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Laser heterodyning

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Abstract. The development of heterodyne detection from the radiowave to the optical region of the electromagnetic spectrum is briefly reviewed. Attention is focused on the submillimetre/far-infrared region. A simple phenomenological model for the transition from electric-field to photon-absorption detection is developed. The relative detected power at double- and sum-frequencies is multiplied by an attenuating coefficient that depends on the incident photon energy and on the effective temperature of the system. Several recent developments in heterodyne detection are mentioned.

1. Brief history

Heterodyne detection has a long and august history in the annals of electrical engineering, reaching back to the earliest years of the century. The term has its roots in the Greek words 'heteros' (other) and 'dynamis' (force).

In 1902, Reginald Fessenden was awarded a United States patent [1] 'relating to certain improvements ... in systems where the signal is transmitted by [radio]waves differing in period, and to the generation of beats by the waves and the employment of suitable receiving apparatus responsive only to the combined action of waves corresponding in period to those generated ... '. The subsequent realization that one of these waves could be locally generated (the development of the local oscillator or LO) provided a substantial improvement in system performance. Practical demonstrations of the usefulness of the technique were carried out between the 'Fessenden stations' of the U.S. Navy at Arlington (Virginia) and the Scout Cruiser *Salem*, between the *Salem* and the *Birmingham* (1910), and at the National Electric Signaling Company. In 1913, John Hogan provided an enjoyable account of the development, use, and performance of the Fessenden heterodyne signaling system in the first volume of the *Proceedings of the Institute of Radio Engineers* [2].

It was not long thereafter, in 1917, that Edwin H. Armstrong of the Department of Electrical Engineering at Columbia University carried out a thorough investigation of the heterodyne phenomenon occurring in the oscillating state of the 'regenerative electron relay' [3]. A major breakthrough in the field, the development of the superheterodyne receiver, was achieved by Armstrong in 1921 [4], and this famous invention is now used in systems as diverse as household radios and microwave Doppler radars. The prefix 'super' refers to 'the super-audible frequency that could be readily amplified'.

In the succeeding years, the application of heterodyne and superheterodyne principles followed the incessant march toward higher frequencies that culminated in the remarkable developments in microwave electronics about the time of World War II.

The marriage of heterodyning and the optical region took place in 1955. In a difficult and now classic experiment, Forrester, Gudmundsen, and Johnson [5] observed the mixing of two Zeeman components of a visible (incoherent) spectral line in a specially constructed photomultiplier tube. But it was the development of the laser twenty-five years ago [6, 7] that allowed high-frequency heterodyning to become easily observable and a practicable technique in the optical and infrared. The first studies were carried out in 1962 by Javan, Ballik, and Bond [8] at 1·15 μm using a He–Ne laser, and by McMurtry and Siegman [9] at 694·3 nm using a ruby laser.

The development of new transmitting and receiving components in the middle infrared region of the electromagnetic spectrum prompted Teich, Keyes, and Kingston [10] in 1966 to carry out a heterodyne experiment using a CO_2 laser at 10·6 μm in conjunction with a copper-doped germanium photoconductive detector operated at 4 K. Subsequent experiments with lead–tin selenide photovoltaic detectors confirmed the optimal nature of the detection process [11–13]. Many effects can diminish the performance of a heterodyne system (for example, amplitude and phase variations, excess noise arising from LO fluctuations). These have received considerable attention in the literature. In particular, van de Stadt experimentally demonstrated that the deleterious effects of LO excess fluctuations are reduced by use of a dual-detector configuration (balanced mixer) in a He–Ne laser heterodyne system [14]. Figures of merit other than the signal-to-noise ratio [15] have been developed to characterize heterodyne system performance [16, 17].

System configurations employing different forms of nonlinear heterodyne detection have been proposed for various applications [18–20]. These make use of multiple frequencies, nonlinear detectors, and/or correlation schemes. All are aimed at increasing signal detectability in situations for which usual operating conditions are relaxed in particular ways. The use of a three-frequency system (with a two-frequency transmitter) [20] has been demonstrated experimentally [21, 22].

Heterodyne detection in the submillimetre/far-infrared region of the spectrum can be understood quite simply by relaxing the assumption that the heterodyning takes place by means of photon absorption, as it does in the middle infrared and optical [23, 24], or by means of electric-field detection as it does in the radiowave and microwave regions. A simple phenomenological treatment valid for a strong coherent local oscillator and a coherent signal (laser heterodyning) has been set forth [25]. It provides insight into the detection process for fields that possess a positive-definite weight function in Glauber's P-representation [26, 27]. The elements of the model are described in the next section. In the final section of the paper, we draw attention to some consequences of mixing laser LO light with special non-laser squeezed light.

2. Heterodyning in the far-infrared transition region

2.1. *Model*

We consider a hypothetical two-level antenna/detection system with an effective temperature T_e. It is assumed that the system responds in proportion to the incident photon flux, and that its interaction with the field is sufficiently weak such that the state of the field is not perturbed by the presence of the detector. Collections of thermally excited atoms are, of course, in mixed states. Nevertheless, we construct the detection system in terms of a pure extended state in order to capture the dependence on a variable parameter (e.g., effective temperature) in a simple way.

We label the initial and final states of the system as $|\alpha\rangle$ and $|\Omega\rangle$ and of the radiation field as $|\mathrm{i}\rangle$ and $|\mathrm{f}\rangle$, respectively. For an electric-dipole transition, the transition probability W_{fi} (which is related to the detected photon flux) is given approximately by

$$W_{\mathrm{fi}} = |\langle \mathrm{f}\Omega | eq(E^+ + E^-) | \alpha \mathrm{i}\rangle|^2. \tag{1}$$

The quantity e is the electronic charge, q is the detector coordinate, and E^+ and E^- are the positive and negative portions of the electric-field operator, respectively. Since E^+ corresponds to photon absorption or annihilation, and E^- corresponds to photon emission or creation, the transition probability may be written as

$$W_{\mathrm{fi}} = |\langle \mathrm{fu}| eqa_{\mathrm{b}} E^+ |\mathrm{bi}\rangle + \langle \mathrm{fb}| eqa_{\mathrm{u}} E^- |\mathrm{ui}\rangle|^2 \tag{2}$$

where $|\mathrm{b}\rangle$ and $|\mathrm{u}\rangle$ represent the lower and upper states of the two-level system, respectively, and a is the probability amplitude. This equation assumes that the detection system is, in general, in a superposition state. Using microscopic reversibility, the quantity $|\langle \mathrm{u}|eq|\mathrm{b}\rangle|^2$, which represents the quantum efficiency η, may be factored our of equation (2). We then sum over the final states of the field [26], which are not observed, to obtain

$$W \propto \langle \mathrm{i}| a_{\mathrm{b}}^* a_{\mathrm{b}} E^- E^+ |\mathrm{i}\rangle + \langle \mathrm{i}| a_{\mathrm{u}}^* a_{\mathrm{u}} E^+ E^- |\mathrm{i}\rangle + \langle \mathrm{i}| a_{\mathrm{b}}^* a_{\mathrm{u}} E^- E^- + a_{\mathrm{b}} a_{\mathrm{u}}^* E^+ E^+ |\mathrm{i}\rangle. \tag{3}$$

The normally ordered first term corresponds to stimulated absorption, the antinormally ordered second term corresponds to photon emission [28], and the third is an interference term.

We now assume that before the interaction, the probability amplitudes of the two possible states of the antenna/detector system, a_{b} and a_{u}, were related by the Boltzmann factor, with lower and upper level energies represented by E_{b} and E_{u}, respectively, and with excitation energy kT_{e} defining the effective temperature of the detector. Thus,

$$|a_{\mathrm{u}}|^2/|a_{\mathrm{b}}|^2 = \exp[-(E_{\mathrm{u}} - E_{\mathrm{b}})/kT_{\mathrm{e}}], \tag{4}$$

yielding

$$a_{\mathrm{b}} = (1 + e^{-x})^{-1/2} \exp(i\phi) \tag{5}$$

and

$$a_{\mathrm{u}} = e^{-x/2}(1 + e^{-x})^{-1/2} \exp(i\theta) \tag{6}$$

with $x \equiv h\nu/kT_{\mathrm{e}}$ and $\exp(i\theta)$, $\exp(i\phi)$ representing phase factors. Generalizing to a field characterized by the density operator ρ, we obtain

$$\begin{aligned} W \propto ([e^{x/2}\,\mathrm{sech}(x/2)]\,\mathrm{Tr}\{\rho E^- E^+\} &+ [e^{-x/2}\,\mathrm{sech}(x/2)]\,\mathrm{Tr}\{\rho E^+ E^-\} \\ &+ [\mathrm{sech}(x/2)]\,\mathrm{Tr}\{\rho(e^{-i\gamma} E^- E^- + e^{i\gamma} E^+ E^+)\}), \end{aligned} \tag{7}$$

with $\gamma \equiv \phi - \theta$.

Considering ideal heterodyne detection, i.e., two parallel, co-polarized, monochromatic, and coherent waves of frequencies ν_1 and ν_2 impinging normally on the detector, and neglecting spontaneous emission so that the antinormally ordered and the normally ordered terms are equal in magnitude [28], the first two terms above generate d.c. and difference-frequency signals, while the third term contributes double- and sum-frequency signals. This may be clearly seen in the strong coherent field limit by explicitly rewriting equation (7) as

$$W \propto |\varepsilon_1^0|^2 + |\varepsilon_2^0|^2 + 2|\varepsilon_1^0|\,|\varepsilon_2^0| \cos[2\pi(\nu_1 - \nu_2)t + (\beta - \alpha)]$$
$$+ [\operatorname{sech}(h\nu/2kT_e)]\{|\varepsilon_1^0|^2 \cos(4\pi\nu_1 t - 2\alpha - \gamma) + |\varepsilon_2^0|^2 \cos(4\pi\nu_2 t - 2\beta - \gamma)$$
$$+ 2|\varepsilon_1^0|\,|\varepsilon_2^0| \cos[2\pi(\nu_1 + \nu_2)t - (\alpha + \beta) - \gamma]\}, \quad (8)$$

where $\varepsilon_1^0 = |\varepsilon_1^0| \exp(i\alpha)$ represents the complex electric-field amplitude of the constituent field with frequency ν_1 and phase α. Double- and sum-frequency terms in the heterodyne signal are therefore multiplied by the factor $\operatorname{sech}(h\nu/2kT_e)$.

For $h\nu/kT_e \to 0$, this factor approaches unity and the classical electric-field heterodyne signal obtains

$$W_{\text{elec}} \propto [|\varepsilon_1^0| \cos(2\pi\nu_1 t - \alpha - \gamma/2) + |\varepsilon_2^0| \cos(2\pi\nu_2 t - \beta - \gamma/2)]^2. \quad (9)$$

For $h\nu/kT_e \to \infty$, $\operatorname{sech}(h\nu/2kT_e) \to 0$ and the photon-absorption (optical) heterodyne signal obtains [23, 24],

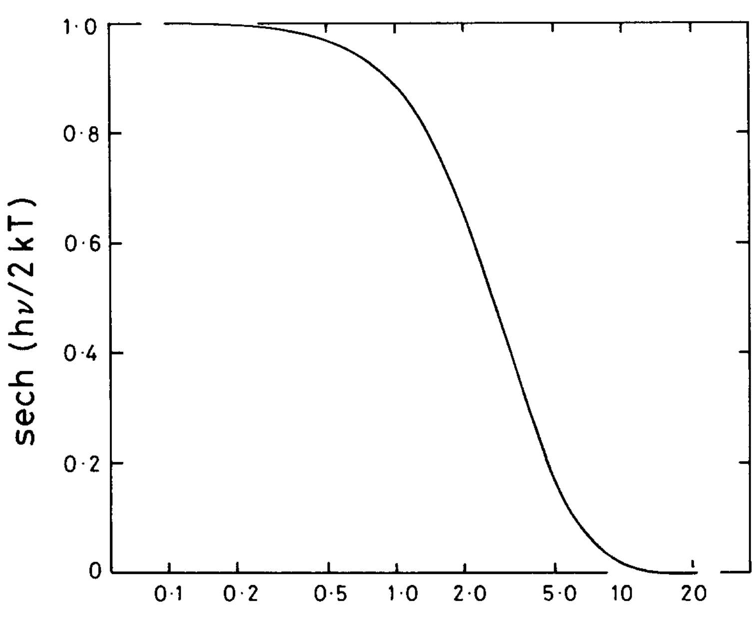
$$W_{\text{abs}} \propto \{|\varepsilon_1^0|^2 + |\varepsilon_2^0|^2 + 2|\varepsilon_1^0|\,|\varepsilon_2^0| \cos[2\pi(\nu_1 - \nu_2)t + (\beta - \alpha)]\}. \quad (10)$$

A graphical presentation of the function $\operatorname{sech}(h\nu/2kT_e)$ versus $(h\nu/kT_e)$ is provided in the figure.

2.2. *Direct detection*

The result for the heterodyne case easily reduces to the direct detection (video) case for a coherent signal by setting $|\varepsilon_2^0| = 0$, yielding

$$W_{\text{dir}} \propto |\varepsilon_1^0|^2 \{1 + [\operatorname{sech}(h\nu/2kT_e)][\cos(4\pi\nu_1 t - 2\alpha - \gamma)]\}. \quad (11)$$



The factor $\operatorname{sech}(h\nu/2kT)$ versus $h\nu/kT$. This quantity appears as a coefficient in the absorption/emission interference term.

Thus, double-frequency intensity fluctuations are discerned for electric-field direct detectors, while they are suppressed for photon-absorption direct detectors which respond simply as $|\varepsilon_1^0|^2$.

2.3. *Discussion*

For heterodyne mixing with strong coherent signals in a special two-level detector, a simple argument indicates that double- and sum-frequency terms in the detected photon flux are multiplied by the factor sech $(h\nu/2kT_e)$, which varies smoothly from unity in the electric-field detection regime to zero in the photon-absorption detection regime. This factor depends on both the incident photon energy $h\nu$ and on the effective excitation energy of the (two-level) detector, kT_e.

The photon-absorption detector is, by definition, initially in its ground state and functions by the annihilation of a single (in general non-monochromatic) photon [23]. The presence of the difference-frequency signal is understood to arise from our inability to determine from which of the two constituent beams the single photon is absorbed [24]. The two-level electric-field detector, on the other hand, has equal probability of being in the lower and in the upper state, so that the pure processes of photon emission and photon absorption occur with equal likelihood, and we must add the effects of both. When we are unable to determine which of these processes is occurring, we must add amplitudes rather than squares of amplitudes, thereby allowing interference to occur. Any attempt, in this case, to determine whether photon emission or photon absorption takes place would randomize the phase γ, and thereby wash out the sum- and double-frequency components. In general, then, a photon incident on a video detector induces upward and downward transitions with different probabilities. The detector response than contains a double-frequency signal. For heterodyne detection, in the general case, sum-frequency signals are observed as well.

The foregoing heuristic model yields a simple result for an idealized two-level system. Replacing the operator eq by the non-relativistic hamiltonian $(\hat{p}-e\hat{A})^2/2m$, where $\hat{p}$ and $\hat{A}$ represent the momentum and vector-potential operators, respectively, would allow transitions more general than electric-dipole, and absorptions of more than one quantum, to occur. A rigorous treatment might consider a collection of elemental two-level systems (as a model for a bulk photodetector or metal antenna), in the presence of a surrounding reservoir, and could be carried out using the density matrix formalism. The effects described here appear to be of interest for a broad range of systems, including the Josephson detector [29].

Tucker [30] has carried out a rigorous analysis of quantum-limited detection in tunnel junction mixers. He demonstrated that nonlinear tunneling devices should undergo a transition from energy detectors to photon counters at frequencies where the photon energy becomes comparable to the voltage scale of the d.c. nonlinearity. The discussion presented here provides a physical picture for his result.

3. Recent advances

Laser heterodyning has found application in many areas, including coherent infrared radar [31], fibre-optic communications [32], space-communications [33], spectroscopy [18], and radiometry [18]. The standard semiclassical treatment is entirely satisfactory for describing most of these systems since the coherent-state family of fields (including the global coherent state) is essentially classical in its behaviour [34].

Of current interest is the possibility of beating coherent LO light with squeezed-state light. It is predicted that radiation with this characteristic is generated in certain multiphoton [35–39] and single-photon [40] processes, but such light has not yet been produced in the laboratory (several experimental efforts are currently underway [41–43]). Yuen and Shapiro, and their collaborators, have studied the theoretical properties of a specific kind of squeezed-state light (the two-photon coherent state [44] or ideal squeezed state). They expect that there would be advantages in employing such light in an optical homodyne or heterodyne communication system [45, 46]. One of the principal difficulties, however, is that absorption and diffraction losses reduce the squeezed nature of a light source; indeed, the reduction factor turns out to be the same as that affecting the optical intensity, as Loudon and Shepherd have recently shown [47]. The transmission of squeezed-state light through an optical fibre is subject to these deleterious effects. We mention that squeezed states offer hope in the area of precision measurements, particularly for the detection of gravitational waves [48].

Submillimetre/far-infrared detection is of great importance for astronomy and astrophysics, where there have been great strides in the past decade [49]. It is in this arena, perhaps, as well as in coherent fibre-optic communication systems, that laser heterodyning may be expected to have its greatest impact in the next decade.

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