Branching Processes in Quantum Electronics

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Invited Paper

***Abstract*—Noise and random fluctuations play an important role in quantum electronic devices and systems. Such fluctuations reside, for example, in the random creation of photons in optical amplifying media and in the generation of electron–hole pairs in ionizing regions of semiconductor devices. We provide an overview of the fundamental random branching processes that underlie optical and electronic gain. We describe branching processes as concatenations of basic elements that comprise filtered Poisson processes (shot noise) driving secondary Poisson processes. In the presence of feedback, these elementary processes become self-exciting in nature; they are then suitable for characterizing squeezed light and sub-Poisson photon emissions.**

***Index Terms*—Avalanche photodiode, branching process, cascaded stochastic process, cascaded-Poisson process, compound-Poisson process, doubly Poisson process, doubly stochastic process, filtered Poisson process, fluctuations, gain, laser, multiply-Poisson process, noise, nonclassical light, optical amplifier, photomultiplier tube, Poisson process, quantum electronics, self-exciting process, shot noise, squeezed light, sub-Poisson light, twin-photon beams.**

I. Introduction

THE OPTICAL amplifier, the laser, and the avalanche photodiode are devices that lie at the heart of quantum electronics. And the heart of these devices lie processes involving the sequential generation of cascades of particles, photons or electrons, in a chain-reaction-like branching fashion. Whether it is photons in an optical amplifying medium, or electrons and holes in a semiconductor device, the underlying cascading process is described by the theory of branching processes [1]. This mathematical construct, first set forth by Irénée Bienaymé in 1845, is probably best known for its successes in describing cosmic-ray showers and nuclear chain reactions. However, it is also ideally suited to describing optical amplification in laser media and electronic gain in avalanche photodiodes (APDs). It permits us to determine the gain, time response, and noise characteristics of these devices. Indeed, various aspects of branching theory have appeared in one form or another in the quantum electronics literature since its inception.

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As Shimoda *et al.* [2] understood early on, photons in an optical amplifying medium may undergo birth (stimulated emission) or death (absorption). The third process attendant to optical amplification is immigration (spontaneous emission). In a traveling-wave configuration, the photon amplifier is described by the birth–death–immigration (BDI) process, a special case of branching theory. The initial photon distribution is determined by the statistical character of the light presented to the amplifying medium [3].

A classical laser oscillator operates on the basis of these same three processes but the photons are trapped by the optical resonator that provides the feedback. The ensuing gain saturation results in a suppression of the exponential intensity growth that characterize the laser amplifier. Nevertheless, the equilibrium photon-number distribution that results bears the stamp of the BDI branching process that underlies it [4].

Photodetectors with gain are also characterized by branching processes. In this case, the birth, death, and immigration processes are associated with charged carriers (electrons and holes) rather than with photons [5]. Indeed, the very first application of branching-process theory to the field of quantum electronics was set forth in the context of photodetection. In 1938, when branching processes first found their way into the physical sciences [1], [6], [7], [8], Shockley and Pierce [9] used a branching-process model to calculate the gain and noise properties of the electron-multiplication cascade in a photomultiplier tube (PMT).

The branching-theory descriptions of these three devices characterize not only the physics underlying their operation, but also provide a measure of their noisiness. From an engineering perspective, this allows the performance of systems incorporating these devices to be determined. One example is an optical receiver incorporating an avalanche photodiode [10]. The beauty of the avalanche process is that every carrier pair generated by a photon incident on the depletion region undergoes a chain-reaction cascade that results in hundreds more carriers being added to it by the time the carriers exit the multiplication region and produce a current in the circuit. For sufficiently small values of the photon flux, this process has the salutary effect of amplifying the photon-induced current so that it exceeds the receiver circuit noise, which thence serves to improve the signal-to-noise ratio (SNR) of the system. For sufficiently large values of the photon flux, on the other hand, the noise associated with the branching of the charged carriers in the device (gain fluctuations) is deleterious and reduces the SNR. Thus, branching theory tells us when the use of

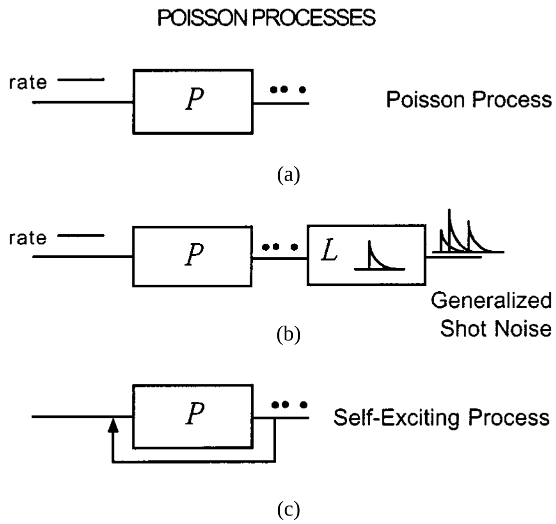

Fig. 1. (a) Poisson generator $\mathcal{P}$ generates a Poisson point process with constant rate μ. (b) A random linear filter $\mathcal{L}$ following $\mathcal{P}$ leads to generalized shot noise. (c) The evolution of a self-exciting point process depends on the occurrence times of past events. An arbitrary point process, with variability greater or less than that of the Poisson, can be cast in the form of a self-exciting process.

an avalanche photodiode in an optical receiver is useful for enhancing performance.

Branching theory can also be of help in the design of superior quantum electronic devices. For APDs, as an example, a dead-space-modified version of branching theory [11] tells us that thin devices should exhibit superior noise properties, and this expectation is indeed borne out in practice [12].

This paper provides an overview of fluctuation processes in quantum electronics that involve cascading and branching. We make use of an approach that we have found to be useful in our own work. Branching processes are considered as concatenations of elements comprising singly and doubly stochastic Poisson processes and shot noises [13]. Along the way, the individual elements themselves provide the photon statistics for various sources of light, which is also of interest in quantum electronics. We begin with the most elemental of such conceptions: the homogeneous Poisson process, shot noise, and self-exciting point processes. We end with cascaded and branching Poisson processes.

II. Poisson Processes

The homogeneous Poisson point process [14], which is illustrated in Fig. 1(a), is perhaps the simplest of random cascade elements. It is characterized by a single quantity, its rate μ, which is constant. Its distinguishing feature is that it is memoryless; the occurrence times and numbers of events before an arbitrary time have no bearing on the subsequent occurrence times and numbers of events. Because of its simple properties, it forms a suitable building block for more complex point processes and cascades. In optics, a light source of constant intensity, such as an ideal amplitude-stabilized laser, leads to photoelectron statistics characterized by the homogeneous Poisson point process [10].

A. Shot Noise

A Poisson process of rate μ passed through a deterministic linear filter with impulse-response function $h(t)$ gives rise to shot noise. Campbell obtained values for the mean and variance of this continuous process in 1909 and used it to characterize the emission of light [15]. The process was extensively studied, and named *shot effect*, by Schottky in 1918 [16]. When the impulse-response function is of finite duration τ_p and the emissions are dense ($\mu\tau_p \gg 1$), the shot-noise amplitude distribution approaches Gaussian statistics [17].

Generalized shot noise arises when a Poisson process is passed through a linear filter whose impulse response is a random function chosen from a common distribution, as illustrated in Fig. 1(b). It has been shown [18], [19] that an ensemble of stochastic impulse-response functions has an equivalent deterministic impulse-response function that is suitable for calculating the first-order statistics of the shot-noise process.

When the impulse-response function assumes the form of a decaying power law, its characteristic time can become arbitrarily large or small. Such fractal (power-law) shot noise can then violate the conditions of the central limit theorem whereupon the amplitude distribution does not approach Gaussian form for any value of the Poisson rate μ. The behavior of fractal shot noise, and its generalized cousin, have been extensively studied for a variety of parameters of the process [20]. For certain parameters, the power spectral density exhibits $1/f$-type behavior over a substantial range of frequencies, so that the process serves as a source of $1/f^{\alpha}$ shot noise for α in the range $0 < \alpha < 2$. For other parameters, the amplitude probability density function is a Lévy-stable random variable with an order parameter less than unity. This process then behaves as a fractal shot noise that fails to converge to a Gaussian amplitude distribution in the asymptotic limit as the driving rate increases. In the domain of optics, fractal shot noise provides a suitable model for describing the intensity statistics of Čerenkov radiation arising from a random stream of charged particles.

B. Self-Exciting Processes

An arbitrary regular point process can be cast in the form of a Poisson process with a rate controlled by past events, as illustrated in Fig. 1(c). In its most general form, the future evolution of such a self-exciting point process depends on the occurrence times of past events as well as on their total number [14].

A special but useful case occurs when the process has limited memory; in particular, the interevent intervals of a homogeneous self-exciting Poisson process with a memory that reaches back exactly one event form a sequence of statistically independent random variables (a renewal process) [14]. The dead-time-modified Poisson process, a renewal process, is of particular interest in quantum electronics [21]. The circuitry at the output of a photodetector, such as a photomultiplier tube operated in the photon-counting mode, generally exhibits a fixed period of time τ_d following the registration of an event, during which it is incapable of registering another event, i.e., it is dead. Whether the dead time assumes nonparalyzable or paralyzable form [22], its presence serves to substantially regularize the photoelectron point process. This, in turn, results in a photoelectron counting

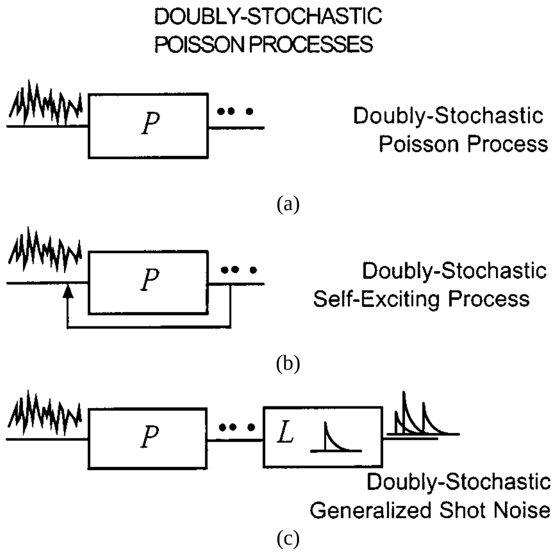

Fig. 2. (a) A Poisson point process $\mathcal{P}$ whose rate is a stochastic process in its own right, is termed a doubly stochastic Poisson point process. (b) A self-exciting point process whose rate is a stochastic process. (c) A random linear filter $\mathcal{L}$ following a doubly stochastic Poisson process leads to doubly stochastic generalized shot noise.

distribution whose normalized variance (count variance divided by count mean) can reach substantially below unity, the value at which it is fixed for a homogeneous Poisson process [22]. Such counting statistics are therefore called *sub-Poisson*.

III. Doubly Stochastic Poisson Processes

The doubly stochastic Poisson process has a rate that takes on a stochastic nature of its own. This process was first examined by Wold [23]. Cox [24] studied this process extensively and provided an example of its use in textile technology. The designation *doubly stochastic* was introduced to emphasize that two kinds of randomness are operative: randomness associated with the point process itself and an independent randomness associated with its rate, as illustrated in Fig. 2(a). The doubly stochastic Poisson process, often called the compound Poisson process, has become the basis for understanding photoelectron statistics of all orders that result from the detection of classical light of all forms [25]. The rate of the point process is the squared magnitude of the complex field. Particular attention was devoted early on to unraveling the photoelectron statistics for thermal light, which is characterized by a circularly symmetric complex Gaussian field [25]–[28], and for interfering superpositions of thermal and amplitude-stabilized (ideal-laser) light [25], [28], [29].

A. Doubly Stochastic Self-Exciting Processes

The effects of dead time on a doubly stochastic Poisson point process, illustrated in Fig. 2(b), are substantial and dramatic. Though the complexity of the calculations quickly escalates, expressions for the dead-time-modified count mean and variance have been obtained for thermal light [30], for shot-noise light [31], which is a doubly Poisson form of light that will be discussed in some detail subsequently, and for fractal-Gaussian-noise driven Poisson light [32].

B. Doubly Stochastic Shot Noise

The calculation of the statistical properties of doubly stochastic generalized shot noise, illustrated in Fig. 2(c), was carried out by Picinbono *et al.* [28]. To incorporate the effects of gain fluctuations inherent in the photomultiplication process, they modeled the photomultiplier-tube anode-current pulses as a sequence of independent nonstationary brief random impulse-response functions drawn from a single probability distribution. They established that in the limit of dense photoemissions, $\mu\tau_p \gg 1$ where μ is the rate of photoelectron arrivals and τ_p is their effective duration at the anode, the asymptotic photocurrent resulting from such a filtered doubly stochastic Poisson point process in general fails to converge to Gaussian form. They therefore concluded that the central-limit theorem is not applicable in this circumstance. They likened this behavior to the result of finding the limit of a random number of independent random variables, and cited Robbins' 1948 study [28].

Among the specific results derived in this paper are the single- and two-fold anode current distributions when thermal light is incident at the faceplate of a photomultiplier tube. The resulting generalized shot noise was found to be characterized by an asymptotic probability distribution proportional to the zeroth-order modified Bessel function of the second kind, K_0. In recent years, a number of important studies have elaborated on the emergence of the K_0 distribution in this context [33]. The same distribution emerges repeatedly in quantum electronics; it is useful for characterizing the field and intensity fluctuations of scattered light, as well as light that has been transmitted through a random medium such as the turbulent atmosphere [34].

IV. Doubly Poisson Processes

The designation *doubly Poisson* indicates the participation of a *pair* of Poisson processes. The simplest of the doubly Poisson processes, illustrated in Fig. 3(a), concatenates a homogeneous Poisson process, a deterministic linear filter, and a second Poisson process. Since the output of the linear filter is shot noise, this construct is given the appelation *shot-noise-driven Poisson process* [35]. It is a special kind of doubly stochastic Poisson process as can be understood by comparing Fig. 3(a) with Fig. 2(a). A representative example of the applicability of this process in optics is provided by cathodoluminescence. Such light is generated when a Poisson stream of electrons (the first Poisson) impinges on a phosphor whereupon it splays out, over a small range of times of duration $\sim \tau_p$ (the linear filter), a random cluster of photons (the second Poisson). The statistical properties of cathodoluminescence photons are well described by the shot-noise-driven Poisson process [35].

A number of variations on this construct have been set forth. A nonstationary version of the shot-noise-driven Poisson process has been developed [36], as has a fractal shot-noise version [37]. Moreover, general analysis, synthesis, and estimation techniques have been developed for such fractal-rate point processes [38].

Unfortunately, expressions for the photocounting distributions associated with shot-noise light are rather complex. This is because they depend on a number of features of the process:

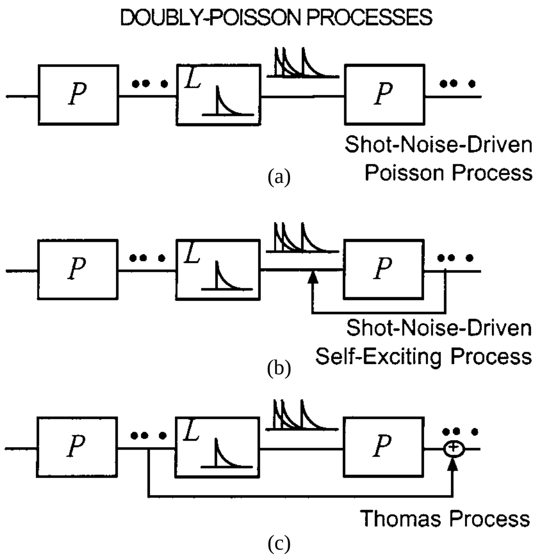

Fig. 3. (a) A Poisson point process $\mathcal{P}$ (right) whose stochastic rate is shot noise is termed a shot-noise-driven Poisson process. (b) A self-exciting process driven by a shot-noise rate is, by analogy, referred to as a shot-noise-driven self-exciting process. (c) The Thomas process emerges when the initial shot events not only excite linear-filter responses but are also carried forward to the output; it is a variation on the shot-noise-driven Poisson process.

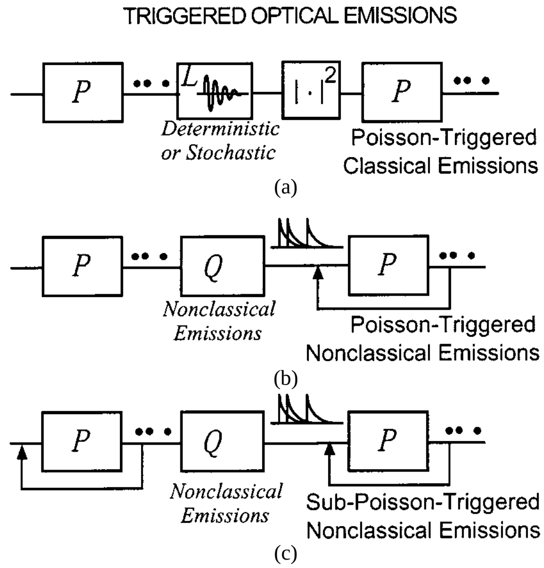

Fig. 4. (a) A homogeneous Poisson point process $\mathcal{P}$ (left) followed by a deterministic (stochastic) linear filter $\mathcal{L}$ generates ordinary (generalized) shot noise. In this case, the shot noise represents the optical *field*, rather than the optical *intensity*, as in Fig. 3, so it must be squared before serving as the rate for the second Poisson process ($\mathcal{P}$ at right). This sequence serves as a model for Poisson-triggered classical emissions and gives rise to photon statistics that are noisier than those of the homogeneous Poisson process. (b) Poisson events can, in the alternative, trigger nonclassical photon emissions $\mathcal{Q}$ (e.g., single photons), modeled by a self-exciting process. This construct cannot generate stationary nonclassical light however. (c) The generation of unconditionally sub-Poisson (photon-number-squeezed) light requires a concatenation in which sub-Poisson events trigger sub-Poisson emissions, both of which are represented by self-exciting processes.

the means of the two Poisson distributions, the spectrum of the light, and the detector counting time. It turns out, however, that a simple two-parameter distribution, the Neyman type-A [39], provides a remarkably good approximation to the photocounting statistics of shot-noise light with arbitrary spectral properties and arbitrary counting times [40]. This distribution therefore plays the role for shot-noise light that the negative-binomial distribution plays for thermal light [41]. An accurate method for computing the tails of the Neyman type-A distribution has been developed [42]. Conditions under which it converges in distribution to the fixed-multiplicative Poisson and to the Gaussian have been established [43].

In the context of quantum electronics, shot-noise light provides a suitable description for the statistical properties of light generated by a number of mechanisms, including cathodoluminescence as described above [35], Čerenkov radiation from a random stream of charged particles [37], [43], betaluminescence photons generated by high-energy electrons at the glass faceplate of a photomultiplier tube [44], beta and radioluminescence noise in star-scanner detection systems operating in ionizing-radiation environments of space [31], [43], and bending-magnet light produced at the Brookhaven National Laboratory vacuum-ultraviolet electron storage ring [45]. In the context of visual neurophysiology, the Neyman type-A distribution is also useful for understanding how a brief flash of Poisson photons at the cornea is transformed into a sequence of neural events in the visual system [46].

In certain cases, such as when detector dead time is present, superior agreement with experiment is obtained by replacing the second Poisson process in Fig. 3(a) with a self-exciting process, as illustrated in Fig. 3(b). This representation is also a special case of Fig. 2(b), since shot noise is a special stochastic rate. Analytical results have been obtained for the count mean and variance [30], [31], and for the interevent-interval distribution [47], of shot-noise light in the presence of dead time. Two optics experiments for which the shot-noise-driven self-exciting process provides an excellent description are betaluminescence in transparent materials [31] and the interspike-interval histogram recorded from a cat retinal ganglion cell in darkness [47].

Another variation, the Thomas process [48], is illustrated in Fig. 3(c). This point process is a modified version of the shot-noise-driven Poisson process in which primary events are carried forward. In the limit of long counting times, it yields the Thomas counting distribution [48], [49], whence its name. Much like the Neyman type-A distribution, the Thomas also converges in distribution to the fixed-multiplicative Poisson and to the Gaussian in certain limits [43].

V. Triggered Optical Emissions

Many processes associated with the generation of light, classical and nonclassical alike, take place via the triggering of optical emissions by point excitation events [50], [51].

The shot-noise rate function considered in the previous section comprised a superposition of brief intensity flashes generated, for example, by luminescence emissions. This construct is satisfactory when interference is absent so that intensities may be added. A more general approach to shot-noise light considers the superposition of brief nonstationary optical-*field* wavepackets which may interfere with each other. The shot-noise intensity rate function is then obtained as the absolute square of the superposed analytic signal, as shown in Fig. 4(a).

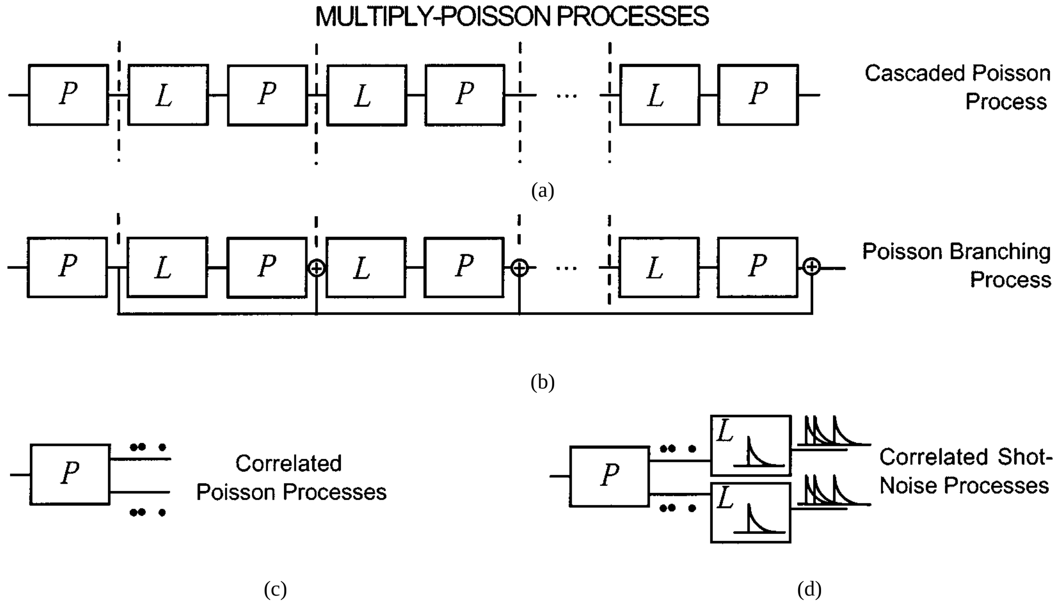

Fig. 5. (a) Series cascade of Poisson processes linked by linear filters. (b) Series cascade of Thomas processes. The trigger events are carried forward so that the result is a Poisson branching process. (c) Parallel excitation of fully correlated Poisson processes such as occurs in spontaneous optical parametric downconversion. (d) Fully correlated parallel shot noises arising from the simultaneous detection of twin photon beams.

The statistical nature of the emission times introduces fluctuations manifested in the relative contributions of different emissions at a given observation time. This results in the introduction of an additional particle-like contribution to the normalized second-order correlation function of the light and a concomitant increase in the photocount variance [50].

The emissions may be deterministic or stochastic, as indicated in Fig. 4(a). For coherent or thermal wavepacket emissions at Poisson trigger times, interference between the randomly delayed emissions produces additional wave-like noise. In the limit when the emissions overlap strongly, $\mu\tau_p \gg 1$, the field exhibits the correlation properties of thermal light, whatever the statistics of the individual emissions. This is a consequence of the central limit theorem. In the opposite limit, when emissions seldom overlap, the light intensity is describable by a superposition of intensities as considered in the previous section, and the photocounts show enhanced particle-like noise which exhibits its largest value when the counting time is long. The photocounts then obey the Neyman type-A and generalized Polya–Aeppli distributions for coherent and thermal emissions, respectively [50].

For nonclassical emissions, neither the intensity representation nor the Poisson-photon generation process embodied in Fig. 4(a) is applicable [51]. Rather, these must be replaced by a quantum photon-generation process $\mathcal{Q}$ represented by a self-exciting process, as shown in Fig. 4(b). This admits the possibility of triggering single photons, or sub-Poisson clusters of photons. However, even when the individual emissions comprise number states, the Poisson trigger times result in the reduction or elimination of their nonclassical character. Indeed, when the emissions overlap strongly the asymptotic behavior of the field is that of thermal light, just as if the individual emissions were classical [50]. This perspective provides a physical underpinning for the ubiquity of Gaussian light, which can be generated in various ways.

It is clear from the foregoing that the direct generation of stationary sub-Poisson (photon-number-squeezed) light cannot be accommodated using Poisson trigger times. We have developed a more general theory in which the trigger times are determined by a self-exciting process, as illustrated in Fig. 4(c). In particular, results have been derived using trigger times that fluctuate in accordance with a stationary renewal point process (which can assume sub-Poisson form), and individual emissions that are coherent, thermal, or n-state in nature [51], [52]. Spatial effects are incorporated into the model by choosing the positions of the emissions to be independent and uniformly distributed over the source volume. The normalized second-order correlation function that emerges from this construct contains the usual form for thermal light, but has two additional terms. The first of these is determined by the statistical nature of the individual emissions (it is positive for coherent and thermal, and zero for single-photon, emissions). The second term is governed by the statistics of the trigger process (it is positive for super-Poisson, zero for Poisson, and negative for sub-Poisson excitations). Both additional terms become small for light with a high degeneracy parameter (many total photons per emission lifetime), in which case the light is asymptotically Gaussian. In the opposite limit, when the degeneracy parameter is small (or the emissions are instantaneous), the correlation properties of the trigger process are directly transferred to the correlation properties of the photons. The first-order spatial-coherence properties of the field are identical to those of thermal light (the van Cittert–Zernike theorem is obeyed), although the second-order properties differ. The photon-counting distribution reflects the character of the correlation function. Thus, sub-Poisson primary excitations, together with single-photon emissions, leads to sub-Poisson photon counts under appropriate conditions. Such nonclassical light may be made arbitrarily intense if interference effects are eliminated by detecting many spatial modes [52].

This theory is applicable to the Franck–Hertz experiment excited by a space-charge-limited electron beam. If the electron excitations are represented as a sub-Poisson renewal point process, and the photon emissions as single-photon states, the light generated should be antibunched and sub-Poisson. This indeed does turn out to be the case [53]. Ultraviolet (253.7-nm) sub-Poisson photons were generated in mercury vapor by inelastic collisions with a space-charge-limited electron beam. This first stationary and unconditionally photon-number-squeezed source, dating from 1985, was only weakly sub-Poisson. However, the same excitation-control approach has been successfully used in a number of laboratories, in the form of suppressed-noise current sources, to produce strongly sub-Poisson light from semiconductor lasers and light-emitting diodes. This and other related techniques for generating nonclassical light have been summarized in several review articles [51], [54]–[57].

VI. Branching Processes

The analysis of doubly Poisson processes can be extended to multiply-Poisson processes. The multifold statistics of the events at the output of a series cascade of an arbitrary number of Poisson processes have been determined [58]. A linear filter following the output of each stage converts the pulsatile sequence of events into a stochastic rate function suitable for driving the next Poisson process, as illustrated in Fig. 5(a). The greater the number of stages of the cascade, the longer the tail of the counting distribution.

If, instead, the cascade comprises Thomas processes, so that trigger events are carried forward as illustrated in Fig. 5(b), the result is the Poisson branching process [59]. This process characterizes electron multiplication at the dynodes of a photomultiplier tube. A useful limiting process of the Thomas cascade is obtained when the number of branching stages is infinite, while the average number of added events per event of the previous stage is infinitesimal. In particular, when the branching is instantaneous, the limit of continuous branching yields the Yule–Furry branching process with an initial Poisson population [1], [59].

Parallel, rather than series, configurations of multiply-Poisson processes can also be constructed, as displayed in Fig. 5(c) (the doubly Poisson form is illustrated). In the context of quantum electronics, photon streams with precisely these properties are generated by optical spontaneous parametric downconversion. A pump laser beam emits photons in accordance with a Poisson process; each of these photons splits into an entangled pair in a nonlinear optical crystal such that energy and momentum are conserved. The resultant twin photon beams are marginally Poisson, but are fully correlated with each other [60], [61]. Filtered versions of these photon streams, illustrated in Fig. 5(d), correspond to correlated shot-noise processes.

VII. Conclusion

The Poisson point process and its variations are useful for describing many phenomena in optics, including the statistics of photon emissions by various sources of light. The cascading and concatenation of two or more such processes with linear filters mediating them has been investigated. Cascaded versions of these elements give rise to a rich hierarchy of branching processes that are suitable for characterizing many important optical and optoelectronic processes in quantum electronics. Among these are photon multiplication in laser amplifiers and oscillators, secondary emission in photomultiplier tubes, and charged–carrier multiplication in avalanche photodetectors.

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