Information Transmission with Photon-Number-Squeezed Light

BAHAA E. A. SALEH, FELLOW, IEEE, AND MALVIN C. TEICH, FELLOW, IEEE,

Invited Paper

Several methods have been proposed for the generation of photon-number-squeezed (sub-Poisson) light by imparting to the photon stream an anticorrelation that regularizes the times of arrival of the photons. This is accomplished by means of control of the excitation or emission process or by feedback, using a copy of the photon point processes in cases the emissions occur in pairs. Possible advantages of communication by photon-number-squeezed light are discussed. For receivers in which the photon arrival times are observed, the channel capacity cannot be improved by modifying an initially Poisson photon stream and making it sub-Poisson. For photon-counting receivers, however, improvement of the channel capacity is possible. The bit-error-rate of an on-off keying communication system using sub-Poisson photons created by introducing anticorrelation into an initially Poisson beam may or may not be smaller than the error rates of the Poisson channel, depending on where the maximum-power constraint is placed.

I. Introduction

The rates at which information can be transmitted using optical beams are governed by random fluctuations (noise). When extraneous noise is eliminated the quantum uncertainties become important, especially at high speeds and high spatial resolutions since fewer photons are available for each bit of information. Fundamental limits on the uncertainty of light have been a subject of considerable and continued interest.

In accordance with the semiclassical theory of light, the optical field is treated classically; it can, therefore, be without uncertainty. The interaction of light with a photodetector, however, generates a stream of photoelectrons with an uncertainty always greater than a certain minimum. Light with nonfluctuating intensity (coherent light) generates photoelectrons described by a Poisson point process. This is characterized by statistically independent events, and a number of counts in any prescribed time interval obeying the Poisson distribution, for which the variance is identically equal to the mean. This is the minimum possible variance. The uncertainty of the corresponding photoelectric current is known as "shot noise." Light with excess intensity fluctuations, such as thermal light, generates photoelectron counts with excess fluctuations; the variance is greater than the mean and the photon counting distribution is said to be super-Poisson. Until recently, Poisson noise (shot noise) has been regarded as an impenetrable noise floor. For all classical light sources, this is indeed the case. All lightwave communication systems that have been developed to date make use of Poisson (or super-Poisson) light.

In the quantum theory of light, however, the two quadratures of the optical field, which represent the real and imaginary parts of the field phasor, are noncommuting operators that satisfy the Heisenberg uncertainty principle, like the position and momentum of a harmonic oscillator. The precision of one quadrature can be enhanced without limit, but this will be accompanied by an unbounded increase of the uncertainty of the other component. Coherent light has quadrature components with equal uncertainties and a minimum uncertainty product. Quadrature-squeezed light is a minimum-uncertainty-product state with nonsymmetric quadrature components, one having uncertainty lower than that of coherent light, and the other has greater uncertainty, as illustrated in Fig. 1(a). Quadrature-squeezed light has been generated in the laboratory using nonlinear optical interactions that provide a simultaneous amplification of one component with an equal reduction of the other [1], [2]. The usefulness of quadrature-squeezed light stems from the possibility of using the precise quadrature to carry information and employing a detection scheme that is insensitive to the uncertain quadrature.

Light can also be described by its amplitude and phase or by the photon number and the phase. Similar uncertainty relations govern the components of this latter pair, so that the two components cannot be simultaneously completely precise. If the uncertainty of the photon number for all (or some) counting times is reduced below that of the coherent state, as illustrated in Fig. 1(b), the light is said to be photon-number squeezed [3]–[5]. The increased precision of the photon number, of course, comes at the expense of an increase of the phase uncertainty. Such light is

Manuscript received December 2, 1990; revised July 5, 1991. This work was supported by the Joint Services Electronics Program through the Columbia Radiation Laboratory and by the National Science Foundation.

B. E. A. Saleh is with the Department of Electrical and Computer Engineering, University of Wisconsin, Madison, WI 53706-1691.

M.C. Teich is with the Department of Electrical Engineering, Columbia University, New York, NY 10027.

IEEE Log Number 9106973.

0018–9219/92$03.00 © 1992 IEEE

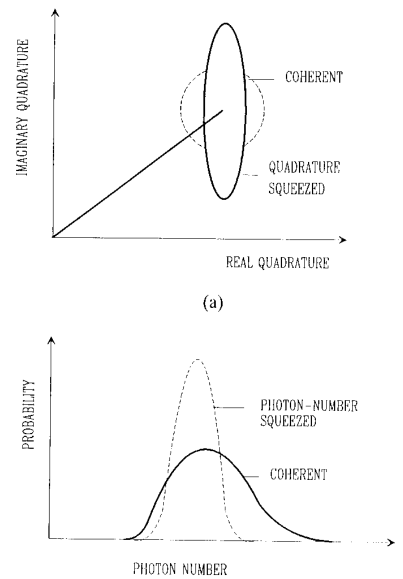


Fig. 1. (a) The circle and the ellipse represent the uncertainties of the quadrature components of coherent and quadrature-squeezed optical fields, respectively. (b) The dashed and solid curves represent the photon-counting distributions for coherent light (Poisson) and photon-number-squeezed (sub-Poisson) light.

intrinsically nonclassical in nature. The photon counts of photon-number-squeezed light have a variance less than the mean, the counting distribution is said to be sub-Poisson, and the photocurrent is sub-shot noise. There is no fundamental limit on squeezing the photon number. The traditional noise floor can, in principle, be lowered to zero. Light in a photon-number-state, for example, has a precise number of photons, but the phase is totally random.

The nature of photon-number-squeezed light has been elucidated in recent years [3]–[16]. This type of light is expected to find use in various disciplines, ranging from optical precision measurement [17] to monitoring the human visual system at the threshold of seeing [18], [19]. In this paper we consider the potential advantages of using photon-number-squeezed light in direct-detection lightwave communication systems and other information-carrying applications [20], [21].

The earliest sources of photon-number-squeezed light exhibited only a slight reduction of the variance [3]–[5]. Far stronger photon-number squeezing has been produced in recent years [22] and continuing advances promise further improvement. It is therefore of interest to examine the advantages to be gained in using photon-number-squeezed light to transmit information.

The channel capacity of optical communication systems has been the subject of a number of studies over the years [8]. These studies have assumed a receiver that observes the number of photon counts in a fixed period of time. The channel capacity of a receiver that observes the times of occurrence of the photoevents (the point process) is evidently greater than or equal to the channel capacity of the photon-counting receiver under the same conditions, because the latter does not record the times of occurrence of the photoevents. In this paper we discuss the channel capacity of both the counting receiver and the point-process receiver for photon-number-squeezed light, assuming direct detection. We also provide an example of the reduction of error probability and improvement of receiver sensitivity of a binary communication system achieved by using photon-number-squeezed light. The channel capacity of systems using quadrature-squeezed light and a homodyne-detection receiver is discussed in [8].

II. Generation of Photon-Number-Squeezed Light

There are essentially three classes of mechanisms by means of which photon-number-squeezed (sub-Poisson) light may be generated.

1) In the first class, sub-Poisson light is produced from a beam of *initially Poisson* (or super-Poisson) *photons*. This can be achieved in a number of ways, e.g., by making use of correlated photon beams [23]. Sub-Poisson photons can be generated from the pairs of correlated photon beams produced in parametric downconversion; one of the twin beams is fed back to control the pump [24] or fed forward to control the other beam.
2) The second class of mechanisms relies on the direct generation of squeezed photons from a beam of *initially sub-Poisson excitations* (e.g., electrons) [4], [5], [25]. This technique was first used in a space-charge-limited version of the Franck–Hertz experiment [4], [5]. Space-charge-smoothing is a well known classical example of a sub-Poisson process. Perhaps the simplest implementation of this principle is achieved by driving a light-emitting diode (LED) with a sub-Poisson electron current [26], [26a] but it is most effectively achieved by the use of a semiconductor injection laser [10], [15], [22].
3) The third method is based on preparing light in a photon-number-squeezed state, in which the uncertainty in the optical phasor shown in Fig. 1(a) is confined in the radial direction. An example is the superposition of light in a quadrature-squeezed vacuum state (a vacuum state with nonsymmetric quadratures) with coherent light of appropriate phase, using a beam splitter. This scheme is used in the homodyne detection of quadrature-squeezed light [1], [2]. Another example is the use of nonlinear interactions in a Kerr medium (which introduces a phase shift proportional to the intensity of the light) [11]. This method cannot be explained in terms of photons described by point processes controlling one another, as in the first and second methods. A quantum mechanical analysis is required to determine the state of the generated light and the statistics of its corresponding photon stream.

The discussion in this paper is generally limited to photon-number-squeezed light generated by the first and second methods and described by the point processes of photon events. Such processes are described by characteristics such as the rate of coincidence of events, and the statistics of the number of counts in prescribed intervals. Light for which the arrival of photon pairs are anticorrelated at small time intervals is termed antibunched light [12], [33]. Photon antibunching was the first form of nonclassical light to be studied in the laboratory. The introduction of photon antibunching may be used to achieve photon-number squeezing (at least for short counting times). The connection between photon-number squeezing and antibunching is subtle [4], [12], [25].

A. Generation of Photon-Number Squeezed Light by Point-Process Control

Consider a point process of excitation and another of emission, as illustrated in Fig. 2. If the excitation process is described by a Poisson point process and each excitation corresponds to an independent emission with some fixed probability, the emissions obey a Poisson point process. The presence of anticorrelations (antibunching) in the excitation process (*excitation control*) results in a more regular stream of photons, exhibiting sub-Poisson behavior. Alternatively, restrictions in the emission mechanism that prohibit, for example, the emission of closely spaced events, could also result in an anticorrelated and more regular photon stream. This mechanism is called *emission control. Feedback control* may also be implemented, whereby the emitted photons control the excitation process or the emission mechanism by imparting anticorrelation. We discuss these three control mechanisms in turn.

1) Excitation Control: Consider the light generated by a collection of atoms excited by inelastic collision with a stream of electrons, as, for example, in the Franck–Hertz experiment illustrated in Fig. 3(a). The electrons are the excitation process and the atoms represent the emission process. Coulomb repulsion can render the electron stream space-charge-limited, so that a regularity is imparted to the electron flow. Such excitation control therefore results in the emission of spontaneous fluorescence photons that are photon-number-squeezed. The space-charge-limited Franck–Hertz experiment in Hg vapor provided the first source of unconditionally photon-number-squeezed light [4], [5].

Unfortunately, the random deletion of photons resulting from imperfect photon collection and detection diminishes the sub-Poisson behavior and these losses account for the small amount of squeezing observed in the Franck–Hertz experiment. Although the photon-number uncertainty can in principle be reduced to zero, the effect is fragile (as is quadrature squeezing), so that loss and the presence of background photons must be assiduously avoided.

To minimize the loss, a number of compact Franck–Hertz-type devices with high-collection-efficiency have been developed. The electrons supplied from a dc source, such

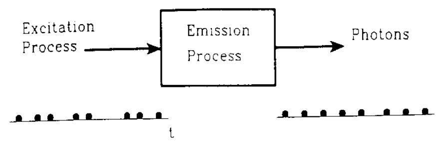


Fig. 2. The components of a photon-generation system: An excitation process, an emission process, and the emitted photons.

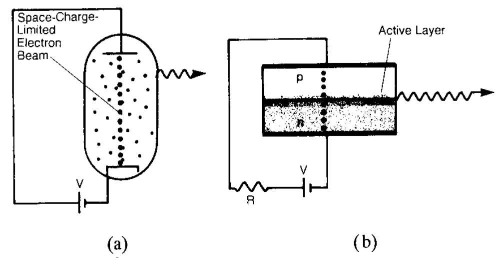


Fig. 3. Photon-number-squeezed light can be generated by means of excitation control. a) The generation of photon-number-squeezed light at a wavelength of 253.7 nm using a space-charge-limited electron beam as the excitation process and the Franck–Hertz effect in Hg vapor as the emission process. (b) A similar scheme in an InGaAsP/InP distributed-feedback semiconductor injection laser. The pump fluctuations are suppressed, so that the electrons and holes (the excitation process) have sub-Poisson distributions. Their recombination in the active region generates 1.56-μm photons with a sub-Poisson distribution. The effect is enhanced by the optical feedback inherent in the laser (after M. C. Teich and B. E. A. Saleh, *Physics Today*, vol. 43, no. 6, 1990).

as a battery, provide a convenient source of sub-Poisson excitations because of their intrinsic Coulomb repulsion (the principal source of noise is Johnson noise). An LED, which ideally emits one photon per injected electron, can serve as the emitter. A solid-state analog of the space-charge-limited Franck–Hertz experiment is therefore provided by a simple LED driven by a constant current source. Indeed, this device has been shown to emit photon-number-squeezed light [26], [26a].

A significant advance was achieved when a semiconductor injection laser diode (ILD) driven by a constant current was used to produce photon-number-squeezed light [10]. This device behaves like a solid-state simulated-emission version of the space-charge-limited Franck–Hertz experiment, as shown in Fig. 3(b). It is compact and produces a large photon flux, and has broadband spectral width and high efficiency. Other semiconductor device structures, employing solid-state space-charge-limited current flow and recombination photons, have also been proposed [12], [16], [49].

2) Emission Control: Several mechanisms can be used to regularize a Poisson sequence of events. Dead time, for example, prohibits a second event from occurring within a fixed time following the occurrence of a given event. It therefore prevents the events from being arbitrarily close to each other and regularizes them. This reduces the uncertainty of the number of events registered in a fixed counting time T. A trigger or firing mechanism that requires a fixed time for resetting between consecutive shots, but is otherwise random, produces a sub-Poisson distribution.

Because isolated atoms subjected to Poisson excitations cannot emit photons during the time they are being re excited, resonance fluorescence emissions are characterized by this description [48]. In the earliest photon-number-squeezed experiments carried out with resonance fluorescence radiation, single atoms could not be isolated, so that the photodetector had to be gated to assure operation with only a single active atom in the apparatus. The resulting light was therefore conditionally photon-number-squeezed [3]. Subsequent experiments were successful in trapping single ions and thus in producing unconditionally photon-number-squeezed resonance-fluorescence radiation.

3) Feedback Control: The control of the excitations or emissions may be derived from the emitted photons themselves by using feedback control. If the arriving photons can be monitored without being destroyed, their arrival times can be used to modify subsequent excitations or emissions. Feedback control of this type can be carried out if the photons are observed by means of a quantum nondemolition (QND) measurement, which allows an observable to be measured without perturbing it. Schemes for implementing QND measurements have been implemented. Feedback control can also be achieved if twin photon streams are available, in which case one of the streams can be annihilated to create the control signal, while the clone stream survives. Configurations of this kind may be useful for generating photon-number-squeezed light with arbitrary photon-number statistics [23].

One suggested photon-feedback configuration makes use of cascaded atomic emissions [34] as portrayed in Fig. 4(a). A Poisson stream of laser-excited Ca^{40} atoms enters the apparatus. Each atom decays by the sequential emission of two photons—one green and one violet. The green photon is detected in a conventional manner to provide a feedback signal. This signal is used to selectively permit some of the violet companion photons to pass through an optical gate. Since the photons are always emitted in correlated pairs, only selected companions survive to produce a sub-Poisson photon stream at the output. The same approach has been implemented by making use of a parametric downconversion experiment [31], [32] as shown in Fig. 4(b). In this case, the feedback signal is used to control the excitation (pump) rather than one of the twin photon beams.

B. Examples of Control Methods

Several examples of the control rule that have been suggested for use in converting the Poisson emissions into sub-Poisson photons are illustrated in Fig. 5 and discussed in the following. It is assumed for simplicity (but without loss of generality) that the various conversions can be achieved in an ideal manner.

1) Dead-Time Deletion: Delete all photons within a prescribed fixed (nonparalyzable) dead time τ_d following the registration of a photon [30]. This approach was implemented to generate photon-number-squeezed light by using one of the twin beams produced in parametric downconversion to selectively gate photons from the other

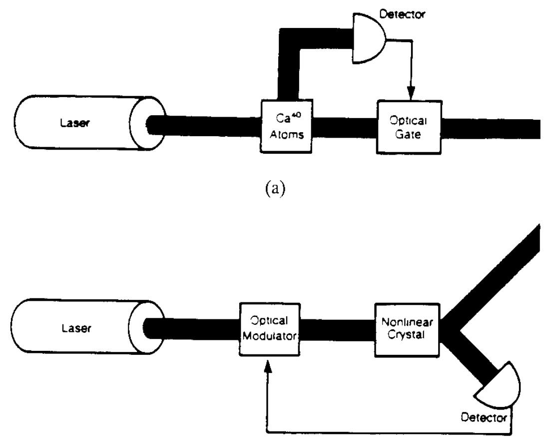


Fig. 4. Feedback control for generating photon-number-squeezed light. (a) The laser excites a beam of Ca^{40} atoms which then decay by the sequential emission of a pair of photons (one green and one violet). The electrical signal produced by the detection of a green photon is fed forward to operate an optical gate that selectively allows certain of the violet companion photons to pass. (b) Parametric downconversion in a nonlinear crystal results in two correlated streams of photons, one of which provides a feedback signal to control the excitations by means of an optical modulator (after M. C. Teich and B. E. A. Saleh, *Physics Today*, vol. 43, no. 6, 1990).

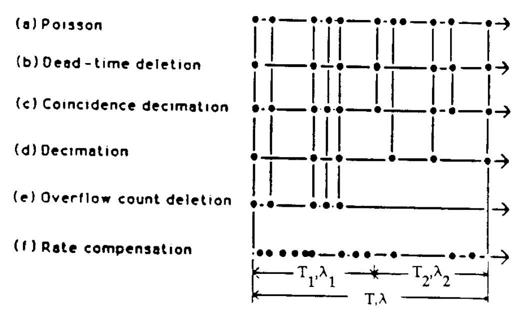


Fig. 5. Several transformations of Poisson photons into sub-Poisson photons that have been suggested for use in quantum optics. (After M. C. Teich and B. E. A. Saleh, in *Progress in Optics*, vol. 26, E. Wolf, Ed., North-Holland, Amsterdam (1988)).

beam via dead-time control [31]. Dead-time deletion could also be used with correlated photon beams produced in other ways.

2) Coincidence Decimation: Remove all pairs of photons separated by a time shorter than a prescribed time interval τ_0. This is achieved, for example, in second-harmonic generation (SHG); two photons closer than the intermediate-state lifetime of the SHG process are exchanged for a third photon (which is at twice the frequency and therefore easily eliminated) [33].

3) Decimation: Select every rth photon ($r = 2, 3, \ldots$) of an initially Poisson photon process, deleting all intermediate photons. In cascaded atomic emissions from Ca^{40}, for example, sequences of correlated photon pairs (green and violet) are emitted. The green photons can be detected and used to operate a gate that passes every rth violet photon

[34]. Decimation control could also be used in conjunction with parametric-downconversion photon twins.

4) Overflow Count Deletion: The number of photons occurring in preselected time intervals $[0, T_0], [T_0, 2T_0], \ldots$, is counted, retaining the first n_0 photons in each time interval (without changing their occurrence times) and deleting the remainder. If the average number of photons in $[0, T_0]$ of the initial process is $>> n_0$, then the transformed process will almost always contain n_0 photons within this time interval. As an example, it was suggested [35] that if a collection of n_0 atoms in the ground state is subjected to a brief, intense, incoherent excitation pulse, all n_0 atoms will become excited with high probability; the radiated optical field would then be describable, to good approximation, by an n_0-photon state. Related schemes [36], [37] have been proposed for use with parametric processes.

5) Rate Compensation: Let each photon registration at time t_i of the control photons cause the rate of the controlled photons to be modulated by a factor $h(t - t_i)$ (which vanishes for $t < t_i$). In linear negative feedback the rate is $\lambda_t = \lambda_0 - \sum_i h(t - t_i)$, where λ_0 is a constant. If the instantaneous photon registration rate happens to be above the average, then it is reduced, and vice versa. This process is schematically illustrated in Fig. 5(f) for two adjacent subintervals T_1 and T_2. The use of rate compensation in conjunction with a QND measurement (using the optical Kerr effect) has been suggested [38], but rate compensation could be used just as well, for example, with correlated photon pairs. Dead-time deletion can be viewed as a special case of rate compensation in which the occurrence of an event zeros the rate of the process for a specified time period τ_d after the registration [23].

III. Communicating with Photon-Number-Squeezed Light and a Point-Process Receiver

The channel capacity of a receiver that observes the times of occurrence of the photoevents (the point process) is evidently greater than or equal to the channel capacity of the photon-counting receiver under the same conditions, because the latter does not record the times of occurrence of the photoevents. Estimates of optical parameters based on the point process of the observed photoevents have been considered in the literature [50]–[52]. In this section we discuss the channel capacity of both the counting receiver and the point-process receiver for photon-number-squeezed light. We demonstrate that the channel capacity of the point-process receiver *cannot* in principle be increased by the use of photon-number-squeezed light [20]. In Section IV we show that the channel capacity of the counting receiver *can* be increased by the use of photon-number-squeezed light [20].

Consider the transmission of information by use of photon-number-squeezed photons generated by modification of a Poisson excitation process or by conversion of an initially Poisson photon stream. The initial Poisson point process has rate μ_t and is represented by the number of events N_t in the time interval $(-\infty, t)$. This process is transformed into a sub-Poisson point process M_t of rate λ_t, in accordance with some deterministic or stochastic rule, as illustrated in Fig. 6. Several examples of transformations of this kind are illustrated in Fig. 15. The events of the initial process N_t are assumed to be observable and their registrations used to operate a mechanism which, in accordance with the specified rule, controls the events of the transformed photon process M_t. The rate λ_t of the process M_t is thereby rendered a function of the realizations of the initial point process N_t at prior times, i.e., $\lambda_t = \lambda_t(N_{t'}; t' \leq t)$. The modified point process describes the photon arrival times of the final photon-number-squeezed beam.

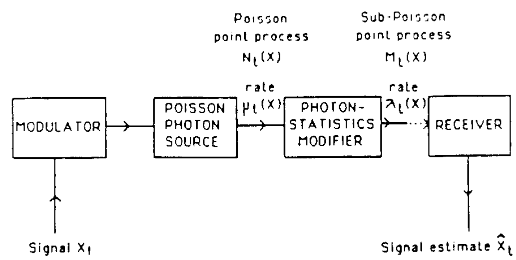


Fig. 6. Idealized lightwave communication system employing a Poisson photon source and a photon-statistics modifier (after M. C. Teich and B. E. A. Saleh, in *Progress Optics*, vol. 26, E. Wolf, Ed., North-Holland, Amsterdam, 1988).

Now consider a communication system in which information is transmitted by modulating the rate μ_t of the initial process N_t with a signal $\hat{X}_t$. The receiver detects the arrival times of the photons of the modified point process M_t and extracts an estimate of the signal X_t. We proceed to show that none of the modification schemes can increase the channel capacity of this communication system above that of a system using simply the initial Poisson point process for communication.

The channel capacity of a communication system using a Poisson point process with a variable rate μ_t controlled by the signal X_t is infinite, if the rate μ is permitted to be increased without bound. If μ_t is constrained to be smaller than a maximum rate $\mu_{\max}$ then the channel capacity is $C = \mu_{\max}/e$. (The base e has been used for simplicity). A key point in determining the channel capacity of the communication system using an initially Poisson point process that is converted into a sub-Poisson process is where the rate constraint is placed.

If a constraint is placed on the rate of the initial Poisson process $\mu_t \leq \mu_{\max}$, then it is obvious that C cannot be increased beyond the value $\mu_{\max}/e$ by any modification $N_t \to M_t$. This is simply a consequence of the definition of channel capacity as the rate of information carried by the system without error, maximized over all coding, modulation, and *modification* schemes. However, can the modification $N_t \to M_t$ increase the channel capacity if the constraint is instead placed on the rate of the modified process λ_t (i.e., $\lambda_t \leq \lambda_{\max}$)?

We address this question by noting that the modified point process M_t is, in general, a self-exciting point process [23], [50] with rate $\lambda_t \ (M_{t'};\, t' \leq t)$. This is a process that contains an inherent feedback mechanism in which present event occurrences are affected by the previous event occurrences of the same point process. Of course, the examples of the modifications $N_t \rightarrow M_t$ introduced in Fig. 5 are special cases of self-exciting point processes.

Now consider a communication system that uses a point process M_t whose rate λ_t is modulated by a signal X_t. The process M_t can be an arbitrary self-exciting point process, including sub-Poisson processes obtained by the feedforward- or feedback-modification of an otherwise Poisson process [23]. Neither feedforward nor feedback transformations can increase the capacity of this channel, as provided by Kabanov's theorem [39] and its extensions [40]–[42]:

Kabanov's Theorem: The capacity of the point-process channel cannot be increased by feedback. Under the constraint $\lambda_0 \leq \lambda_t \leq \lambda_{\max}$, the channel capacity C is

$$C = \lambda_0 \left[\frac{1}{e}\left(1+\frac{s}{\lambda_0}\right)^{1+\lambda_0/s} - \left(1+\frac{\lambda_0}{s}\right)\ln\left(1+\frac{s}{\lambda_0}\right)\right] \tag{1}$$

where $s = \lambda_{\max} - \lambda_0$. When $\lambda_0 = 0$ (no dark counts), this expression reduces to

$$C = \lambda_{\max}/e. \tag{2}$$

When the capacity is achieved, the output of the zero-dark-count point-process channel ($\lambda_0 = 0$) is a Poisson process with rate $\lambda_t = \lambda_{\max}/e$. The channel capacity has also been determined under added constraints on the mean rate. A coding theorem has also been proved. Kabanov's theorem is analogous to the well known result that the capacity of the white Gaussian channel cannot be increased by feedback [43].

In summary, no increase in the channel capacity of a point-process lightwave communication system may be achieved by using photons that are first generated with Poisson statistics and subsequently converted into sub-Poisson statistics, regardless of whether the power constraint is placed at the Poisson photon source or at the output of the conversion process; nor may an increase in channel capacity be achieved by using feedback to generate a self-exciting point process.

We now examine the applicability of the foregoing conclusions to photon-number-squeezed light generated by mixing an initially coherent beam (local oscillator) with squeezed vacuum using a beam splitter. Clearly the wave mixing process is quantum in nature, and does not correspond to a classical (deterministic or stochastic) rule for conversion of the Poisson arrival times of the coherent photons into the arrival times of the squeezed photons. In this mixing process the photons of the coherent beam are actually never observed. The information may be imparted to the coherent field by means of intensity modulation or phase modulation, but the point process of the coherent photons does not directly mediate the observed point process, so that the foregoing ideas are not applicable.

If the coherent beam is intensity modulated by the signal X_t, the information is completely contained in the intensity μ_t, which is assumed not to exceed a maximum value $\mu_{\max}$. If detected with a point-process receiver prior to mixing with the squeezed vacuum, a Poisson point process of rate μ_t would be observed, so that the channel capacity of the system would be $\mu_{\max}/e$. If the coherent light is mixed with the squeezed vacuum and detected, the arrival of photons is described by a self-exciting point process that depends on the rate λ_t, but not on the realizations of its corresponding point process, which are never observed. However, since the observed point process is a self-exciting process of some type, the channel capacity cannot exceed $\lambda_{\max}/e$ where $\lambda_{\max}$ is the maximum rate of the detected superposition. Again, it appears that the use of photon-number-squeezing by mixing with squeezed vacuum cannot improve the channel capacity of a communication system using intensity modulation and a point-process receiver.

IV. Communicating with Photon-Number-Squeezed Light and a Photon-Counting Receiver

The conclusions reached in the previous section are valid only when there are no restrictions on the receiver structure. The conclusion is different if the receiver is operated in the photon-counting regime, in which information is carried by the random variable n representing the number of photoevents registered in time intervals of prescribed duration T (rather than by the photon occurrence times).

A. Channel Capacity

The channel capacity of optical communication systems using counting receivers has been the subject of a number of studies over the years [27]–[29], [8]. The information was often assumed to be carried by light with Poisson statistics in background light with Bose–Einstein photon statistics (thermal radiation). More recently, the channel capacity of a sub-Poisson (photon-number-squeezed) counting system was examined [8].

The capacity of the photon-counting channel is given by [28].

$$C = B[\bar{n}\ln(1+1/\bar{n}) + \ln(1+\bar{n})] \tag{3}$$

where $\bar{n}$ is the mean number of counts in T and $B = 1/T$ is the bandwidth. Two limiting expressions emerge:

$$\begin{aligned} C &= B\bar{n}\ln(1/\bar{n}), & \bar{n} &<< 1 \\ C &= B\ln(\bar{n}), & \bar{n} &>> 1. \end{aligned} \tag{4}$$

If an added condition requires that the photon counts obey the Poisson counting distribution, the capacity is further reduced. In that case, the limiting results analogous to (4) are

$$\begin{aligned} C &= B\bar{n}\ln(1/\bar{n}), & \bar{n} &<< 1 \\ C &= (1/2)B\ln(\bar{n}), & \bar{n} &>> 1. \end{aligned} \tag{5}$$

The capacity in the region $\bar{n} >> 1$ is a factor of 2 smaller in (5) than in (4). The capacity-to-bandwidth ratio C/B is plotted versus $\bar{n}$, for both the unrestricted and Poisson photon-counting channels, in Fig. 7.

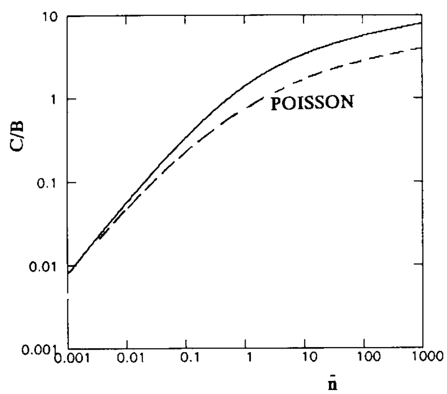


Fig. 7. Comparison of the capacity of the unrestricted photon-counting channel (solid curve) with the Poisson photon-counting channel (dashed curve).

In the case of photon counting, therefore, an increase in the channel capacity *can* in principle be realized by using photon-number-squeezed light. However, in the small mean-count limit $\bar{n} << 1$ (when the counting time T is very short), the capacity of the Poisson counting channel approaches that of the unrestricted counting channel, and the advantage of photon-number squeezing disappears. This is not unexpected in view of the result obtained for the point-process channel.

B. Error Rate of a Binary OOK Photon-Counting Receiver

The channel capacity provides a limit on the maximum rate of error-free information transmission for all codes, modulation formats, and receiver structures [27]–[29]. As such, it does not specify the performance (error probability) achievable by a communication system with prescribed coding, modulation, and receiver structure.

It is therefore of interest to discuss the performance of a system with specified structure. We consider a binary on–off keying (OOK) photon-counting system [21]. The information is transmitted by selecting one of two values for the photon rate λ_t in time slots of (bit) duration T. The receiver operates by counting the number of photons received during the time interval T and then deciding which rate was transmitted in accordance with a likelihood-ratio decision rule (threshold test). For simplicity, it is assumed that background light, dark noise, and thermal noise are absent, so that photon registrations are not permitted when the keying is OFF (i.e., false-alarms are not possible). Furthermore, the detector quantum efficiency is taken to be unity so that system performance is limited only by the quantum fluctuations of the light.

A measure of performance for a digital system such as this is the error probability P_e. In the simplified system described above, errors are possible only when the keying is ON and 0 photons are received (a miss). For a Poisson transmitter, with equal *a priori* probabilities for ON and OFF, P_e is [21]

$$P_e(\text{Poisson}) = (1/2)\exp(-\bar{n}) \tag{6}$$

where $\bar{n}$ denotes the mean number of emitted photons in time T.

The receiver sensitivity S is the mean number of photons per bit necessary to achieve a prescribed probability of error, $P_e = 10^{-9}$ for example. For an OOK optical link using a Poisson photon stream, (6) yields $\bar{n} \approx 20$ so that the mean number of photons per bit $S = \bar{n}/2 \approx 10$ [12].

The receiver sensitivity of the Poisson channel is compared with that obtained for photon-number-squeezed light derived from an initially Poisson source. The outcome will depend on whether the sensitivity is measured at the input or output of the modification process, i.e., on where the mean photon-number constraint is placed. As an example we will use dead-time deletion to demonstrate that the system performance can be enhanced by the use of photon-number-squeezed light, provided that the constraint is applied to the squeezed light. No enhancement of system performance emerges in converting Poisson photons into squeezed photons when the constraint is at the Poisson source.

C. Dead-Time-Modified-Poisson Photon Counts

For a nonparalyzable dead-time modifier that is always *blocked* for a dead-time period τ_d at the beginning of the counting interval T, the passage of 0 photons arises from the emission of 0 photons in the time $T - \tau_d$, independent of the number of emissions during τ_d. The error probability for this system is therefore

$$P_e(\text{dead time}) = (1/2)\exp[-\bar{n}(1 - \tau_d/T)] \tag{7}$$

where $\bar{n}$ is the mean number of photons at the input to the modifier. The mean number of photons at the output of the modifier is reduced to [30], [44]

$$\bar{m} \approx \bar{n}/(1 + \bar{n}\tau_d/T). \tag{8}$$

The receiver sensitivity at the input to the modifier (the value of $\bar{n}/2$ at which $P_e = 10^{-9}$) is, using (7),

$$S_i \approx 10/(1 - \tau_d/T)$$

whereas the receiver sensitivity at the output of the modifier

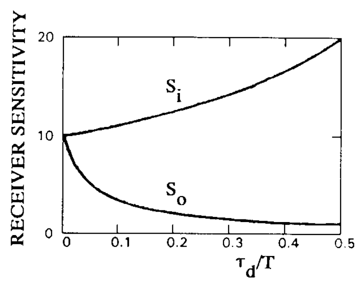


Fig. 8. The receiver sensitivity (mean number of photons necessary to achieve a probability of error $P_e = 10^{-9}$) for dead-time-modified-Poisson light measured at the input of the modifier (S_i) and at the output of the modifier (S_o), as a function of the ratio of the dead time to the counting time τ_d/T. The well-known "quantum limit" (10 photons/bit) emerges as $\tau_d/T \to 0$.

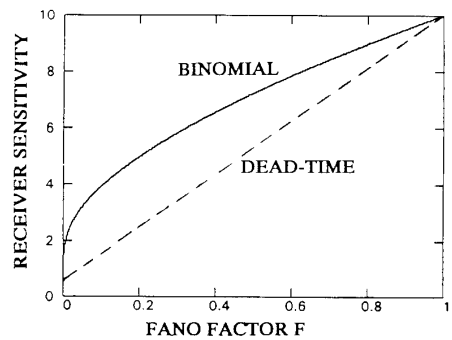


Fig. 9. Receiver sensitivity S as a function of the Fano factor F for the binomial channel (solid) and for the dead-time-modified Poisson channel (dashed). The quantum limit of 10 photons/bit emerges as $F \to 1$.

(the value of $\bar{m}/2$ at which $P_e = 10^{-9}$) is, using (8) and (9),

$$S_o \approx 10/(1 + 19\tau_d/T).$$

These sensitivities are plotted in Fig. 8 as a function of τ_d/T. If $\tau_d/T = 0.1$, for example, a minimum of 11.1 photons (instead of 10) are necessary at the input to the modifier; but only 3.45 photons are needed at the output of the modifier. Thus the sensitivity is enhanced or reduced, depending on where it is measured.

D. Receiver Sensitivity of the Photon-Number-Squeezed Link

It is instructive to examine the relation between the receiver sensitivity S and the Fano factor F, which is the ratio of the variance to the mean. The Fano factor is a measure of the degree of photon-number squeezing ($F = 1$ for Poisson light and < 1 for sub-Poisson light). Since the relation between S and F depends on the counting distribution, we illustrate this relation with two examples, the binomial distribution and the dead-time-modified Poisson distribution.

Consider light with photons of a binomial counting distribution,

$$p(n) = \binom{N}{n} \eta^n (1-\eta)^{N-n}.$$

This light is obtained when an ideal source that generates a deterministic photon number N suffers loss, resulting in the random deletion of each photon with probability $(1-\eta)$ [45]. Random photon deletion is inevitable; it results from absorption, scattering, and the finite quantum efficiency of the detector [12]. The binomial distribution is sub-Poisson with mean $\langle n \rangle = \eta N$, variance $\eta(1-\eta)N$, and Fano factor $F = (1-\eta) \leq 1$. It has been shown that the information rate per symbol carried by such a counting channel will be greater than that for the Poisson channel, but will approach the latter as $\eta \to 0$ [35]. A source that emits a binomial number at the outset [46] retains its binomial form, but exhibits reduced mean in the presence of random deletion [45].

The performance of such a binary OOK photon-counting receiver, in the absence of background, is limited by the binomial fluctuations of the detected photons. In this case, it is easily shown from the binomial distribution that [12], [47]

$$P_e = (1/2)F^{\langle n \rangle/(1-F)}. \tag{12}$$

Solving (12) for the receiver sensitivity $S = \langle n \rangle/2$ with $P_e = 10^{-9}$ yields

$$S \approx 10\frac{1-F}{\ln(1/F)}. \tag{13}$$

The receiver sensitivity S versus the Fano factor F is illustrated by the solid curve in Fig. 9.

A similar analysis for the sensitivity of the dead-time-modified-Poisson photon-counting distribution measured at the output of the modifier yields the dashed curve in Fig. 9. In both cases, the Poisson limit of $S \approx 10$ photons per bit emerges in the limit $F = 1$ where these distributions go over to the Poisson. Fewer photons are needed to achieve the same error probability when F is smaller than unity. For example, $F = 0.5$ corresponds to a receiver sensitivity of 7.22 photons per bit for the binomial case and 5.25 photons per bit for the dead-time case.

Under idealized conditions, a distribution for which $p(0) = 0$ yields zero probability of error and can operate with a mean number of photons close to 1/2 (i.e., an average close to one photon in the ON bit). To reduce the error rates of binary optical communications, therefore, a counting distribution must be found with the smallest $p(0)$, instead of a distribution with the smallest variance (or Fano factor). In general, one should seek to squeeze the tails of the probability distribution, which are better described by parameters such as the fourth-order cumulants.

REFERENCES

[1] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, "Observation of squeezed states generated by four-wave mixing in an optical cavity," *Phys. Rev. Lett.*, vol. 55, pp. 2409–2412, 1985.

[2] L. Wu, H. J. Kimble, J. L. Hall, and H. Wu, "Generation of squeezed states by parametric downconversion," *Phys. Rev. Lett.*, vol. 57, pp. 2520–2524, 1986.

[3] R. Short and L. Mandel, "Observation of sub-Poissoian photon statistics," *Phys. Rev. Lett.*, vol. 51, pp. 384–387, 1983.

[4] M. C. Teich, B. E. A. Saleh, and D. Stoler, "Antibunching in the Franck–Hertz experiment," *Opt. Commun.*, vol. 46, pp. 244–248, 1983.

[5] M. C. Teich and B. E. A. Saleh, "Observation of sub-Poisson Franck-Hertz light at 253.7 nm," *J. Opt. Soc. Amer. B*, vol. 2, pp. 275–282, 1985.

[6] D. F. Smirnov and A. S. Troshin, "Sub-Poisson statistics of photons in laser emission," *Opt. Spectrosc.*, vol. 59, pp. 1–3, 1985.

[7] ——, "New phenomena in quantum optics: Photon antibunching, sub-Poisson photon statistics, and squeezed states," *Sov. Phys.–Usp.*, vol. 30, pp. 851–874, 1987.

[8] Y. Yamamoto and H. A. Haus, "Preparation, measurement and information capacity of optical quantum states," *Rev. Mod. Phys.*, vol. 58, pp. 1001–1020, 1986.

[9] J. E. Carroll, "A proposal for quiet photon streams from injection lasers," *Opt. Acta (J. Mod. Opt.)*, vol. 33, pp. 909–918, 1986.

[10] S. Machida, Y. Yamamoto, and Y. Itaya, "Observation of amplitude squeezing in a constant-current-driven semiconductor laser," *Phys. Rev. Lett.*, vol. 58, pp. 1000–1003, 1987.

[11] Y. Yamamoto, S. Machida, N. Imoto, M. Kitagawa, and G. Björk, "Generation of number-phase minimum-uncertainty states and number states," *J. Opt. Soc. Amer. B*, vol. 4, pp. 1645–1662, 1987.

[12] M. C. Teich and B. E. A. Saleh, "Photon bunching and antibunching," in *Progress in Optics*, vol. 26. E. Wolf, Ed. Amsterdam, The Netherlands: North-Holland, 1988, pp. 1–104.

[13] M. C. Teich and B. E. A. Saleh, "Squeezed states of light," *Quantum Optics*, vol. 1, pp. 153–191, 1989.

[14] M. C. Teich and B. E. A. Saleh, "Squeezed and antibunched light," *Physics Today*, vol. 43, no. 6, pp. 26–34, 1990.

[15] Y. Yamamoto, S. Machida, and O. Nilsson, "Squeezed-state generation by semiconductor lasers," in *Coherence, Amplification and Quantum Effects in Semiconductor Lasers*, Y. Yamamoto, Ed. New York: Wiley, 1991, pp. 461–537.

[16] M. C. Teich, B. E. A. Saleh, and F. Capasso, "Generation of photon-number-squeezed light by semiconductor incoherent light sources," in *Coherence, Amplification and Quantum Effects in Semiconductor Lasers*, Y. Yamamoto, Ed. New York: Wiley, 1991, pp. 539–560.

[17] E. Jakeman and J. G. Rarity, "The use of pair production processes to reduce quantum noise in transmission measurements," *Opt. Commun.*, vol. 59, pp. 219–223, 1986.

[18] M. C. Teich, P. R. Prucnal, G. Vannucci, M. E. Breton, and W. J. McGill, "Multiplication noise in the human visual system at threshold: 3. The role of non-Poisson quantum fluctuations," *Biol. Cybern.*, vol. 44, pp. 157–165, 1982.

[19] B. E. A. Saleh and M. C. Teich, "Multiplication and refractoriness in the cat's retinal-ganglion-cell discharge at low light levels," *Biol. Cybern.*, vol. 52, pp. 101–107, 1985.

[20] ——, "Can the channel capacity of a light-wave communication system be increased by the use of photon-number-squeezed light?" *Phys. Rev. Lett.*, vol. 58, pp. 2656–2659, 1987.

[21] M. C. Teich and B. E. A. Saleh, "Information transmission with squeezed light," in *Quantum Aspects of Optical Communications*, C. Bendjaballah, O. Hirota, and S. Reynaud, Eds. Berlin, Germany: Springer, 1991, pp. 261–275.

[22] W. H. Richardson, S. Machida, and Y. Yamamoto, "Squeezed photon-number noise and sub-Poissonian electrical partition noise in a semiconductor laser," *Phys. Rev. Lett.*, vol. 66, pp. 2867–2870, 1991.

[23] J. H. Shapiro, G. Saplakoglu, S.-T. Ho, P. Kumar, B. E. A. Saleh, and M. C. Teich, "Theory of light detection in the presence of feedback," *J. Opt. Soc. Amer. B*, vol. 4, pp. 1604–1619, 1987.

[24] P. R. Tapster, J. G. Rarity, and J. S. Satchell, "Use of parametric down-conversion to generate sub-Poissonian light," *Phys. Rev. A*, vol. 37, pp. 2963–2967, 1988.

[25] M. C. Teich, B. E. A. Saleh, and J. Peřina, "Role of primary excitation statistics in the generation of antibunched and sub-Poisson light," *J. Opt. Soc. Amer. B*, vol. 1, pp. 366–389, 1984.

[26] P. R. Tapster, J. G. Rarity, and J. S. Satchell, "Generation of sub-Poissonian light by high-efficiency light-emitting diodes," *Europhys. Lett.*, vol. 4, pp. 293–299, 1987.

[26a] P. J. Edwards, "Sub-Poisson light from GaAlAs infrared emitting diodes," *Int. J. Optoelectron.*, vol. 6, pp. 23–28, 1991.

[27] T. E. Stern, "Some quantum effects in information channels," *IRE Trans. Inform. Theory*, vol. IT-6, pp. 435–440, 1960.

[28] J. P. Gordon, "Quantum effects in communications systems," *Proc. IRE*, vol. 50, pp. 1898–1908, 1962.

[29] J. R. Pierce, E. C. Posner, and E. R. Rodemich, "The capacity of the photon counting channel," *IEEE Trans. Inform. Theory*, vol. IT-27, pp. 61–77, 1981.

[30] M. C. Teich and G. Vannucci, "Observation of dead-time-modified photocounting distributions for modulated laser radiation," *J. Opt. Soc. Amer.*, vol. 68, pp. 1338–1342, 1978.

[31] J. G. Rarity, P. R. Tapster, and E. Jakeman, "Observation of sub-Poissonian light in parametric downconversion," *Opt. Commun.*, vol. 62, pp. 201–206, 1987.

[32] A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Camy, "Observation of quantum noise reduction on twin laser beams," *Phys. Rev. Lett.*, vol. 59, pp. 2555–2557, 1987.

[33] R. Loudon, *The Quantum Theory of Light*, 2nd ed. Oxford, U.K.: Clarendon, 1983.

[34] B. E. A. Saleh and M. C. Teich, "Sub-Poisson light generation by selective deletion from cascaded atomic emissions," *Opt. Commun.*, vol. 52, pp. 429–432, 1985.

[35] L. Mandel, "Ideal light source for an optical communication channel," *J. Opt. Soc. Amer.*, vol. 66, pp. 968–970, 1976.

[36] H. P. Yuen, "Generation, detection, and application of high-intensity photon-number-eigenstate fields," *Phys. Rev. Lett.*, vol. 56, pp. 2176–2179, 1986.

[37] D. Stoler and B. Yurke, "Generating antibunched light from the output of a nondegenerate frequency converter," *Phys. Rev. A*, vol. 34, pp. 3143–3147, 1986.

[38] Y. Yamamoto, N. Imoto, and S. Machida, "Amplitude squeezing in a semiconductor laser using quantum nondemolition measurement and negative feedback," *Phys. Rev. A*, vol. 33, pp. 3243–3261, 1986.

[39] Yu. M. Kabanov, "The capacity of a channel of the Poisson type," *Theory Prob. Appl.*, vol. 23, pp. 143–147, 1978.

[40] M. H. A. Davis, "Capacity and cutoff rate for Poisson-type channels," *IEEE Trans. Inform. Theory*, vol. IT-26, pp. 710–715, 1980.

[41] A. A. Lazar, "On the capacity of the Poisson type channel," in *Proc. 14th Annual Conf. Information Sciences and Systems*, Princeton, NJ, Mar. 26–28, 1980.

[42] P. Brémaud, "A coding theorem for continuous-time point-process channels," Sept. 1983, unpublished.

[43] T. T. Kadota, M. Zakai, and I. Ziv, "Mutual information of the white Gaussian channel with and without feedback," *IEEE Trans. Inform. Theory*, vol. IT-17, pp. 368–371, 1971.

[44] M. C. Teich and B. I. Cantor, "Information, error, and imaging in deadtime-perturbed doubly stochastic Poisson counting systems," *IEEE J. Quantum Electron.*, vol. QE-14, pp. 993–1003, 1978.

[45] M. C. Teich and B. E. A. Saleh, "Effects of random deletion and additive noise on bunched and antibunched photon-counting statistics," *Opt. Lett.*, vol. 7, pp. 365–367, 1982.

[46] D. Stoler, B. E. A. Saleh, and M. C. Teich, "Binomial states of the quantized radiation field," *Opt. Acta (J. Mod. Opt.)*, vol. 32, pp. 345–355, 1985.

[47] K. Yamazaki, O. Hirota, and M. Nakagawa, "Optimum condition of squeezed state signal for optical communication with energy loss," *Trans. IEICE (Japan)*, vol. 71, pp. 775–783, 1988.

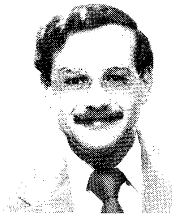
[48] H. J. Kimble, M. Dagenais, and L. Mandel, "Photon antibunching in resonance fluorescence," *Phys. Rev. Lett.*, vol. 39, pp. 691–695, 1977.

[49] M. C. Teich, F. Capasso, and B. E. A. Saleh, "Photon-number-squeezed recombination radiation in semiconductors," *J. Opt. Soc. Amer. B*, vol. 4, pp. 1663–1666, 1987.

[50] R. L. Snyder, *Random Point Processes.* New York: Wiley, 1975.

[51] B. E. A. Saleh, "Estimations based on instants of occurrence of photon counts of low level light," *Proc. IEEE*, vol. 62, pp. 530–531, 1974.

[52] ——, "Joint probability of instants of occurrence of photon events and estimation of optical parameters," *J. Phys. A*, vol. 7, pp. 1360–1368, 1974.



Bahaa E. A. Saleh (Fellow, IEEE) received the B.S. degree from Cairo University, Cairo, Egypt, in 1966 and the Ph.D. degree from the Johns Hopkins University, Baltimore, MD, in 1971, both in electrical engineering.

From 1971 to 1974 he was an Assistant Professor at the University of Santa Catarina, Brazil. Thereafter, he joined the Max Planck Institute in Göttingen, Germany, where he was involved in research in laser light scattering and photon correlation spectroscopy. He is presently Professor and Chairman of the Department of Electrical and Computer Engineering at the University of Wisconsin, Madison, where he has been since 1977. He held visiting appointments at the University of California, Berkeley, in 1977, and the Columbia Radiation Laboratory of Columbia University, New York, in 1983. He is currently involved in research in image processing, optical information processing, statistical optics, optical communication, and vision. He is the author of *Photoelectron Statistics* (Springer, 1978), and the coauthor of *Fundamentals of Photonics* (Wiley, 1991). He is currently the Editor of the *Journal of the Optical Society of America A* and a member of the Editorial Board of *Quantum Optics*.

Dr. Saleh is a Fellow of the Optical Society of America and the John Simon Guggenheim Foundation, and a member of Phi Beta Kappa and Sigma Xi.

Malvin C. Teich (Fellow, IEEE) was born in New York City. He received the S.B. degree in physics from the Massachusetts Institute of Technology, Cambridge, MA, in 1961, the M.S. degree in electrical engineering from Stanford University, Stanford, CA, in 1962, and the Ph.D. degree in quantum electronics from Cornell University, Ithaca, NY, in 1966.

After receiving the doctorate, he joined MIT Lincoln Laboratory, Lexington, MA, where he was engaged in work on coherent infrared detection. In 1967, he became a member of the faculty in the Department of Electrical Engineering, Columbia University, New York, where he is now teaching and pursuing his research interests in the areas of quantum optics, optical and infrared detection, and sensory perception. He served as Chairman of the Department from 1978 to 1980. He is also a member of the faculty in the Department of Applied Physics, and a member of the Columbia Radiation Laboratory and the Center for Telecommunications Research. He has authored or coauthored some 175 technical publications and holds one patent. He is the coauthor of *Fundamentals of Photonics* (Wiley, 1991).

Dr. Teich was the recipient of the IEEE Browder J. Thompson Memorial Prize Award for his paper "Infrared Heterodyne Detection" in 1969, and in 1981 he received the Citation Classic Award of the Institute for Scientific Information for this work. He was awarded a Guggenheim Fellowship in 1973. He is a Fellow of the American Physical Society, the Optical Society of America, and the American Association for the Advancement of Science. He is a member of Sigma Xi, Tau Beta Pi, the Acoustical Society of America, the Association for Research in Otolaryngology, and the New York Academy of Sciences. He served as a member of the Editorial Advisory Panel for the journal *Optics Letters* from 1977 to 1979. He is currently a Member of the Editorial Board of the *Journal of Visual Communication and Image Technology* and Deputy Editor of the *Journal of the European Optical Society B: Quantum Optics*.