INFORMATION TRANSMISSION WITH SQUEEZED LIGHT

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INTRODUCTION

The nature of photon-number-squeezed light (also called sub-Poisson light) has been elucidated in recent years.[1-13] This type of light is expected to find use in the study of optical interactions in various disciplines, ranging from the behavior of the human visual system at the threshold of seeing[14] to optical precision measurement.[15] In this paper we consider the potential use of photon-number-squeezed light in direct-detection lightwave communication systems and other information-carrying applications.[16]

All lightwave communication systems that have been developed to date make use of Poisson (or super-Poisson) light.[17] For Poisson light, the variance of the photon number is identically equal to its mean for all values of the counting time T. Photon-number-squeezed light, on the other hand, has a photon-number variance that is less than its mean for all or some values of T.[1-3] Such light is intrinsically nonclassical in nature. The earliest sources of photon-number-squeezed light exhibited only a slight reduction of the variance.[1,2] Far stronger photon-number squeezing has been produced in recent years,[18] and continuing advances promise further improvement. It is therefore of interest to examine the advantages to be gained in using photon-number-squeezed light in a direct-detection lightwave communication system.

There are essentially two classes of mechanisms by means of which unconditionally photon-number-squeezed light may be generated. In the first class, squeezed photons are produced from a beam of **initially Poisson** (or super-Poisson) **photons**. This can be achieved in a number of ways, e.g., by mixing coherent light with quadrature-squeezed vacuum photons or by making use of correlated photon beams.[19] An experiment of

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this kind was carried out by Tapster, Rarity, and Satchell.[20] Squeezed photons were generated from the pair of correlated photon beams produced in parametric downconversion; one of the twin beams was then fed back to control the pump.

The second class of mechanisms relies on the direct generation of squeezed photons from a beam of initially sub-Poisson excitations (e.g., electrons).[2,21] This technique was first used by Teich and Saleh in a space-charge-limited version of the Franck-Hertz experiment.[3] Perhaps the simplest implementation of this principle is achieved by driving a light-emitting diode (LED) with a sub-Poisson electron current,[22] but it is most effectively achieved by the use of a semiconductor injection laser.[7,12,18]

We discuss calculations of the channel capacity of a lightwave communication system based on the observation of the photoevent point process, demonstrating that it cannot in principle be increased by the use of photon-number-squeezed light.[16] We also discuss calculations that show that the channel capacity of a photon-counting system can be increased by the use of photon-number-squeezed light.[16] The channel capacity is the maximum rate of information that can be transmitted through a channel without error. The capacity of the photon channel has been the subject of a number of studies over the years.[23,5] We also discuss an example in which the use of photon-number-squeezed light produced from Poisson light either degrades or enhances the error performance of a simple binary ON-OFF keying photon-counting system, depending on where the average power constraint is placed.

COMMUNICATING WITH MODIFIED POISSON PHOTONS

Consider the transformation of a Poisson beam of photons (represented by a Poisson point process N_t of rate μ_t) into a sub-Poisson beam of photons represented by a point process M_t of rate λ_t, as illustrated in Fig. 1.

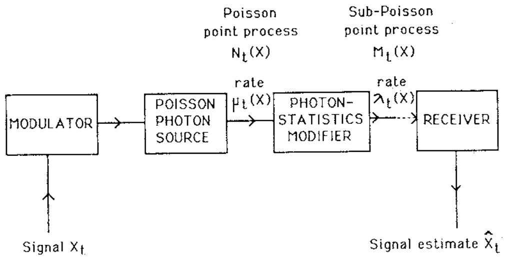

Fig. 1. Idealized lightwave communication system employing a Poisson photon source and a photon-statistics modifier. [After M. C. Teich and B. E. A. Saleh, in Progress in Optics, vol. 26, E. Wolf, ed., North-Holland, Amsterdam (1988)].

The events of the initial process N_t are assumed to be observable [e.g., by the use of correlated photon beams or a quantum-nondemolition (QND) measurement] and their registrations used to operate a mechanism which, in accordance with a specified rule, leads to the events of the transformed photon process M_t. The rate λ_t of the process M_t is thereby rendered a function of the realizations of the initial point process N_t at prior times, i.e., $\lambda_t = \lambda_t(N_{t'}; t' \leq t)$.

Several examples of transformations of this kind that have been suggested for use in quantum optics are illustrated in Fig. 2 and discussed below. It is assumed for simplicity (but without loss of generality) that the various conversions can be achieved in an ideal manner.

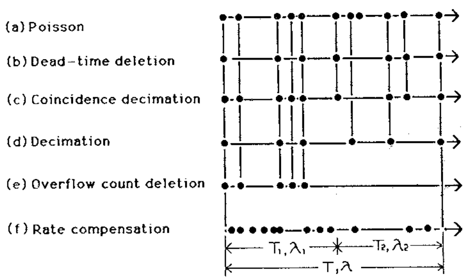

Fig. 2. Several transformations of Poisson photons into sub-Poisson photons that have been suggested for use in quantum optics. [After M. C. Teich and B. E. A. Saleh, in Progress in Optics, vol. 26, E. Wolf, ed., North-Holland, Amsterdam (1988)].

(1) Dead-time deletion: Delete all photons within a prescribed fixed (nonparalyzable) dead time τ_d following the registration of a photon.[24]

Rarity, Tapster, and Jakeman[25] generated photon-number-squeezed light by using one of the twin beams produced in parametric downconversion to selectively gate photons from the other beam via dead-time control. Dead-time deletion could also be used with correlated photon beams produced in other ways.

(2) Coincidence decimation: Remove all pairs of photons separated by a time shorter than a prescribed time interval τ'. This is achieved, for example, in second-harmonic generation (SHG); two photons closer than the intermediate-state lifetime of the SHG process are exchanged for a third photon (which is at twice the frequency and therefore easily eliminated).[26]

(3) Decimation: Select every rth photon ($r = 2,3,\ldots$) of an initially Poisson photon process, deleting all intermediate photons. Saleh and Teich[27] suggested using correlated photon beams to implement this technique. In cascaded atomic emissions from ^{40}Ca, for example, sequences of correlated photon pairs (green and violet) are emitted. The green photons can be detected and used to operate a gate that passes every rth violet photon. Decimation control could also be used in conjunction with parametric-downconversion photon twins.

(4) Overflow count deletion: The number of photons occurring in preselected time intervals $[0,T_0]$, $[T_0,2T_0],\ldots$, is counted, retaining the first n_0 photons in each time interval (without changing their ocurrence times) and deleting the remainder. If the average number of photons in $[0,T_0]$ of the initial process is $\gg n_0$, then the transformed process will almost always contain n_0 photons within this time interval. As an example, Mandel[28] suggested that if a collection of n_0 atoms in the ground state are subjected to a brief, intense, incoherent excitation pulse, all n_0 atoms will become excited with high probability; the radiated optical field would then be describable, to good approximation, by an n_0-photon state. Related schemes have been proposed by Yuen[29] and by Stoler and Yurke[30] for use with parametric processes.

We proceed to illustrate that none of these modifications can increase the channel capacity of a communication system based on photoevent point-process observations.

If a constraint is placed on the rate of the initial Poisson process $\mu_t \le \mu_{max}$, then it is obvious that C cannot be increased by the modification $N_t \longrightarrow M_t$. This is simply a consequence of the definition of channel capacity: it is the rate of information carried by the system without error, maximized over all coding, modulation, and **modification** schemes.

However, can the modification $N_t \longrightarrow M_t$ increase the channel capacity if the constraint is instead placed on the rate of the modified process λ_t (i.e., $\lambda_t \leq \lambda_{max}$)? We address this question for an arbitrary self-exciting point process in the next section.

COMMUNICATING WITH PHOTONS DESCRIBED BY A SELF-EXCITING POINT PROCESS

Consider a self-exciting point process M_t of rate $\lambda_t(M_{t'};\ t'\leq t)$. This is a process that contains an inherent feedback mechanism in which present event occurrences are affected by the previous event occurrences of the same point process. Of course, the modified Poisson processes $N_t \longrightarrow M_t$ introduced above are special cases of self-exciting point processes.

An example of a system that generates a self-exciting point process is that of rate compensation (by linear feedback) of a source which, without feedback, would produce a Poisson process. Let each photon registration at time t_i cause the rate of the process to be modulated by a factor $h(t-t_i)$ (which vanishes for $t < t_i$). In linear negative feedback the rate is $\lambda_t = \lambda_0 - \Sigma_i h(t-t_i)$, where λ_0 is a constant. If the instantaneous photon registration rate happens to be above the average then it is reduced, and vice versa. This process is schematically illustrated in Fig. 2(f) for two adjacent sub-intervals T_1 and T_2. Yamamoto, Imoto, and Machida[31] suggested the use of rate compensation in conjunction with a QND measurement (using the optical Kerr effect) but rate compensation could be used just as well, for example, with correlated photon pairs. Dead-time deletion can be viewed as a special case of rate compensation in which the occurrence of an event zeros the rate of the process for a specified time period τ_d after the registration.[19]

Now consider a communication system that uses a point process $M_t(X)$ whose rate $\lambda_t(X)$ is modulated by a signal X_t. The process $M_t(X)$ can be an arbitrary self-exciting point process (e.g., it can be photon-number-squeezed) which includes processes obtained by the feedforward- or feedback-modification of an otherwise Poisson process.[19]

Neither feedforward nor feedback transformations can increase the capacity of this channel, as provided by Kabanov's theorem[32] and its extensions[33]:

Kabanov's Theorem — The capacity of the point-process channel cannot be increased by feedback. Under the constraint $\lambda_0 \le \lambda_t \le \lambda_{max}$, the channel capacity C is

$$C = \lambda_0 \left[\frac{1}{e} \left(1 + \frac{s}{\lambda_0}\right)^{1+\lambda_0/s} - \left(1 + \frac{\lambda_0}{s}\right) \log\left(1 + \frac{s}{\lambda_0}\right) \right], \qquad (1)$$

where $s = \lambda_{max} - \lambda_0$. When $\lambda_0 = 0$ (no dark counts), this expressions reduces to

$$C = \lambda_{max}/e. \qquad (2)$$

When the capacity is achieved, the output of the zero-dark-count point-process channel is a Poisson process with rate $\lambda_t = \lambda_{max}/e$ (the base e has been used for simplicity). The channel capacity has also been determined under added constraints on the mean rate. A coding theorem has also been proved. Kabanov's theorem is analogous to the well-known result that the capacity of the white Gaussian channel cannot be increased by feedback.[34]

In summary, no increase in the channel capacity of a point-process lightwave communication system may be achieved by using photons that are first generated with Poisson statistics and subsequently converted into sub-Poisson statistics, regardless of whether the power constraint is placed at the Poisson photon source or at the output of the conversion process. Nor may an increase in channel capacity be achieved by using feedback to generate a self-exciting point process.

COMMUNICATING WITH SQUEEZED PHOTON COUNTS

These conclusions are valid only when there are no restrictions on the receiver structure. The conlusion is different if the receiver is operated in the photon-counting regime, in which information is carried by the random variable n representing the number of photoevents registered in time intervals of prescribed duration T (rather than by the photon occurrence times).

The capacity of the photon-counting channel is given by[23]

$$C = B[\bar{n}\ln(1 + 1/\bar{n}) + \ln(1 + \bar{n})], \tag{3}$$

where $\bar{n}$ is the mean number of counts in T and B = 1/T is the bandwidth. Two limiting expressions emerge:

$$\begin{aligned} C &= B\bar{n}\ln(1/\bar{n}), & \bar{n} &\ll 1 \\ C &= B\ln(\bar{n}), & \bar{n} &\gg 1. \end{aligned} \tag{4}$$

If an added constraint is applied to the photon counts, such that they must obey the Poisson counting distribution, the capacity is further reduced. In that case, the limiting results analogous to Eq. (4) are

$$\begin{aligned} C &= B\bar{n}\ln(1/\bar{n}), & \bar{n} &\ll 1 \\ C &= \tfrac{1}{2}B\ln(\bar{n}), & \bar{n} &\gg 1. \end{aligned} \tag{5}$$

The capacity in the region $\bar{n} \gg 1$ is a factor of 2 smaller in Eq. (5) than in Eq. (4). The capacity-to-bandwidth ratio C/B is plotted versus $\bar{n}$, for both the unrestricted and Poisson photon-counting channels, in Fig. 3.

In the case of photon counting, therefore, an increase in the channel capacity <u>can</u> in principle be realized by using photon-number-squeezed light. However, in the small mean-count limit $\bar{n} \ll 1$ (when the counting time T is very short), the capacity of the Poisson counting channel approaches that of the unrestricted counting channel, and the advantage of photon-number squeezing disappears. This is not unexpected in view of the result obtained from Kabanov's theorem for the point-process channel.

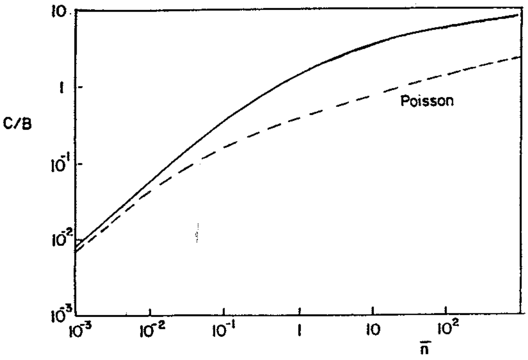

Fig. 3. Comparison of the capacity of the unrestricted photon-counting channel (solid curve) with the Poisson photon-counting channel (dashed curve).

PERFORMANCE OF A BINARY OOK PHOTON-COUNTING RECEIVER

The channel capacity provides a limit on the maximum rate of error-free information transmission for all codes, modulation formats, and receiver structures.[23] As such, it does not specify the performance (error probability) achievable by a communication system with prescribed coding, modulation, and receiver structure.

It is therefore of interest to discuss the performance of a system with specified structure. We consider a binary on-off keying (OOK) photon-counting system.[17] The information is transmitted by selecting one of two values for the photon rate λ_t, in time slots of (bit) duration T. The receiver operates by counting the number of photons received during the time interval T and then deciding which rate was transmitted in accordance with a likelihood-ratio decision rule (threshold test). For simplicity, it is assumed that background light, dark noise, and thermal noise are absent so that photon registrations are not permitted when the keying is OFF (i.e., false-alarms are not possible). Furthermore, the detector quantum efficiency is initially taken to be unity so that system performance is limited only by the quantum fluctuations of the light.

A measure of performance for a digital system such as this is the error probability P_e. In the simplified system described above, errors are possible only when the keying is ON and 0 photons are received (a miss). For a Poisson transmitter, P_e is[17]

$$P_e(\text{Poisson}) = \tfrac{1}{2}\exp(-\bar{n}), \qquad (6)$$

where $\bar{n}$ denotes the mean number of emitted photons. To minimize P_e, $\bar{n}$ is made equal to its maximum allowed value $\bar{n}_{max}$. This result is now compared with those obtained for photon-number-squeezed light derived from an initially Poisson source. The outcome will depend on where the mean photon-number constraint is placed. Two transformations are explicitly considered: dead-time deletion and decimation.

It will become evident from these examples that system performance can be enhanced by the use of photon-number-squeezed light, provided that the constraint is applied to the squeezed light. No enhancement of system performance emerges in converting Poisson photons into squeezed photons when the constraint is at the Poisson source.

Dead-Time-Modified-Poisson Photon Counts — For a nonparalyzable dead-time modifier that is always **blocked** for a dead time period τ_d at the beginning of the counting interval T, the passage of 0 photons arises from the emission of 0 photons in the time T - τ_d, independent of the number of emissions during τ_d. The error probability for this system is therefore

$$P_e(\text{dead-time}) = \tfrac{1}{2}\exp[-\bar{n}(1 - \tau_d/T)]. \qquad (7)$$

To minimize error under the mean photon-number constraint $\bar{n} \leq \bar{n}_{max}$, we take $\bar{n} = \bar{n}_{max}$. The error is larger than that for the Poisson channel [Eq. (6)], as illustrated in Fig. 4, so no performance enhancement can be achieved by use of this modifier.

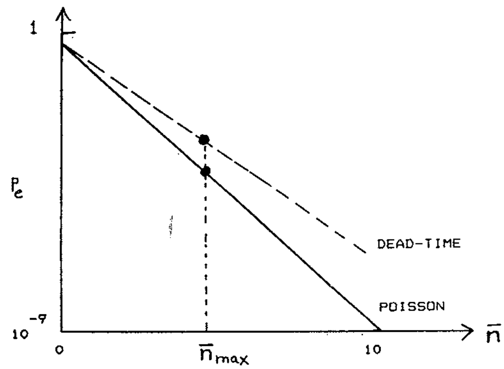

Fig. 4. Comparison of the error probabilities for the Poisson (solid curve) and blocked dead-time (dashed curve) channels with the constraint at the input. No performance improvement is possible.

If, instead, the dead-time modifier is always **unblocked** at the beginning of each bit then the passage of 0 photons arises from the emission of 0 photons in the time T, and the dead-time has no effect on the error rate in this simple system. Calculations for the unblocked counter in the presence of false alarms, however, demonstrate that the presence of dead time always does, in fact, degrade system performance with such a constraint.[35] Although the detailed calculations were carried out for electrical dead time, the results are also applicable for optical dead-time when the photon detection efficiency $\eta = 1$.

On the other hand, if the constraint is placed on the mean photon count $\bar{m}$ after dead-time modification ($\bar{m} \leq \bar{m}_{max}$), it can be shown that there exists a value of $\bar{m}_{max}$ below which performance is degraded, and above which performance is improved, relative to the Poisson channel. This is illustrated in Fig. 5.

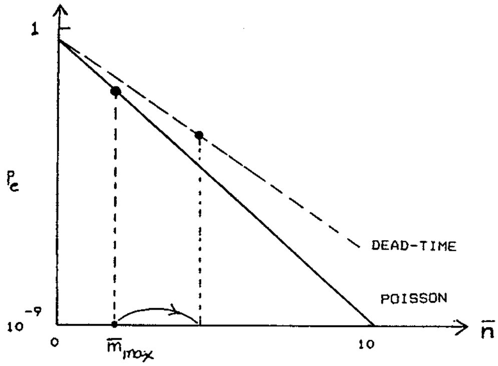

Fig. 5. Comparison of the error probabilities for the Poisson (solid curve) and blocked dead-time (dashed curve) channels with the constraint at the output. Performance improvement is possible.

Decimated-Poisson Photon Counts — We assume that the decimation parameter is $r = 2$ (i.e., every other photon of a Poisson sequence of events is selected) and that the decimation process is reset at the beginning of each bit (i.e., the first photon in each bit is not selected). The error probability is then

$$P_e(\text{decimation}) = \tfrac{1}{2}(1 + \bar{n})\exp(-\bar{n}), \tag{8}$$

which again represents a degradation of performance in comparison with the Poisson channel (under a constraint $\bar{n} \le \bar{n}_{max}$). In this case, the error rate is increased because there are two ways for the passage of 0 photons to arise in the time T: from the emission of 0 photons or from the emission of 1 photon. However, if the constraint is placed on the modified process then, once again, there exists a value of $\bar{m}_{max}$ below which performance is degraded and above which it is improved, relative to the Poisson channel.

PERFORMANCE DEGRADATION ARISING FROM PHOTON LOSS

We conclude by discussing the effects of photon loss (random deletion).[8] We do this in the context of an ideal source that generates a deterministic photon number. This is an important consideration because random photon deletion is inevitable; it results from absorption, scattering, and the finite quantum efficiency of the detector.[8] It is well-known that such deletions will transform a deterministic photon number into a binomial photon number, which always remains sub-Poisson but approaches the Poisson boundary as the random deletion probability η decreases.[36] It has been shown that the information rate per symbol carried by such a counting channel will be greater than that for the Poisson channel, but will approach the latter as $\eta \longrightarrow 0$.[28] A source that emits a binomial number at the outset[37] retains its binomial form, but exhibits reduced mean, in the presence of random deletion.[36]

The performance of such a binary OOK photon-counting receiver, in the absence of background, is limited by the binomial fluctuations of the detected photons. In this case, it is easily shown from the binomial distribution that[9,38]

$$P_e(\text{binomial}) = \tfrac{1}{2} F_n^{[2\langle n'\rangle/(1-F_n)]} \tag{9}$$

where $F_n = 1 - \eta$ is the Fano factor of the photon-counting distribution and where $\langle n'\rangle$ represents the mean number of photons/bit (note that $2\langle n'\rangle = \langle n\rangle$ since there are two bits per pulse in OOK). The Poisson result in Eq. (6) is recovered as $F_n \longrightarrow 1$. The probability of error represented by Eq. (9) is plotted as a function of the mean number of photons per bit $\langle n'\rangle$, with the Fano factor F_n as a parameter, in Fig. 6. System performance improves as F_n decreases.

Solving Eq. (9) for the mean number of photons per bit $\langle n'\rangle$ provides

$$\langle n'\rangle = \tfrac{1}{2}[(1-F_n)/\ln(1/F_n)]\ln(1/2P_e), \tag{10}$$

which leads to a direct-detection quantum limit that is < 10 photons/bit (< 20 photons per pulse) for OOK, if $F_n < 1$ and $P_e = 10^{-9}$. The mean number of photons per bit $\langle n'\rangle$ is plotted as a function of F_n in Fig. 7. The usual quantum limit ($\langle n'\rangle = 10$ photons/bit) emerges in the limit $F_n = 1$ where the binomial distribution goes over to the Poisson.

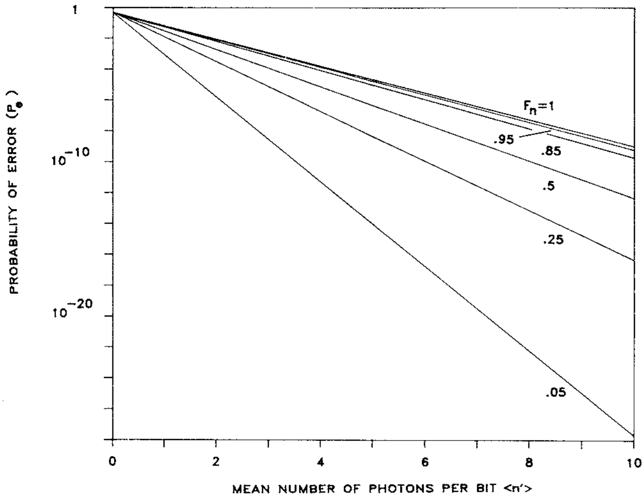

Fig. 6. Error probability (P_e) vs mean number of photons per bit <n'> for the binomial channel, with the Fano factor F_n as a parameter. System performance clearly improves as F_n decreases below unity.

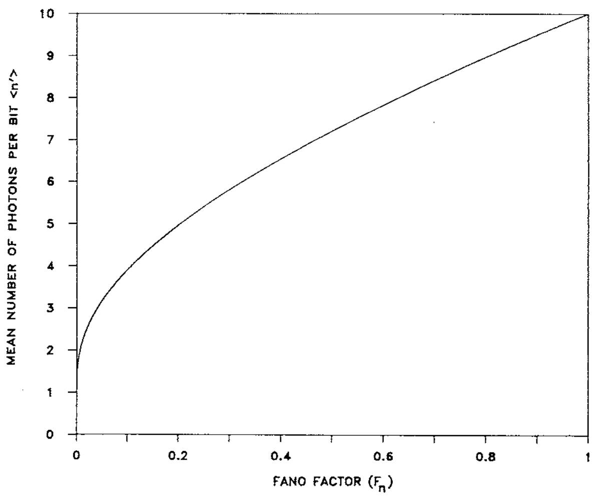

Fig. 7. Mean number of photons per bit <n'> as a function of the Fano factor F_n for the binomial channel. The well-known "quantum limit" (10 photons/bit) emerges as the binomial distribution goes over to the Poisson distribution (F ——> 1).

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