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Book Review

A Unique Approach to Stimulus Detection Theory in Psychophysics Based upon the Properties of Zero-Mean Gaussian Noise

DONALD LAMING. **Sensory Analysis.**
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1. INTRODUCTION

This is a very important book, perhaps the most significant advance in psychophysical detection theory since the now-classical collection of papers, *Signal Detection and Recognition by Human Observers*, published under the editorial direction of John A. Swets in 1964.

The importance of Laming's work derives from a multiplicity of virtues. Foremost is the quality of his analysis, combining technical skill, breadth of scholarship, and a tough advocative stance on key mechanisms of sensory detection. Laming's unblushing advocacy brings fresh vitality to a theoretical area grown moribund in recent years.

The principal virtue here is the prospect of reviving an immensely important part of sensory psychology. The original work pioneered by W. P. Tanner, J. A. Swets,

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and D. M. Green had this bold quality. It challenged the very idea of a threshold, pointed to the critical importance of outcomes or payoffs, and gave us a vision of sensory processes as approximations to a mathematically defined "ideal observer." To be frank, the ideal observer idea did not fly. It failed to suggest easily identifiable mechanisms by which its properties might be realized or approximated. Thus, depicting the auditory nervous system as a cross-correlator did not suggest much that is practical in the interpretation of neurophysiological data.

Moreover, few serious attempts were made beyond the research of Barlow (1957) and Nachmias and Steinman (1963) to bridge the gap between detection theories that had grown up independently in vision and audition. Many believed (and still believe) that the phenomena of the two major senses are so dominated by mechanisms in the receptors themselves as to be unamenable to a single detection theory.

And so the early vitality of signal detection theory, a powerful driving force in auditory research of the 1960s, dissipated slowly into a methodological literature not unlike Thurstone scaling—an important tool but essentially a curiosity. The task of describing how information passes from sense organ to sensorium edged subtly back to a concentration on sense organs, at least among sensory psychologists. Studies of information flow drifted over into the so-called cognitive areas where the utterly complex adaptive mechanisms of perception, language, and memory had begun to challenge the best mathematical minds. Incremental detectability in the eye, the ear, and the skin senses was, of course, not abandoned; nothing of the sort. But it came to be seen as less interesting than the more complex forms of cognition. Intensity discrimination couched in the framework of a single broad detection theory simply lost its fastball and was relegated to footnotes in papers on parallel processing midway through the 1980s.

Then along came Laming in 1986 with his breath of fresh air regenerating the excitement of the search for missing pieces of the puzzle surrounding sensory communication.

> The idea is wide spread that the sensations we experience correspond nicely in magnitude to the level of neural activity at some ... part of the sensory apparatus. I happen not to share that belief ...
>
> The neural quantum theory ... (Stevens, Morgan, and Volkmann (1941)) ... is a patent red-herring ....
>
> Another belief is ... (that) ... the sensory response to small perturbations about an equilibrium point is linear. That idea also has to go.
>
> My objective is not to achieve a balanced appraisal of ... opinions which may reasonably be held in the light of present experimental findings but to present my own conception of how sensory analysis works.

Isn't it refreshing? You need not agree with Laming's conception of sensory analysis at every disputed point, and believe us we do not, in order to appreciate the force of his arguments or the thrust of this criticisms. The point is that you finish the book with furious marginalia on virtually every page, and yet you know that detection theory is alive again.

2. Laming's Sensory Analysis

The sophisticated reader will benefit by turning at once to Chapter 13, *Some Principles of Sensory Analysis*, before beginning at the beginning with Laming's summary of the psychophysical foundations of Weber's Law.

His principles set forth the key points of his position. There are, he argues, basic functional differences in data on difference discrimination (two distinct stimuli separated by a silent interval or gap) and increment detection (continuous background with stimulus briefly superimposed).

In difference discrimination:

(a) Weber's Law holds down nearly to absolute threshold.

(b) The psychometric function is a normal integral linear with stimulus magnitude (amplitude).

(c) The ROC curve derives from the Gaussian and is symmetric.

In increment detection:

(a) Weber's Law at high magnitudes gives way to a square-root law for low-intensity stimuli.

(b) The psychometric function is a normal integral with the *square* of the increment in stimulus amplitude (i.e., low-increment suppression).

(c) The ROC curve is asymmetric and biased toward detection of an increment when it is present.

The discovery of these regularities is not without potential for dispute about the reality or importance of some of the claims made, but taken together such regularities offer a striking tribute to Laming's observational acumen. It is perfectly evident that something is going on in such data and Laming has seized upon it.

Laming lays out his basic arguments on visual and auditory detection in Chapter 4. He reviews the classical psychometric functions for pure noise (vision, audition), pure tones in noise (audition), pedestals in noise (audition), and modulation of visual itensity via sinusoidal gratings. The linchpin of Laming's analysis is the finding, well-documented but thus far unexplained, that the slope of the psychometric function changes sharply from condition to condition. Why should the slope of the psychometric function undergo a marked steepening as we move from difference discrimination to increment detection in studies of luminance? More important, why should the detectability of a sinusoid in wideband noise accord well with energy detection, while the same function for increments in pure noise does not? The slope of the psychometric function in the latter case is roughly double that of an energy detector.

Laming relies heavily on a remarkable paper by Green (1960; reprinted in Swets (Ed.), 1964) in which virtually every property of incremental detection with band-limited noise checks out nicely against energy detection, but, unaccountably, the psychometric functions are much too steep. Laming shows that a normal integral against the *square* of the signal-to-noise ratio fits Green's data well, and then Laming offers his own explanation of why this must be so.

It is at this point that Laming decides to jettison energy detection and sets forth his position that the "proper psychological measure of auditory intensity is *amplitude*." It is an important decision fraught with danger for the unwary. Auditory amplitude and auditory power or energy are related by a square law, but moving from one unit system to the other can produce a quagmire in which the inexperienced reader can sink into total confusion. Laming shows himself never to be confused but he moves abruptly without warning, and it is often difficult to track him.

The next chapters (5 and 6) are devoted to the heart of Laming's analysis. He postulates that in steady state the level of incoming stimulation feeds into a mechanism that divides a stimulus into positive and negative inputs each perturbed independently by Gaussian noise. The so-called "negative" input inhibits its counterpart, producing a result resembling algebraic addition when the inputs are combined. Hence the combined input generates a new Gaussian noise with zero mean, and variance proportional to the level of the input. The magnitude of the combined input fades to zero in steady state stimulation, but magnitudes are implicitly reflected in variances. This is a well-known property of linearly filtered Poisson events, so-called "shot-noise"; i.e., the mean and variance are proportional. Moreover, such mixtures of excitatory and inhibitory inputs reflect a familiar principle of sensory physiology (Kuffler, Nicholls, and Martin, 1984).

It follows that Laming's mechanism, coupled with the idea that detectability is fixed when the ratio of variances is fixed, will produce Weber's Law in any sense modality for stimuli at any intensity level. Weber's Law is a property of such zero-mean Gaussian noise. Laming then argues that a brief rectangular change in level (i.e., a "boxcar") will be differentiated by the mechanism if the latencies of response to the positive and negative inputs are slightly different. The noisy derivative is fed via half-wave rectification and integration to a fixed observation window, generating the data on which detection decisions are based.

Evidently with only Gaussian internal noise and the derivative of the signal in increment detection, a careful study of the properties of Gaussian noise enables Laming to compute the behavior of the mechanism under various forms of visual, auditory, and other sensory stimulation. He shows (Chapter 6) that a square-law transform must occur with brief incremental inputs. The latter should then be governed by the properties of non-central χ^2 (non-central because of the brief positive jump as the stimulus increment is differentiated). Laming proceeds to calculate the behavior of the mechanism in threshold experiments on spatial and temporal summation (Chapter 7), as well as in studies of negative masking (Chapter 8).

Negative masking is one of the puzzling ambiguities of auditory research. Partial suppression of internal noise by stimuli at or just above threshold is known to occur in sensory neurophysiology. One example is the reduction of firing rate during the "clipped" portion of the cycle in primary auditory nerve fibers. Does it also occur in psychophysical data? Laming reviews auditory data on pure-tone and noise detection as well as visual contrast observed with sinusoidal gratings, aiming to show the effect. He then deduces the shapes of the masking curves from the properties of his "differential coupling" mechanism.

In both these areas, summation and negative masking, Laming's systematic derivations of $I \times T$ laws, square-root law summation in vision and audition, as well as negative masking curves and even $\frac{1}{4}$th power laws for sinusoidal gratings, constitute a dazzling tour-de-force on the power of zero-mean Gaussian noise as a tool of detection theory.

Chapter 9 outlines data in which a square-root law is found to characterize increment detection over the first 2–4 log units of intensity. The result is a classical one in luminance discrimination where it has become known as the deVries–Rose Law (operating over some 5–6 log units of intensity above threshold.) But Laming manages also to extract a square-root law at low intensities from pure tones and incremental noise detection. He does this by reworking data of Harris (1950, 1963) into amplitude units.

The deVries–Rose Law is ordinarily viewed as a bulwark of the argument that at low levels the eye is, in effect, an energy detector. This is because light energy flows essentially as a Poisson process where square-root law detectability results from changes in level. (Equation (5.5) is missing a square-root sign on the right-hand side.)

Laming rejects such ideas. His differential coupling mechanism should follow Weber's Law all the way down to threshold. Hence square-root law detectability poses a problem. Laming solves it by devising a second mechanism which introduces no latency difference in positive and negative inputs. There is perfect symmetry. This second mechanism sits in front of the differential coupler. At low intensities where shot noise breaks down into its discrete Poisson counts, half-wave rectification simply wipes out the negative input. Hence a Poisson process at half-rate forms the input to the differential coupler which is in effect transparent to it since there is nothing to differentiate. To take care of other difficulties, Laming views the mechanism as having three distinct stages: a pair of "exact" (i.e., symmetrical) analyzers surrounding the "differential" (asymmetrical) analyzer. Incremental changes at low intensity levels are detected primarily by the first stage; at medium and high levels by the second stage. A third "exact" stage is required to produce Weber's Law all the way down to absolute threshold in difference discrimination (non-contiguous stimuli).

3. The Units Problem in Audition

The foregoing non-quantitative summary of Laming's position is intended to convey both the flavor of his argument and the sophisticated character of his handling of zero-mean Gaussian noise as a detection device. It is, to say the least, a vast improvement over the ideal-detection schemes that Laming wants to put behind us.

But advocative force, however necessary and attractive, sometimes causes him to soft pedal arguments on which other workers experience difficulty. What follows is the reviewers' effort to illuminate some of those difficulties in greater detail.

Laming's commitment to amplitude as the proper measure of the stimulus in any sense modality, but particularly in audition, has profound consequences that are not fully acknowledged. At first blush one is inclined to react with indifference. Who cares? Power and amplitude determine each other. Laws stated in terms of one variable also determine their opposite numbers completely. One of the reviewers (W.J.M.) recalls long arguments with L. Jeffress on this topic. Jeffress (1968) devised an electronic model, not cited by Laming, resembling Laming's differential coupler closely although lacking the latter's zero-mean characteristic. Jeffress, too, believed that sensation is an amplitude-like event. He took his commitment to the extreme of rederiving the statistics of his device via the distribution of χ (rather than χ^2). The argument centered on whether or not such precautions made any difference since each probability distribution determined the other completely.

It does make a difference. Consider the relation between peak amplitude and power in a sinusoid,

$$I = A^2/2,$$

where I is intensity or power and A is peak amplitude. Now differentiate and divide both sides by $A^{1+\beta} = (2I)^{(1+\beta)/2}$:

$$\frac{dI}{I^{(1+\beta)/2}} = \frac{dA}{A^{\beta}} \cdot 2^{(1+\beta)/2}.$$

So, to a good approximation, if a small increment ΔI is negligible in relation to I,

$$\frac{\Delta I}{I^{(1+\beta)/2}} \simeq \frac{\Delta A}{A^{\beta}},$$

where the relation is one of proportionality.

Laws stated in terms of amplitude are not identical to laws stated in power units. The quantity β is a dummy variable intended to reveal such differences. For example, suppose that over a range of amplitudes ΔA remains constant (see Laming's Fig. 1.10). Set $\beta = 0$ and we see that ΔI will follow a square-root law over the same range.

Similarly, if ΔI is nearly constant, as it is near absolute threshold, set $\beta = -1$ and note that $\Delta A \cdot A$ is constant over the same region. As amplitude increases, ΔA decreases, and we see that negative masking may arise in part from the choice of amplitude units.

Laming is fully aware of these problems. He chooses simply not to draw attention to them, leaving them to a footnote on p. 14. A more interesting issue is why auditory research persists in reporting results in terms such as "ΔA in dB." The decibel scale was devised so as to convert measurements to power units precisely to avoid such confusions, but 20 log ΔA is not a quantity expressing the spirit of the decibel scale. The answer, of course, is that researchers generally measure voltages into earphones and then use the earphone calibration to convert to sound pressure levels. Effects that seem linked to changing voltage may be severely attenuated or even wiped out by conversion to power units (cf. negative masking). The creation of bastardized decibel measures is understandable and perfectly acceptable except when one unit system is looked upon as right and the other wrong. In fact, they are transforms of each other.

The potential for confusion posed by the choice of a unit system is nowhere clearer than in noise intensity discrimination. Miller's (1947) classical study (ignored by Laming) was reported in power units (ΔI in dB vs I in dB). It displays unit slope (i.e., Weber's Law) over a range of 70–80 dB and then coasts smoothly toward zero (ΔI constant) below about +10 dB re absolute threshold. Subsequently Raab, Osman, and Rich (1963), extensively reported by Laming, reran Miller's experiment with a cute gimmick. They produced noise increments in two ways: via changes in gain and via the addition of an independent noise generator. Raab *et al.* (1963) argued that a gain change would produce correlated noise and should be reported in pressure units whereas an independent noise should be plotted as additive power units. The so-called correlated noise showed the negative masking effect. Independent noise did not.

The difficulty here is that the two methods must produce nearly the same noise and we appear to be looking at a units problem. This was pointed out by Green (1966) and eventually straightened out. But in the meantime Miller's original article was reprinted in the Luce, Bush, and Galanter (1963) handbook, and Miller (p. 134) attached a footnote acknowledging an error that, in the opinion of these reviewers, he had not committed. To demonstrate the point consider the moment generating function $F_x(\theta)$ of a narrowband Rayleigh noise

$$F_x(\theta) = \frac{1}{1 - N_0\theta},$$

where x is measured in power or energy units, N_0 is the noise power per unit bandwith, and θ is the variable in the moment generating function domain. Changing the gain takes us to

$$F_x(\theta) = \frac{1}{1 - (S_0 + N_0)\theta},$$

and S_0 is taken to be the increment in power in the so-called correlated case. Now suppose an independent noise generator adds an energy increment x to the base-band noise. Hence

$$G_x(\theta) = \frac{\exp(x\theta/1 - N_0\theta)}{1 - N_0\theta}.$$

The result is clearly different, but x is a random variable. We proceed to integrate it out,

$$G_x(\theta) = \int_0^\infty \frac{1}{S_0} e^{-x/S_0} \left(\frac{\exp(x\theta/1 - N_0\theta)}{1 - N_0\theta} \right) dx$$

$$= \frac{1}{1 - (S_0 + N_0)\theta} = F_x(\theta).$$

The two noise processes (gain change and independent generator) are identical in energy content. If detectable differences are to be found, they will come via changes in the autocorrelation function, not in the way noise increments are formed.

Now of course this does not suggest that negative masking is wholly an artifact of the unit system. Repeated encounters with such masking in the psychoacoustic literature and its plausible basis in auditory electrophysiology suggest that the phenomenon is real. But the effects are small and, for practical purposes, dependent on the choice of amplitude units. If a function exists mapping stimulus energy into impulse counts in the auditory nervous system, then we (the reviewers) prefer to work in power units because they are additive. Laming is entitled to argue for a different view, but we think his book would have benefited from a chapter devoted to the units problem in audition, and perhaps also in those areas of vision research where light undergoes sinusoidal modulation.

4. Square-Law Transform

Perhaps Laming's most remarkable achievement in the book is his derivation of the square-law transform property of the differential coupler. It enables Laming to predict the steep slopes of Green's (1960) psychometric functions in noise-increment detection and to explain related phenomena in receiver-operating characteristic (ROC) plots. Moreover, the square-law transform provides the principal basis for Laming's rejection of energy detection.

It is not altogether graceful for the reviewers to react to these unquestioned advances by saying that, of course, no one ever took the energy-detection hypothesis literally in any case. Vision and audition span some 12–13 log units of intensity sensitivity whereas the nervous system is hardput to range over as much as 3 log units. Even allowing for volume measurements of activity rather than single fibers, there is just no way to avoid some compression when stimulus energy is mapped into neural counting. So linear energy mapping was never supposed, even over the relatively short ranges spanned by the psychometric function.

Moreover, Green's (1960) data depart from the ideal energy detector in two respects. They are shifted toward fewer degrees of freedom (i.e., narrow bandwidths), and the psychometric functions are steeper. The big difficulty for the theorist is to explain both of these effects simultaneously. The shift toward fewer degrees of freedom is easily handled by supposing the listener's bandwidth to be less than ideal, but in that event the psychometric function will be flatter not steeper. Laming's differential coupler deals with the steepened psychometric function but not with the degrees-of-freedom problem.

We have computed psychometric functions supposing that stimulus energy sets off a birth–death process in the auditory tract. Even with this additional flexibility the steepness of the pure-noise psychometric function is unchanged from that found in simple energy detection. But we also observed that any fractional power of the signal-to-noise ratios calculated from the birth–death process shifts the psychometric function toward fewer degrees of freedom and makes it steeper. In other words both requirements can be met by a mapping restriction on stimulus energy.

We hold no brief at this point for the birth–death process over the differential coupler. The intent of the remark is that there may well be other ways of mapping energy into neural counting that will generate suitably steep psychometric functions located in the place at which Green (1960) found them. It is a bit too early to declare the battle ended and to cast all forms of energy detection into exterior darkness. But we agree with Laming that simple energy detection will not do the trick.

Finally, we remark that, precise as psychophysical data tend to be, they are insufficiently precise to meet the demands that Laming's generalizations place upon them. We have already discussed the problem posed by negative masking in audition. To these we would add that 2 or 3 log units are generally not enough to establish a square-root law trend. If you study the data Laming produces to support his contention that a square-root law is a characteristic feature of low levels of incremental detectability and if you bring to the task a suitably jaundiced eye, you find the argument a little fuzzy. Laming argues, almost passionately, for more precise data in order to test his theoretical views. We certainly agree, but it may be that some of these theoretical points turn on questions of such subtlety we may have to wait a long time before the data to settle them become available. Scientific disputes, sad to say, are rarely settled by data.

We cannot close this review on a note in which we appear to cavil. Laming has written a very important book. He has breathed fresh life into detection theory. Every serious experimental psychologist, every mathematical psychologist, will want to own it not only for the pleasure of reading (and quarreling with) Laming's theoretical arguments but also for the treasure of functional data he provides over the whole range of modern psychophysics. It is hard to imagine anyone else so competent to handle so much so well. It is, when all else is said and done, a very remarkable achievement.

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