SELF-SIMILARITY IN SENSORY NEURAL SIGNALS

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A variety of statistical measures are useful for identifying the form of the point process that describes the maintained discharge of retinal-ganglion-cell (RGC) and lateral-geniculate-nucleus (LGN) neurons in the cat visual system. These measures are based on both interevent intervals and counts, and include the interevent-interval histogram, rescaled range analysis, the event-number histogram, Fano factor, Allan factor, and periodogram. All of these measures have been applied to data sets derived from both RGC and LGN neurons, and to surrogate versions thereof generated by randomly shuffling the order of the interevent intervals. The counting statistics reveal multiscale (1/*f*-type) fluctuations in the data (long-duration power-law correlation), which can be identified with neuronal facilitation. These fractal fluctuations are eliminated by the process of shuffling. Certain short-term features of the RGC and LGN spike trains are well described by the gamma-*r* renewal process that is often used to model visual-system action-potential interevent intervals. However, this model fails to accommodate the long-duration correlation. We present a new model for visual-system action-potential firings comprising a gamma-*r* renewal process compounded with fractal binomial noise. This fractal doubly stochastic point process characterizes the statistical behavior of both RGC and LGN spike trains exceedingly well.

INTRODUCTION

A number of studies of the nerve-spike trains recorded from mammalian retinal ganglion cells (RGCs) and lateral geniculate nucleus (LGN) neurons have made use of the gamma renewal process for describing the data. With appropriate choice of parameters, this simple model satisfactorily describes several statistical measures of the spike trains, including the interevent-interval histogram and the periodogram (power spectral density estimate), at least for frequencies above 0.5 Hz.

However, the gamma renewal process fails to describe the multiscale fluctuations present in both RGC and LGN spike trains. Indeed, no renewal process can account for such fluctuations since they require memory. We construct a fractal doubly stochastic point process which imposes fractal fluctuations on a gamma-*r* renewal process. This point process provides an excellent description for the maintained-firing characteristics of RGC and LGN neurons [1].

Interestingly, multiscale fluctuations are not unique to RGC and LGN spike trains. They have also been found in cat striate-cortex neurons [2] and in a visual interneuron in the locust [3]. Such behavior is also exhibited in other sensory systems, *viz.* the sequence of action potentials in primary afferent auditory neurons in the cat [4-6], chinchilla [7], and chicken [8]; and neurons in the cat mesencephalic reticular formation [9].

Perhaps the simplest measure that reveals the presence of fractal fluctuations is the estimated rate of neural firing. Fractal signals exhibit slow convergence. As the counting time T used to compute this rate is increased, the magnitude of the fluctuations decreases slowly, as a power-law function of the counting time, or even persists at the same value. The magnitude of nonfractal signals, on the other hand, decreases quickly as T is increased. This latter behavior characterizes the shuffled neural spike trains (in which the interevent intervals have been randomly reordered). This is because the shuffling process destroys the long-term correlation in the spike train and thereby removes its fractal character.

METHODS

Previous studies of visual-system spike trains have focused primarily on analyses of the sequence of *intervals* between action potentials. Our analysis [1] joins this approach with information obtained from *count* and *rate* estimates of the neural spike train, which preserve real time on the abscissa and thereby allow a direct interpretation of the observed correlation.

The details of the preparation and surgical procedures, along with the methods used in acquiring and processing the nerve-spike trains from the thalamus of the adult cat, are described in detail in [1].

RESULTS

We examined two measures obtained from the sequence of interevent intervals of the RGC and

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LGN spike trains. The interevent-interval histograms for both types of cell are well modeled by the gamma renewal process. On the other hand, rescaled-range analysis demonstrates strong correlation in the sequence of intervals for both kinds of cells, which clearly cannot be accounted for by a renewal model.

Several event-number-based measures were also used to analyze the data. The event-number histogram, constructed for sufficiently large counting times, indicates large rate fluctuations in both RGC and LGN neurons. The Fano factor and Allan factor also shows power-law correlation in the sequence of counts for counting times in excess of about 1 second. Finally, the periodogram reveals power-law behavior for frequencies less than about 0.5 Hz, for most of the cells examined.

Thus, all of the event-number-based measures of RGC- and LGN-neuron firing patterns we investigated reveal nonrenewal behavior and the presence of multiscale correlations.

DISCUSSION

We have constructed a fractal doubly stochastic point process to accurately model all of the nerve-spike trains observed from RGC and LGN visual-system neurons in the cat [1]. We previously considered a similar model for primary afferent auditory nerve-fiber firing patterns [4, 10]. In the auditory case we posited that a cochlear nerve fiber would generate a homogeneous Poisson point process in the presence of a putative steady concentration of neurotransmitter. Because the neurotransmitter concentration is well described by fractal binomial noise (generated by the superposition of ion-channel openings/closings in the cell membrane [11] or fractal neurotransmitter exocytosis [12]), however, the net result is a compounding of the Poisson process with these fractal rate fluctuations. Applying similar reasoning to the visual-system neural-firing patterns, we posit that the cells would generate a gamma-r renewal (rather than a Poisson) process in the presence of constant neurotransmitter concentration. The fractal binomial-noise-driven gamma-r point process therefore serves as a plausible model for RGC and LGN firing patterns.

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REFERENCES

[1] M. C. Teich, C. Heneghan, S. B. Lowen, T. Ozaki, and E. Kaplan, "Fractal character of the neural spike train in the visual system of the cat," *J. Opt. Soc. Am. A*, submitted for publication, 1995.

[2] M. C. Teich, R. G. Turcott, and R. M. Siegel, "Variability and long-duration correlation in the sequence of action potentials in cat striate-cortex neurons," *IEEE Eng. Med. Biol. Mag.* vol. **15**, in press, 1996.

[3] R. G. Turcott, P. D. R. Barker, and M. C. Teich, "Long-duration correlation in the sequence of action potentials in an insect visual interneuron," *J. Statist. Comput. Simul.*, vol. **52**, pp. 253–271, 1995.

[4] M. C. Teich, "Fractal character of the auditory neural spike train," *IEEE Trans. Biomed. Eng.*, vol. **36**, pp. 150–160, 1989.

[5] M. C. Teich, "Fractal neuronal firing patterns," in *Single Neuron Computation* (T. McKenna, J. Davis, and S. Zornetzer, Eds.). Boston: Academic Press, 1992, pp. 589-625.

[6] S. B. Lowen and M. C. Teich, "The periodogram and Allan variance reveal fractal exponents greater than unity in auditory-nerve spike trains," *J. Acoust. Soc. Am.*, in press, 1996.

[7] N. L. Powers, R. J. Salvi, and S. S. Saunders, "Discharge rate fluctuations in the auditory nerve of the chinchilla," in *Abstracts of the XIV Midwinter Research Meeting, Assoc. for Res. in Otolaryngology* (D. J. Lim, Ed.). Abstract No. 411, p. 129, 1991.

[8] N. L. Powers and R. J. Salvi, "Comparison of discharge rate fluctuations in the auditory nerve of chickens and chinchillas," in *Abstracts of the XV Midwinter Research Meeting, Assoc. for Res. in Otolaryngology* (D.J. Lim, Ed.). Abstract No. 292, p. 101, 1992.

[9] F. Grüneis, M. Nakao, Y. Mizutani, M. Yamamoto, M. Meesmann, and T. Musha, "Further study on *1/f* fluctuations observed in central single neurons during REM sleep," *Biol. Cybern.*, vol. **68**, pp. 193–198, 1993.

[10] S. B. Lowen and M. C. Teich, "Estimation and simulation of fractal stochastic point processes," *Fractals*, vol. **3**, pp. 183–210, 1995.

[11] 65. S. B. Lowen and M. C. Teich, "Fractal auditory-nerve firing patterns may derive from fractal switching in sensory hair-cell ion channels," in *Noise in Physical Systems and 1/f Fluctuations* (AIP Conference Proceedings 285, P. H. Handel and A. L. Chung, Eds.). New York: American Institute of Physics, 1993, pp. 781–784.

[12] S. B. Lowen, S. S. Cash, M.-M. Poo, and M. C. Teich, "Neuronal and non-neuronal exocytosis exhibits fractal properties," in preparation, 1996.