plex that it has never been successfully used for photomultiplier dynodes.

The "bialkali" (K_2CsSb) photocathode, first described by this author,[2] has apparently never been examined for its secondary emission characteristics although it is widely used as a blue-sensitive cathode of high efficiency. Experiments to measure the secondary emission gain from K_2CsSb have now been performed, and the results indicate that this material has much higher gain factors than Cs_3Sb. Figure 1 shows curves of gain as a function of primary energy for four materials, viz., the conventional Cs_3Sb and S-20 [(Cs)Na_2KSb] cathodes and the new K_2CsSb material both before and after superficial oxidation. The two other conventionally used dynode materials, MgO and BeO (used in the form of oxidized Ag-Mg and Cu-Be alloys, respectively), have even lower gain factors than Cs_3Sb.

In a qualitative way, the higher gain of K_2CsSb, as well as that of the S-20 cathode material, as compared with that of Cs_3Sb is consistent with the photoemissive characteristics of these materials since the peak quantum efficiency of both "multialkali" materials is two or three times greater than that of Cs_3Sb.

The delay in the discovery of the high gain of K_2CsSb has reduced the impact that such an improvement over commercially used dynode materials would have had as recently as three years ago. Simon and Williams,[3] using the new concept of "negative effective electron affinity", demonstrated that heavily p-type doped GaP, after a surface activation with Cs, has extremely high secondary emission gains at high primary energies. Figure 2 shows a comparison of a typical GaP(Cs) dynode with a K_2CsSb dynode. It is apparent that the two materials are quite similar at low primary energies but that the gain of GaP continues to rise almost linearly with primary energy up to much higher values. This, as explained in Ref. 3, is due to the much greater escape depth from materials with negative electron affinity. At present, the GaP(Cs) dynode is used in photomultipliers[4,5] at primary electron energies of approximately 600 V. From Fig. 2 it is apparent that at this voltage the gain of superficially oxidized K_2CsSb is almost equally high.

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On the Signal-to-Noise Ratio for Optical Heterodyne Detection*

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The conventional result for the signal-to-noise ratio in an optical heterodyne detector is shown to apply to the mixing of a stable local oscillator with any radiation field which gives rise to a stationary narrow-band i.f. signal. The result is explicitly demonstrated for radiation scattered from a diffuse target, which may be represented as a narrow-band Gaussian random process.

It has been shown elsewhere[1-3] that the ac response for an optical or ir heterodyne detector may be written as

$$r_{i.f.} \propto \mathcal{L}\{(E_{LO}\cos\omega_{LO}t)(E_S(t)\cos[\omega_S t+\delta(t)])\} \quad (1a)$$

$$=\beta E_{LO}E_S(t)\cos[\omega_{i.f.}t+\delta(t)], \quad (1b)$$

provided that the heterodyne signal is a narrow-band process. Here, $r_{i.f.}$ represents the photodetector response at the intermediate frequency (i.f.), E_{LO} and E_S represent the magnitude of the electric field for the local oscillator (LO) and signal (S) beams, respectively, ω is the angular frequency of the particular radiation beam, δ is a phase angle, and β is a proportionality constant containing the detector quantum efficiency. The operator $\mathcal{L}$ stands for "the low frequency part of", and the quantity $\omega_{i.f.}$ is defined as $|\omega_S-\omega_{LO}|$. If the LO is a well-stabilized single-mode laser operated well above threshold, E_{LO} and ω_{LO} are strictly constant, and the quantities $E_S(t)$ and $\delta(t)$ reflect the amplitude and phase fluctuations of the signal beam. For simplicity, spatial coherence has been assumed over the detector aperture, and the addition of a constant phase factor has been omitted.

To obtain the power signal-to-noise ratio $(S/N)_{power}$ for a narrow-band stationary process such as described by Eq. (1b), we first square and then time average this equation to obtain

$$\langle r_{i.f.}^2\rangle=\tfrac{1}{2}\beta^2E_{LO}^2\{\langle E_S^2(t)\rangle+\langle E_S^2(t)\cos[2\omega_{i.f.}t+2\delta(t)]\rangle\}. \quad (2)$$

Because $E_S(t)$ and $\delta(t)$ are independent random variables, the second term above may be written as

$$\langle E_S^2\cos[2\omega_{i.f.}t+2\delta]\rangle=\langle E_S^2\rangle\langle\cos[2\omega_{i.f.}t+2\delta]\rangle=0. \quad (3)$$

since $\langle\cos[2\omega_{i.f.}t+2\delta]\rangle=0$ for the phase uniformly distributed over $(0, 2\pi)$. Furthermore, for a strong LO, which we assume here, the dc part of the detector response may be written as $r_{dc}\simeq\frac{1}{2}\beta E_{LO}^2$. Thus, Eq. (2) becomes

$$\langle r_{i.f.}^2\rangle=\tfrac{1}{2}\beta^2E_{LO}^2\langle E_S^2\rangle=2\langle E_S^2\rangle r_{dc}^2/\langle E_{LO}^2\rangle, \quad (4)$$

which is also given by

$$\langle r_{i.f.}^2\rangle=2P_Sr_{dc}^2/P_{LO}, \quad (5)$$

where the quantities $P_S\propto\langle E_S^2\rangle$ and $P_{LO}\propto\langle E_{LO}^2\rangle$ represent

the radiation powers of the signal and LO beams, respectively.

If we now consider the noise response in the detector to arise from a simple shot process, the mean-square noise response $\langle r_n^2 \rangle$ is given by the well-known shot noise formula[1,4]

$$\langle r_n^2 \rangle = 2e r_{dc} \Delta f, \tag{6}$$

where Δf is the bandwidth of the receiver. Thus, the $(S/N)_{power}$ may be written as

$$\left(\frac{S}{N}\right)_{power} = \frac{\langle r_{i.f.}^2 \rangle}{\langle r_n^2 \rangle} = \frac{P_S}{e\Delta f} \frac{r_{dc}}{P_{LO}}. \tag{7}$$

However, since r_{dc} arises from the comparitively large LO, it is related to the LO beam power P_{LO} by the detector quantum efficiency η:

$$r_{dc} = (\eta e/h\nu) P_{LO}. \tag{8}$$

Therefore, the signal-to-noise ratio becomes

$$(S/N)_{power} = \eta P_S / h\nu \Delta f, \tag{9}$$

which is the usual result obtained for the mixing of two sinusoidal signals. Thus, the validity of Eq. (9) has been extended to the mixing of a stable (sinusoidal) LO with a narrow-band stationary signal. Actually, the optimum S/N for a detector other than a photoemitter (e.g., a photoconductor) may differ by a constant from this expression.[1,4] Quite recently, van der Ziel[5] has reported an expression for $(S/N)_{power}$ which takes into account the effect of mixing of the LO with the fluctuations in the incoming radiation, and differs from Eq. (9) by the factor $(1+2\eta)^{-1}$. These constant factors apply equally well to sinusoidal mixing and to mixing with a narrow-band signal, however.

One particular narrow-band signal of interest arises from the heterodyning (or homodyning) of radiation scattered from a diffuse target with a stable LO, in which case Eq. (1b) would represent a narrow-band Gaussian process centered at $\omega_{i.f.}$.[1,2] It is not difficult to explicitly show that Eq. (4), and therefore Eq. (9), are valid for this case. Letting $y = r_{i.f.}^2$, the probability density function $p(y)$ is then first-order chi-square and is given by[6]

$$p(y) = [(2\pi y)^{1/2} \beta E_{LO} \sigma_S]^{-1} \exp(-y/2\beta^2 E_{LO}^2 \sigma_S^2). \tag{10}$$

Here σ_S is the variance of the narrow-band Gaussian signal. The mean $\langle y \rangle$ is readily evaluated to be

$$\langle y \rangle = \langle r_{i.f.}^2 \rangle = \beta^2 E_{LO}^2 \sigma_S^2. \tag{11}$$

Since the square of the signal radiation envelope $E_S^2(t)$ is exponentially distributed with a mean $\langle E_S^2 \rangle = 2\sigma_S^2$, we have

$$\langle r_{i.f.}^2 \rangle = \tfrac{1}{2} \beta^2 E_{LO}^2 \langle E_s^2 \rangle, \tag{12}$$

in agreement with Eq. (4).

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