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Homodyne Detection of Infrared Radiation from a Moving Diffuse Target

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Abstract—Experiments have been performed in which the radiation from a CO_2 laser was coherently detected after being scattered from a moving diffuse reflector. This is generally the configuration of an infrared laser radar. The power-spectral-density of the heterodyne signal was measured and its width was shown to agree with the calculated value based on a theoretical model for the process. Expressions are obtained for the ratio of heterodyne signal bandwidth to heterodyne frequency for the cases of focused radiation, unfocused radiation, and for a typical radar configuration. In most cases, the heterodyne signal is found to possess a narrow-band character. The probability density of the signal envelope was also measured for a known scatterer (providing Gaussian scattered radiation) and was found to be Rayleigh distributed, as expected. The power-spectral-density and envelope probability distribution provide information about a scattering medium or target which cannot be obtained from average-value measurements of the heterodyne signal-to-noise ratio. This information is useful for communications applications, infrared radar, and heterodyne spectroscopy experiments. Finally, a simple and direct method of obtaining information about the statistics of an incident radiation field (which does not involve photocounting) is discussed.

I. Introduction

COHERENT detection in the infrared and optical has received much attention over the past several years [1]–[7]. The technique is particularly useful at 10.6 μm (in the middle infrared) because of the high radiation power available from the CO_2 laser and because of the 8 to 14 μm atmospheric window [1], [2]. A quantum theory of photomixing [8] has recently been developed, and it has been shown that optimum sinusoidal heterodyne detection occurs for a first-order coherent total incident field with stationary constituent beams. The primary difference between this theory and the previous classical treatment is that double- and sum-frequency components of the photoelectric signal do not appear when $h\nu \gg kT$, where T is the temperature of the detector. Infrared homodyne[1] experiments have been performed at 10.6 μm using $Pb_{1-x}Sn_xSe$ photovoltaic and Ge:Cu photoconductive detectors. Agreement with the theoretical predictions for signal-to-noise ratio and minimum detectable power for near-optimum receiver operation was quite good [1], [2].

All of these previously reported measurements, however, have been concerned with a mean detection rate or a time-averaged value of the signal-to-noise ratio. They are, therefore, related only to the first-order coherence properties of the incident radiation [8]. Information other than average count rates, such as the spectral distribution of the mixing signal or the probability density of its envelope are of interest and may also be examined experimentally. Quantities such as these may be shown to depend on correlation

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[1] Only a single radiation source (CO_2 laser) is used in the experiments described here so that the terms homodyne and heterodyne are used interchangeably throughout this paper.

functions [9] of the radiation field higher than first order, however. Freed and Haus [10], for example, have related the power-spectral-density of the photocurrent for a direct (nonheterodyne) detector to the second factorial moment of the photocounting distribution, and thus to a second-order correlation function of the radiation field. The spectral-density for the photomixing signal has been considered by Forrester [11] who obtained an expression for this quantity in terms of the spectral-densities of the light beams, for the case of Gaussian radiation. Mandel [12] has considered the mixing of two independent *laser* modes and has arrived at an expression similar to that given by Forrester. The spectrum for the photomixing experiment, in contrast to that for direct detection however, is not *strongly* dependent on the higher order coherence properties of the individual sources.

In this paper, we experimentally investigate the fluctuation properties of a homodyne signal arising from infrared radiation scattered from a moving diffuse metallic surface. This is generally the configuration of an infrared laser radar [13]. In particular, we investigate the power-spectral-density of the heterodyne signal and the probability density of its envelope when the radiation source is fully coherent, i.e., a single-mode stabilized laser operated well above threshold. The parameters we study provide direct information about a target, such as its velocity and its statistical nature. They are also useful in the optimum processing or transmission to a distant point of the heterodyne signal. As a simple example, the signal amplifier should be designed for minimum noise figure and its bandwidth chosen sufficiently large to pass the heterodyne signal. Such design will, in general, depend on both the fluctuation and spectral properties of the input signal. Information about the nature of the scattering medium may also be obtained from careful examination of the details of the homodyne signal. This is the basis of the use of the technique for heterodyne spectroscopy. For example, the homodyne return from a moving puff of steam [2], [13] is considerably broader in frequency than the return from a diffusely reflecting moving metallic target. This results, of course, from the large velocity spread of the constituent water molecules. Further information may be obtained, in a similar way, by studying the probability density of the homodyne signal or its envelope. These quantities are much more strongly dependent on the higher order correlation functions of the radiation field than is the photomixing spectrum. In fact, the electric field probability distribution of an unknown radiation source may be determined by the heterodyne mixing of this source with a stable local oscillator, as will be shown.

We first proceed to describe the details of the experimental arrangement to measure these parameters, and then present our results for the power-spectral-density and probability distribution of the envelope of the homodyne signal.

II. Experimental Arrangement

The experimental arrangement for infrared homodyne measurements with a photovoltaic lead-tin selenide detector is illustrated in Fig. 1. The radiation from a CO_2-N_2-He laser, emitting approximately 10 watts at 10.6 μm, was incident on a modified Michelson interferometer. One mirror of the conventional interferometer was replaced by an off-center rotating (3600 r/min) aluminum wheel with a roughened surface. The granularity of the wheel was of the order of 10 μm, the wavelength of the radiation. The diffusely scattered radiation from the wheel provided a Doppler-shifted signal which was recombined at the beam splitter with the unshifted local oscillator (LO) radiation reflected from the mirror of the other interferometer leg.

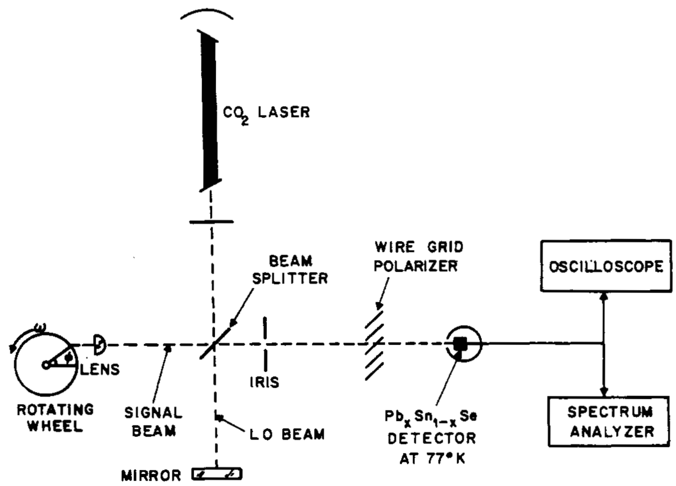

Fig. 1. Experimental arrangement for infrared heterodyne measurements with a lead-tin selenide photovoltaic detector and a sandblasted aluminum scattering wheel.

The experimental setup, with the exception of the rotating wheel, was mounted on a heavy table supported by compressed fiberglass blocks. To further minimize the effect of acoustic vibrations, the 1.25-meter-long Brewster-window laser tube was enclosed in a shield constructed of acoustic tile. The laser was operated well above threshold and was tuned to operate on a single line and mode. An uncoated Kodak Irtran II flat (of thickness 0.64 cm) served as a beam splitter, and front surface mirrors were of standard aluminum-coated glass. For most of the experiments, a 2.54-cm-focal-length Irtran II lens inserted in the signal beam focused the radiation to a single point on the rim of the rotating wheel. One function of the lens was to insure spatial coherence of the scattered radiation at the detector. An iris and a stop in front of the detector were used to maintain the angular alignment of the wave fronts of the two beams to ~2 mrad, which is well within the required angular tolerance for optimum photomixing [6] ($\lambda/a \simeq 5$ mrad for a detector aperture $a = 2$ mm). A Perkin-Elmer wire-grid polarizer insured that the recombined beams had a common linear polarization.

The $Pb_{1-x}Sn_xSe$ diode used as a heterodyne detector was fabricated from a Bridgman-grown crystal by Melngailis and by Calawa *et al.* [14]–[16]. The bandgap of these diffused p-n junction devices varies with composition (x) so that the wavelength for peak responsivity may be adjusted by varying x. The device used achieved its maximum responsivity ($\simeq 1$ V/W, 77 K) at the CO_2 laser wavelength, and had the composition $Pb_{0.936}Sn_{0.064}Se$. The diode had a 1-mm-diameter active area and was operated in the photovoltaic mode. The quantum efficiency of the detector was determined by direct measurement to be 8.5 percent. Its zero-current impedance, which was essentially independent of the presence of the LO, was about 1.5 ohms. The radiation impinged on the thin ($\simeq 10$ μm) n-type layer.[2]

The output of the detector was fed into a Tektronix type 585A oscilloscope for the probability density measurements, and into a Panoramic type SPA-3a spectrum analyzer for the power-spectral-density measurements. In distinction to experiments designed to measure signal-to-noise ratios [1], the heterodyne signal was displayed without amplification. In these experiments, the LO power was maintained at a level of approximately 18 mW, while the signal beam radiation power was sufficient ($\gtrsim 10^{-7}$ watts) to provide a very high signal-to-noise ratio. The heterodyne signal was centered at about 2.0 MHz and had a mean voltage level of about 0.03 volt.

III. Power-Spectral-Density of the Heterodyne Signal

The time trace of a typical heterodyne signal and its envelope, obtained from an oscilloscope photograph, is represented in Fig. 2. It has the appearance of a narrow-band random process, i.e.,

$$B/\nu_D < 1, \tag{1}$$

where B is the heterodyne signal frequency bandwidth, and ν_D is the heterodyne or Doppler frequency. The quantities B and B/ν_D are easily calculated for three cases: 1) focused radiation on the rotating wheel, 2) unfocused radiation on the rotating wheel, and 3) a typical radar experiment.

For a radiation spot of diameter d on the wheel, a completely new area of the wheel is illuminated every $d/v_\perp$ seconds, giving scattered radiation which we assume to be uncorrelated with that of the previous time interval.[3] Thus, in this case, the heterodyne signal frequency bandwidth may be written as

$$B \simeq v_\perp/d. \tag{2}$$

The quantity $v_\perp$ represents the wheel velocity perpendicular to the beam axis and is equal to v cos ϕ, where v is the tangential velocity of the wheel and ϕ is the central angle shown in Fig. 1. The relationship for the tangential wheel velocity ($v = r\omega$) may be combined with the lens equation for *focused* radiation $d \simeq F\lambda/D$ (ω is the angular velocity of the wheel, r its radius, F the focal length of the focusing lens, and D the radiation beam diameter incident on the lens) to yield a heterodyne signal bandwidth B_{foc} given by

$$B_{\text{foc}} \simeq (r\omega D/F\lambda) \cos \phi. \tag{3}$$

For any reasonable value of ϕ, the contribution to the bandwidth arising from the finite spot size on the scattering wheel may be neglected in this case.

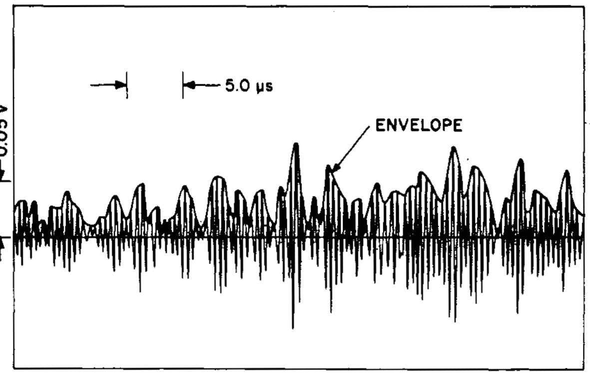

Fig. 2. Time trace of a typical heterodyne signal and its envelope.

The Doppler frequency ν_D, as is well known, is given by the relationship

$$\nu_D = 2v_\parallel/\lambda = (2v/\lambda) \sin \phi \tag{4}$$

where $v_\parallel$ is the wheel velocity component parallel to the radiation beam axis. The ratio of bandwidth to heterodyne frequency B_{foc}/ν_D may then be written

$$B_{\text{foc}}/\nu_D \simeq (D/2F) \cot \phi. \tag{5}$$

This ratio depends only on geometrical factors, as has been pointed out previously [1]. For moderate values of ϕ, this quantity will be less than or of the order of unity in most cases, although it may be seen that the narrow-band nature of the signal will be destroyed for sufficiently small values of ϕ.

For the case of *unfocused* radiation and the rotating wheel, there are two individual contributions to the finite bandwidth: 1) the $v_\perp/d$ component as in the last case, and 2) the contribution arising from the spread in Doppler frequencies over the finite spot size on the wheel. We denote this latter quantity by $\Delta\nu_D$. From (4), is it easily seen that

$$\Delta\nu_D = (2v\Delta\phi/\lambda) \cos \phi \tag{6}$$

for the usual case of $\Delta\phi \ll 1$ and thus

$$\Delta\nu_D/\nu_D \simeq \cot \phi \Delta\phi. \tag{7}$$

But, since $\Delta\phi$ is given by the relation $\Delta\phi \simeq d/(r \cos \phi)$, we obtain

$$\Delta\nu_D/\nu_D \simeq (d/r) \csc \phi. \tag{8}$$

The $v_\perp/d$ contribution is easily seen to be

$$\frac{v_\perp/d}{\nu_D} = (\lambda/2d) \cot \phi, \tag{9}$$

so that the total fractional frequency spread B_{unfoc}/ν_D may be written as

$$B_{\text{unfoc}}/\nu_D \simeq \left[(d/r)^2 \csc^2 \phi + (\lambda/2d)^2 \cot^2 \phi\right]^{1/2}. \tag{10}$$

[2] $Pb_{1-x}Sn_xSe$ and $Pb_{1-y}Sn_yTe$ detectors with considerably superior characteristics have since been fabricated (see [14]).

[3] It should be pointed out that only the uncorrelated case is considered here, which is equivalent to taking an infinite variance for the surface roughness distribution. This model could be modified (to include a time-dependent mean and a finite variance) in order to allow for a determination of the target's mean path, or small-scale surface shape, which might be possible in some applications. I would like to thank one of the reviewers of this paper for suggesting such an addition.

For most situations, this quantity will be smaller than unity for moderate values of ϕ (e.g., for $d/r \sim 0.1$, and $\lambda \ll d$, $B_{unfoc}/\nu_D < 1$ provided only $\phi > 5°$), so that the signal will usually possess a narrow-band character in this case as well.

We now consider the return from an infrared *radar* beam tracking a moving solid target. If it is assumed that the beam size d is of the order of the target size, then the frequency broadening arising from the target's diffuse nature will be negligible. But, in analogy with the previous case, there will be a contribution arising from the *spin* or *rotation* of the target about an axis perpendicular to the beam direction, which gives rise to a Doppler frequency spread. In this case, then, the center frequency of the mixing signal is given by $\nu_D = 2v_{\parallel}/\lambda$, where now $v_{\parallel} = v_r$ is the radial velocity of the target as a whole. Then, an order-of-magnitude estimate of the bandwidth may be given by

$$B_{radar} \sim 2(2v_{rot}/\lambda) \simeq 4r\omega_{\perp}/\lambda \tag{11}$$

where r is the "radius" of the target, v_{rot} is its rotational velocity, and $\omega_{\perp}$ is the component of angular velocity perpendicular to the beam direction. Therefore, the bandwidth to Doppler frequency ratio may be written as

$$B_{radar}/\nu_D \simeq 2r\omega_{\perp}/v_r, \tag{12}$$

which indicates a narrow-band signal when $2r\omega_{\perp} < v_r$.

Thus, for the radar configuration discussed, the center frequency of the heterodyne signal determines the radial velocity of the target (v_r) while the bandwidth of the signal may provide information about the spin or rate of rotation of the target. Coupled with the time dependence of the amplitude of the return (reflecting the infrared radar cross section), specific information may also be obtained, in principle, about the nature of the surface and shape of the target. For a beam size which is smaller than the target, on the other hand, one can scan the target (e.g., the moon) to determine its velocity profile and, thus, its rate of rotation. The contributions would be similar to those observed for unfocused radiation falling on the scattering wheel, with the additional consideration that a center-of-mass translational radial velocity will increase the center heterodyne frequency ν_D. Therefore, in analogy with a microwave radar, a good deal more may be learned about a target than just the magnitude of a single one of its velocity components.

The validity of (3) and (5) above has been demonstrated experimentally with the rotating scattering wheel. For a 5.05-cm radius wheel rotating at 3600 r/min ($\omega = 120\pi$ second^{-1}), with $F = 2.54$ cm, $D \simeq 0.5$ cm, and $\phi \simeq 30°$, we calculate the values $B_{foc} = 0.3 \pm 0.1$ MHz, and $B_{foc}/\nu_D = 0.17 \pm 0.05$ from (3) and (5), respectively. The experimentally measured (relative) power-spectral-density under these conditions is shown in Fig. 3. In this figure, the power-spectral-density scale is linear and the frequency resolution is approximately 0.05 MHz. The measured values of $B_{foc} = 0.3$ MHz (FWHM) and $B_{foc}/\nu_D = 0.15$ are in good agreement with the predicted values above. The narrow-band nature of the signal for these parameter values is most clearly displayed on a multiple-sweep display, such as shown in Fig. 4. The loss of definition in the fifth cycle reflects the ratio B_{foc}/ν_D.

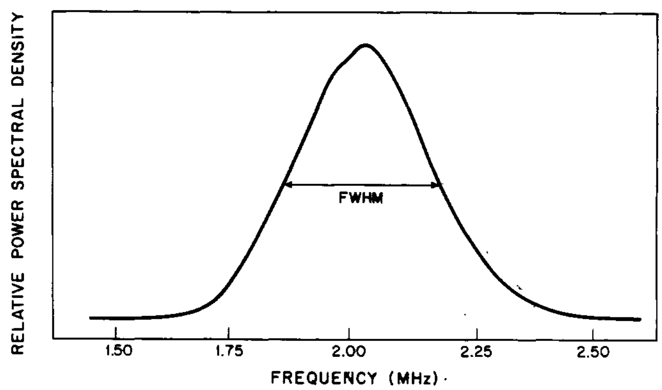

Fig. 3. Experimentally measured power-spectral-density of the heterodyne signal as a function of frequency. Also shown is the full-width at half-maximum (FWHM) of the curve.

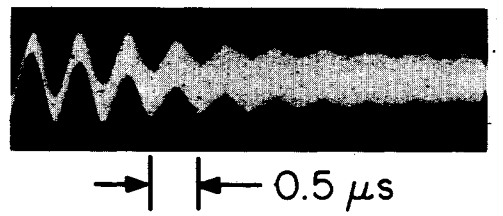

Fig. 4. A multiple-sweep display of the heterodyne signal.

As the angle ϕ (see Fig. 1) is decreased, maintaining focusing of the beam on the wheel rim and the same experimental configuration, the number of cycles before loss of definition decreases, reflecting the increasing value of B_{foc}/ν_D ($\propto \cot \phi$). For sufficiently small values of ϕ, the narrow-band nature of the signal disappears, as expected, and the multiple-sweep display loses all resemblance to the kind of picture seen in Fig. 4. On the other hand, a dramatic *decrease* in the ratio B/ν_D has been observed by simply removing the focusing lens from the experimental arrangement, leaving ϕ unaltered. This operation had the effect of adding many cycles to a representation such as that shown in Fig. 4. This effect is understood on the basis of (10), keeping in mind that $d/r \gg \lambda/2d$ and that d is limited to 2 mm by the iris aperture for these experiments. The heterodyne signal amplitude may decrease in this case, however, if the detector resolves the illuminated spot on the wheel. This has been discussed previously [1]. For optimum detection in a real radar experiment, analogously, the receiver aperture must be adjusted so as not to resolve the return signal [17]–[19], in order to maintain spatial coherence.

We have discussed the power-spectral-density of the homodyne signal in terms of the size, granularity, and configuration of the scattering target. It has not been necessary to refer specifically to the coherence properties of the scattered radiation. Such is not the case, however, if we investigate the statistical nature of the heterodyne signal or its envelope. For these parameters, it is necessary to have direct information about the statistical nature of the scattered radiation signal, or about its higher order correlation functions. This is discussed in the next section.

IV. Probability Density of the Signal Envelope

A knowledge of the statistical behavior of the heterodyne signal is useful for the optimum processing and transmission of the signal, as well as to provide information concerning the nature of the scattering medium. Because of the narrow-band nature of the homodyne signal in many cases of interest, it is useful (and practically speaking, simpler) to investigate the form of the envelope probability density function. We may then compare the theoretically expected results with those obtained from experiments with a known scatterer, and thus verify the validity of our theoretical model and method of calculation. We proceed to do this by first considering the detection rate for a heterodyne receiver and the coherence properties of the radiation scattered from the moving metallic wheel. From this, we obtain an expected probability distribution for the signal envelope which we compare with experiment.

It has been shown elsewhere that double- and sum-frequency heterodyne terms do not appear in the properly formulated quantum theory of infrared heterodyne detection [2], [8]. For radiation fields which possess a positive definite weight function in Glauber's P-representation [20], which is the case for all fields considered here, the count rate for the heterodyne detector may be written in terms of a semiclassical representation as

$$R_{\text{IF}} \propto \mathscr{L}[E_{\text{LO}} \cos \omega_{\text{LO}} t \cdot E_S \cos(\omega_S t + \delta)]. \qquad (13)$$

Here R_{IF} represents the heterodyne signal voltage at the intermediate frequency[4] (IF); E_{LO} and E_S represent the the magnitude of the electric field for the local oscillator (LO) and the scattered beams, respectively; ω is the angular frequency of the particular radiation beam; and δ is a phase angle. The operator $\mathscr{L}$ stands for the "low-frequency part of."

Now, if the LO arises from a well-stabilized single-mode laser above threshold, as assumed earlier, then E_{LO} and ω_{LO} are strictly constant. The addition of a constant phase has been omitted for simplicity. The random scattering from the rotating wheel (see Fig. 1) has the effect of converting the "coherent" incident LO radiation to narrow-band Gaussian [20] radiation. This conversion of radiation statistics is similar to that obtained by inserting a rotating ground-glass screen in the transmission path of a laser beam. Such experiments have been performed by Martienssen and Spiller [21] to deliberately convert a coherent laser mode to narrow-band chaotic radiation, in order to observe a positive Hanbury Brown-Twiss correlation. Thus, the scattered radiation differs from the LO radiation in two respects: 1) its frequency is altered (Doppler shifted), and 2) its statistical properties are changed.

As a consequence, the scattered beam *radiation field* may be represented as a narrow-band Gaussian random process (centered in the infrared) and may be written in standard form [22] as $E_S(t) \cos[\omega_S t + \delta(t)]$. From this, we rewrite R_{IF} as

$$R_{\text{IF}} = \beta E_{\text{LO}} E_S(t) \cos\left[|\omega_{\text{LO}} - \omega_S| t + \delta(t)\right], \qquad (14)$$

where β represents the quantum efficiency of the detector. Since $|\omega_{\text{LO}} - \omega_S| \equiv 2\pi\nu_D$, where ν_D is the Doppler or heterodyne frequency, we obtain finally[5]

$$R_{\text{IF}} = \beta E_{\text{LO}} E_S(t) \cos\left[2\pi\nu_D t + \delta(t)\right]. \qquad (15)$$

But this expression for the homodyne *signal voltage* is itself, as well, in the form of a narrow-band Gaussian random process.[6] Now, however, it is centered at the Doppler frequency. It is therefore seen that for an experimental arrangement such as described here, the heterodyning process effectively translates the fluctuation properties of the scattered field down to the Doppler frequency. Stated differently, the heterodyne voltage accurately reflects the scattered beam electric field distribution in a beating experiment performed with an amplitude-stabilized LO without fluctuation. Indeed, another example of this is the mixing of two coherent fields. Hinkley, Harman, and Freed [23] have recently mixed the radiation from a single-mode CO_2 laser with that of a single-mode $Pb_{1-x}Sn_xTe$ semiconductor laser and obtained a sinusoidal beat signal with almost no fluctuation. The envelope of the heterodyne signal in this case has a delta-function voltage probability distribution, reflecting the absence of fluctuations, and, therefore, the coherent nature of the signal beam. Thus, information about a scatterer may be obtained if the statistics of the radiation beam incident on the scatterer are known, or the statistics of an unknown radiation source may be determined by mixing with a stable LO.

We now direct our attention to the probability density function for the homodyne signal envelope in the case of scattered radiation from the metallic wheel. As is well known, for a narrow-band Gaussian random process (NBGRP) [22], this should be Rayleigh distributed. A typical trace (sample function) of the homodyne signal and its envelope has been shown in Fig. 2. The probability density of interest was experimentally obtained by sampling the envelope at 1.0 μs time intervals. Some 15 oscilloscope photographs similar to the one represented in Fig. 2 were analyzed in this fashion, providing 754 data points. The envelope voltage was always taken to the nearest 0.01 volt. This data is presented in the histogram of Fig. 5 where, on the relative envelope voltage scale, 1 volt actually represents 0.01 volt.

[4] For experiments in which one beam is Doppler shifted with respect to another, the IF is just equal to the Doppler frequency ν_D.

[5] Although the interference term will properly vanish through the ensemble average, the interference is present in any individual experiment. We assume that we select an ensemble, in analogy with the procedure used for spatial interference (see [20]).

[6] Although both constituent beams (LO and scattered) are stationary, optimum *sinusoidal* photomixing is not obtained because the additional requirement of first-order coherence for the total incident field [8] is satisfied only for time intervals well under a coherence time. The detection has, nevertheless, been shown to be optimum [1], [2].

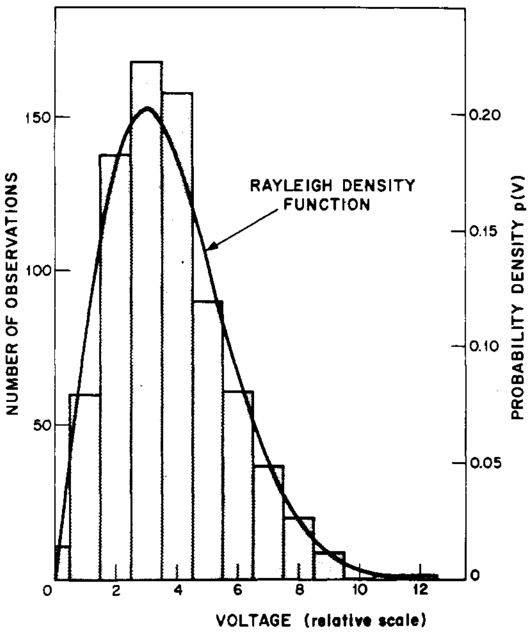

Fig. 5. The heterodyne signal envelope probability density vs. voltage. The experimental result (histogram) and the theoretical prediction (Rayleigh density function) are both shown.

Also plotted in the same figure is the Rayleigh density function

$$p(V) = (V/\alpha)\exp(-V^2/2\alpha) \tag{16}$$

which, as may be seen by inspection, fits the experimental data very well. This expected fit was confirmed by performing a chi-squared test [24]. A value of $\chi^2 = 8.28$ with 7 degress of freedom was obtained, giving a probability $P = 0.3$ that the deviations from the Rayleigh density function would be expected to be greater than those here observed on repeating the series of measurements. This result provides strong evidence that the signal envelope may indeed be fit by a Rayleigh distribution.

The single parameter α in the distribution $p(V)$ above was chosen by setting the observed average envelope voltage $\langle V_{\mathrm{obs}}\rangle$ equal to the average calculated from the Rayleigh distribution $\langle V_{\mathrm{Ray}}\rangle$. Performing the average,

$$\langle V_{\mathrm{Ray}}\rangle = \int_0^\infty Vp(V)\,dV \tag{17}$$

and setting

$$\langle V_{\mathrm{Ray}}\rangle = \langle V_{\mathrm{obs}}\rangle, \tag{18}$$

we obtain

$$\alpha = (2/\pi)\langle V_{\mathrm{obs}}\rangle^2. \tag{19}$$

Thus, taking $\langle V_{\mathrm{obs}}\rangle = 3.73$ relative voltage units from our data (its actual value for the series of experiments performed was 0.0373 volt, as may be approximately seen from Fig. 3), we obtain a value $\alpha = 8.9$ in units of volts squared. The particular distribution plotted in Fig. 5 may therefore be written as

$$p(V) = 0.112\,V\exp(-0.0562\,V^2). \tag{20}$$

The most-probable voltage V_p is found from the relation

$$\left.\frac{\partial p(V)}{\partial V}\right|_{V=V_p} = 0 \tag{21}$$

which, for the Rayleigh distribution, gives the prescription

$$V_p = \sqrt{\alpha}. \tag{22}$$

For the experiments described here, $V_p = 2.98$, and $p(V_p) = 0.203$.

These results are consistent with those obtained by Gould *et al.* [17], who studied the heterodyne signal obtained by scattering visible radiation from different portions of a piece of white bond paper. Finally, it should be mentioned that the statistical performance of an optical *energy-detection* radar has been treated previously, in great detail, by Goodman [25].

V. Conclusion

Previous infrared heterodyne experiments were concerned with measurements of average values of the heterodyne signal and signal-to-noise ratio. A good deal of information is also contained in other parameters, however, such as the spectral distribution of the mixing signal and the probability density of the signal or the signal envelope.

Expressions for the ratio of bandwidth to heterodyne frequency have been obtained for both focused and unfocused radiation incident on a rotating scattering wheel, as well as for a typical infrared radar configuration. Agreement with experiments using a focused radiation beam was good. Knowledge of the center frequency, bandwidth, and time-dependence of an infrared radar signal are shown to provide information about the radial velocity, spin, surface properties, and shape of the target.

The probability density function for the heterodyne signal from a known scatterer is of importance for optimum signal processing and transmission. Alternatively, information may be obtained about an unknown scatterer from an examination of the statistics of the heterodyne signal. The envelope probability distribution was investigated for the sandblasted rotating wheel experiment, and was found to be Rayleigh distributed. This is consistent with the expected distribution for the envelope of a narrow-band Gaussian random process.

It has also been seen that the process of heterodyning an unknown radiation source with a constant amplitude-stabilized laser provides a valuable technique for determining the statistical properties of the unknown source. In the Gaussian case examined here, as well as in the coherent case investigated by Hinkley, Harman, and Freed [23], the statistics of the radiation electric field are directly and simply obtained by analyzing the statistical properties of the heterodyne signal. The usual way of obtaining this information is by photoelectron counting experiments, which are considerably more complicated, and cannot be performed in the infrared in any case.

The "higher order" properties of the infrared heterodyne signal which are discussed here are therefore seen to be useful either in the processing of a known signal, or in obtaining information about an unknown target or signal which could not be obtained from measurements of average signal values. Clearly, the same considerations apply to the optical heterodyne receiver.

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