Generalized Entangled-Photon Imaging

Bahaa E. A. Saleh, Sandu Popescu, and Malvin C. Teich

Department of Electrical & Computer Engineering, Boston University, Boston, MA 02215

ABSTRACT - When one of a pair of entangled photon beams is transmitted through an object, coincidences with photons in the other beam contain information about the object. We present a generalized formulation for a coded-aperture imaging system using this effect.

Entangled-photon beams generated by parametric downconversion have been used in numerous experiments to test the foundations of quantum mechanics. A variety of practical applications have also been demonstrated using such beams, including absolute radiometric measurements, quantum cryptography, and quantum imaging [1,2]. This paper provides a generalized formulation of the quantum imaging problem. We derive an expression for the photon coincidence rate after each of a pair of entangled photon beams has been transmitted through a separate general linear optical system, one of which contains the unknown object. We examine the retrievability of the object information from a measurement of the coincidence rate.

Consider the overall system illustrated in Fig. 1. A source $\mathcal{S}$ emits pairs of entangled photons in two beams, the signal and the idler. The signal photons are transmitted through an object O and are collected by a detector $\mathcal{D}_1$ after passing through an arbitrary optical system $\mathcal{A}$. The idler photons are transmitted through an optical system $\mathcal{B}_n$ and are collected by a detector $\mathcal{D}_2$. The photon coincidence rate C_n is measured with various optical systems $\mathcal{B}_n$, $n = 1,2,..,$ in place. The systems $\mathcal{B}_n$ may represent, for example, a set of coded apertures, or a displaced pinhole. The idea is to extract information about the object O from measurements of the coincidence rate C_n for various optical systems $\mathcal{B}_n$, $n = 1,2,...$

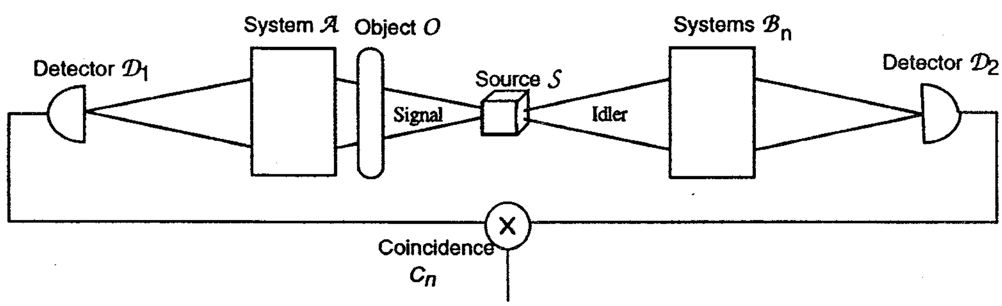

Fig. 1 Generalized entangled-photon imaging system.

To be specific, asssume that O is a thin planar object with complex amplitude transmittance $f(x)$, and for simplicity assume that the object and the optical systems are both one-dimensional. Classically, the fields E_1 and E_2 at the detectors $\mathcal{D}_1$ and $\mathcal{D}_2$ are related to the fields $E_s(x)$ and $E_i(x)$ at the inputs to the object O and to the system $\mathcal{B}_n$, respectively, by the linear integrals

$$
\begin{aligned}
E_1 &= \int A(x)f(x)E_s(x)dx \\
E_2 &= \int B_n(x)E_i(x)dx,
\end{aligned}
\tag{1}
$$

where $A(x)$ and $B_n(x)$ are appropriate weighting factors determined from the impulse response functions of the systems $\mathcal{A}$ and $\mathcal{B}_n$. The most general state of a photon pair at these input planes is

$$|\Psi\rangle = \iint dx dx' g(x,x') \hat{a}_s^+(x) \hat{a}_i^+(x') |0,0\rangle, \tag{2}$$

where $|0,0\rangle$ represents the vacuum state and $\hat{a}_s^+(x)$ and $\hat{a}_i^+(x')$ are the photon creation operators for the signal and idler at positions x and x', respectively. For simplicity we have assumed further that the signal and the idler are monochromatic. All of the characteristics of the source are encoded in the correlation function $g(x,x')$. If we are not interested in resolution on the scale of the wavelength of the photons, we may consider that the creation and annihilation operators for different space points commute, i.e., $\left[\hat{a}_s^+(x), \hat{a}_s^-(x')\right] = \delta(x-x')$, and similarly for the idler photons.

The coincidence rate $C_n = \left\langle \Psi \middle| \hat{E}_1^- \hat{E}_2^- \hat{E}_2^+ \hat{E}_1^+ \middle| \Psi \right\rangle$ can now be determined by straightforward use of the previous equations. This readily leads to the principal equation of this paper:

$$C_n = \left| \iint g(x,x') f(x) A(x) B_n(x') dx dx' \right|^2. \tag{3}$$

This equation relates the coincidence rate C_n to the unknown object transmittance $f(x)$, through the known functions $A(x)$, $B_n(x)$, and $g(x,x')$, which determine the kernel of the transformation. The coincidence rates $\{C_n\}$ are therefore the squared magnitudes of linear projections of the unknown function $f(x)$.

The ideal case in which $g(x,x') = \delta(x-x')$, i.e., the photons are perfectly entangled, is particularly revealing. In this case, Eq. (3) gives

$$C_n = \left| \int f(x) A(x) B_n(x) dx \right|^2. \tag{4}$$

Moreover, since $A(x)$ and $B_n(x)$ are simply multiplied under the integral, the systems $\mathcal{A}$ and $\mathcal{B}_n$ are exchangeable. This may be important in practical situations where optical components cannot be placed in the vicinity of the object. The entangled-photon imaging system therefore offers the flexibility of obtaining the same effect by placing a complex optical apparatus in the idler beam, away from the object.

Suppose now that $A(x) = 1$, i.e., that the $\mathcal{A}$ system simply directs the rays of the signal beam onto $\mathcal{D}_1$. Equation (4) then becomes

$$C_n = \left| \int f(x) B_n(x) dx \right|^2 \tag{5}$$

If the $\{B_n(x)\}$ form a complete set of orthonormal functions, then the $\{C_n\}$ are simply the squared magnitudes of the coefficients of an expansion of the unknown function $f(x)$ in this basis. Under special conditions, the phases can be retrieved, and the function $f(x)$ completely reconstructed.

The special case in which $B_n(x) = \delta(x - x_n)$, i.e., when the system $\mathcal{B}_n$ samples the idler field at positions x_n, has been demonstrated experimentally by using a scanning system [1]. In our case, Eq. (5) then provides $C_n = |f(x_n)|^2$ so that the coincidence rate yields the *intensity* transmittance of the object.

Consider now a system $\mathcal{B}_n$ that collects light from two pinholes: $B_n(x) = \delta(x) + \delta(x - x_n)$. Such system has been recently demonstrated [2]. Here Eq. (5) gives $C_n = |f(0) + f(x_n)|^2$. As x_n is scanned, the coincidence rate C_n provides the same interference pattern that would have been obtained if the object were illuminated with coherent light and viewed through pinholes located at $x = 0$ and $x = x_n$. The phase of the function $f(x)$ can in principle be reconstructed up to a constant phase, the phase of $f(0)$. With the magnitude and phase of $f(x)$ determined, complete knowledge of the object is retrieved.

In conclusion, Eq. (3) shows that complex object information is encoded in the measured coincidence rate. By appropriate choices of the systems functions $A(x)$ and $B_n(x)$, such information can be extracted. Deviation of the function $g(x,x')$ from a delta function, resulting from partial entanglement [3], plays a key role in limiting the resolution of this imaging system. This effect is mathematically analogous to the effect of partial coherence in ordinary imaging systems.

References

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