A. Saleh, D. Stoler and M.C. Teich, Phys. Rev. A27 (1983) 360.
[16] B.E.A. Saleh and M.C. Teich, IEEE Trans. Inf. Theory IT-29 (1983) 939.
[17] M.C. Teich and B.E.A. Saleh, J. Mod. Opt. 34 (1987) 1169.
[18] M.C. Teich and B.E.A. Saleh, Phys. Rev. A24 (1981) 1651.
[19] K.J. Kim, Nucl. Instr. and Meth. A246 (1986) 71.
[20] A. Bhattacharjee and I. Gjaja, in: Physics of Particle Accelerators, eds. M. Month and M. Dienes, AIP Conf. Proc. 184 (1989) 1776.