

# Photon Point Process for Traveling-Wave Laser Amplifiers

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**Abstract**—The authors determine the evolution of the photon point process of a light beam as it passes through a traveling-wave laser amplifier (TWA). In particular, when coherent light is presented to the input of the amplifier, the output photon statistics are characterized by a marked-Poisson (MP) point process, which has a noncentral-negative-binomial (NNB) output photon-number distribution (PND). Using this distribution we calculate the probability of error (PE) for an ON-OFF keying (OOK) direct-detection photon-counting communication system, and show that the results differ somewhat from those obtained when the Gaussian-PND approximation is used. It is shown that receiver performance is optimized by filtering the amplifier output. Analysis of the point process is of interest because it permits the time response of the amplifier to be determined; this, in turn, allows the effects of intersymbol interference to be calculated.

## I. INTRODUCTION

OPTICAL-FIBER and semiconductor-laser amplifiers have become ever more commonplace in optoelectronic systems [1]. Such amplifiers are typically operated either as traveling-wave or resonant devices, depending on the application [2], [3]. Amplification results from the interaction of photons with a large number of atoms for which a population inversion is externally maintained. The randomness of the photon generation process in these devices is an important feature to be considered in systems applications. This randomness results from the characteristics of the amplification mechanism and the spontaneous emission. Mathematically, it may be described in terms of a random point process describing the time course of the photon events.

Optical amplifiers have been examined in the context of quantum optics by a number of authors. Louisell and his collaborators [4], [5] developed an early quantum model of a linear, single-mode, phase-insensitive (intensity) optical amplifier. The spatial propagation of the optical field through the amplifying medium was replaced by a time-dependent growth of the optical intensity. This model has provided the basis for a number of generalizations [6]–[15]. In several of these more general models,

the photon-number distribution (PND) at the output of the amplifier has been obtained in terms of the distribution at the input.

Though it has its limitations, the population-statistical approach first used by Shimoda, Takahasi, and Townes [16] generally provides a suitable point of departure for characterizing the photon-number statistics associated with laser amplification [12], [17]. This approach has its origins in the branching-process models developed long ago for use in cosmic rays and population biology. It relies on the birth-death-immigration (BDI) process, which is well known in the theory of stochastic processes [18]–[20].

Various versions of the BDI model have been used to represent the processes of absorption, stimulated emission, and spontaneous emission in a cavity [21]–[25]. In particular, Schell and Barakat [22] examined the approach to equilibrium of the photon-number distribution for single-mode cavity radiation, given a variety of initial PNDs. They found that a Poisson initial distribution resulted in a final distribution described by the noncentral-negative-binomial (NNB) distribution with one degree of freedom ( $M = 1$ ). Shepherd and Jakeman [12] obtained the same results from a quantum point of view by considering a Poisson number of photons coupled to the cavity radiation by means of the immigration.

The BDI model has also been used to determine the photon-number distribution at the output of a *traveling-wave amplifier* (TWA). When the initial photon population at the input to the amplifier is Poisson distributed (as for coherent light), the PND at the output turns out to be the NNB distribution, but with  $M$  degrees of freedom [26]. This distribution has been used as a point of departure for calculating the performance of a simple photon-counting lightwave communication system incorporating a TWA [26], as well as that of a cascade of TWAs [27]. Results for the PND at the output of a TWA have been derived for a broad range of photon-number distributions at the input [28].

In this paper we study the evolution of the photon *point process* of a beam of light as it passes through a traveling-wave amplifier, thereby generalizing the birth-death immigration *number*-statistics approach used by Shimoda, Takahasi, and Townes [16] and others [21]–[28]. The merit of our generalization lies in the fact that it reveals the time *dynamics* of the amplification process, enabling us to determine how the photons are splayed out along the

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time axis. In the number-statistics approach, in contrast, time is integrated out so that only fluctuations of the photon number in a specified time interval are determined. The point-process model is seen to provide a point of departure for determining the amplifier response to short pulses of light, enabling the degradation of receiver performance arising from intersymbol interference to be calculated.

BD process, the probability that a particle is born (or destroyed) during a short period  $\Delta t$ , about the time  $t$ , is proportional to the system's population  $n$  at that time as well as to the duration  $\Delta t$ . The probability that more than one particle is born (or destroyed) during  $\Delta t$  is  $o(\Delta t)$ , i.e., is of higher order and is ignored. With  $P(n, t)$  representing the probability of finding a total population (number) of  $n$  particles at time  $t$ , we have [29]

$$\begin{aligned} P(n, t + \Delta t) &= \text{Prob} \{(\text{finding } n \text{ at } t) \cap (\text{no transitions during } \Delta t)\} \\ &+ \text{Prob} \{(\text{finding } n - 1 \text{ at } t) \cap (1 \text{ birth during } \Delta t)\} \\ &+ \text{Prob} \{(\text{finding } n + 1 \text{ at } t) \cap (1 \text{ death during } \Delta t)\} \\ &= P(n, t) \{1 - [na(t)\Delta t + nb(t)\Delta t + o(\Delta t)]\} \\ &+ P(n - 1, t) [(n - 1)a(t)\Delta t] + P(n + 1, t) [(n + 1)b(t)\Delta t] \end{aligned} \quad (1)$$

Each of the input photons in our traveling-wave configuration can be viewed as initiating its own birth-death process. In Section II, we obtain the output number probability distribution (and probability generating function) for a birth-death (BD) process with given (but possibly varying) birth and death rates, for a single initial input particle. In Section III, this BD process is used to characterize the photon statistics of the amplified signal (AS) of a TWA with a single photon at its input. The case of coherent input light is also considered in this section; since the AS behaves as a marked-Poisson (MP) point process, the PND of the AS is shown to be a special case of the NNB distribution with  $M = 0$ . In Section IV, we analyze the amplified spontaneous emission (ASE) from a TWA, which can be understood in terms of a sum of equivalent AS components corresponding to the spontaneously emitted photons. The ASE is also represented by a MP point process, and its PND turns out to be the negative-binomial (NB) distribution which is also a special case of the NNB. In Section V, we provide the overall output photon point process for the TWA. Using the results of Sections III and IV, we confirm earlier results that the PND for a TWA with coherent light at the input is described by the NNB distribution. In Section VI, the probability of error (PE) for a binary on-off keying (OOK) direct-detection photon-counting communication system incorporating a TWA is calculated using the exact NNB number distributions, and the results are compared with those obtained using Gaussian approximations for the PNDs. It is shown that optimal performance of the receiver is attained by designing it so that  $M$  is minimized. Finally, the conclusion is presented in Section VII.

## II. BIRTH-DEATH NUMBER STATISTICS

A BD process describes the population statistics of a system in which each of the particles has a birth rate  $a(t)$  representing the probability density ( $s^{-1}$ ) of producing another particle, and a death rate  $b(t)$  representing the probability density ( $s^{-1}$ ) of being destroyed. In a linear

so that

$$\begin{aligned} \frac{\partial P(n, t)}{\partial t} &= \lim_{\Delta t \rightarrow 0} \frac{P(n, t + \Delta t) - P(n, t)}{\Delta t} \\ &= -n[a(t) + b(t)]P(n, t) \\ &+ (n - 1)a(t)P(n - 1, t) \\ &+ (n + 1)b(t)P(n + 1, t). \end{aligned} \quad (2)$$

This is known as the forward Kolmogorov equation for a linear birth-death process.

The relationship between a probability distribution (PD)  $P(n)$  and its probability generating function (PGF)  $G(s)$  is defined in terms of

$$G(s) = \sum_{n=0}^{\infty} P(n)s^n. \quad (3)$$

For a single initial particle, the PGF for the BD process satisfies

$$G_{BD}(s) = \frac{1 + (K - \langle n_0 \rangle)(s - 1)}{1 - \langle n_0 \rangle(s - 1)}, \quad (4)$$

where

$$K = \exp \left[ \int_0^t [a(x) - b(x)] dx \right], \quad (5)$$

$$\langle n_0 \rangle = K \int_0^t \frac{a(x)}{K} dx. \quad (6)$$

In the special case where  $a(t)$  and  $b(t)$  are the constants  $a$  and  $b$ , respectively, these formulas become

$$K = e^{(a-b)t}, \quad (7)$$

$$\langle n_0 \rangle = \frac{a}{a-b} (K - 1). \quad (8)$$

Using (4) and the relationship

$$P(n) = \frac{1}{n!} \left[ \frac{\partial^n G(s)}{\partial s^n} \right]_{s=0}, \quad (9)$$

we obtain the probability distribution of a linear BD process with a single initial particle,

$$P_{BD}(n) = \begin{cases} \frac{1 - K + \langle n_0 \rangle}{1 + \langle n_0 \rangle}, & n = 0 \\ \frac{K \langle n_0 \rangle^{n-1}}{(1 + \langle n_0 \rangle)^{n+1}}, & n > 0. \end{cases} \quad (10)$$

The mean and variance of this distribution are

$$\bar{n} = K \quad (11)$$

and

$$\text{Var}(n) = K + 2K \langle n_0 \rangle - K^2, \quad (12)$$

respectively.

### III. AMPLIFIED SIGNAL IN A TRAVELING-WAVE AMPLIFIER

#### A. Output Point Process for a Single Input Photon

We first develop the point process at the output of a TWA in the absence of spontaneous emission. Consider a TWA of length  $L$  and cross sectional area  $A$  containing atoms with an energy-level difference that matches the input signal photon energy  $h\nu$ , where  $h$  is Planck's constant and  $\nu$  is the optical frequency. For each interaction between a photon and an atom in the upper energy level inside the TWA, there is a probability  $\pi_{st}$  that the atom will be stimulated (induced) to undergo a transition to the lower energy level and, in the process, to emit a duplicate (clone) photon with precisely the same characteristics as the original photon. Similarly, for each interaction between a photon and an atom in the lower energy level, there is a probability  $\pi_{ab}$  that the atom will be induced to undergo a transition to the upper energy level and, in the process, to absorb the original photon. In the TWA, external pumping maintains a higher density of atoms in the upper energy level than in the lower level. This population inversion enables the TWA to amplify the input light. The number of atoms per unit volume in the upper and lower energy levels at position  $z$  are denoted  $N_2(z)$  and  $N_1(z)$ , respectively. We assume that the atomic densities in both the upper and lower energy levels are uniform over the cross sectional area of the TWA.

Photons travel along the axis of the amplifier ( $z$ -direction) at the speed of light  $c$ , so that the photon population  $n$  in the TWA is specified by two variables, representing the position and the time at which the photons appear, and denoted  $z$  and  $t$ , respectively. The PD that describes the evolution of the photon population in the amplifier is written as

$$\begin{aligned} P(n, z, t) \\ = \text{Prob} \{ \text{finding } n \text{ photons at position } z \text{ at time } t \}. \end{aligned} \quad (13)$$

Using the approach of Shimoda, Takahasi, and Townes [16], we assume that a transition (stimulated emission or

absorption) occurs instantaneously when a photon interacts with an atom. In the context of this model, an initial signal photon entering the amplifier at the input  $z = 0$  at time  $t = 0$  gives rise to photons at position  $z$  only at time  $t = z/c$ . Thus,

$$P(n, z, t) = \begin{cases} P(n, z), & t = \frac{z}{c} \\ \delta(n), & t \neq \frac{z}{c} \end{cases}$$

where  $\delta(n)$  is the Dirac delta function

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases}$$

This is illustrated in Fig. 1.

During its transversal through a small width  $\Delta z$  in the amplifier, a photon encounters an average of  $N_2(z)A\Delta z$  atoms occupying the upper energy level. Therefore, the probability of a photon being emitted by stimulated emission from  $\Delta z$ , during the time  $\Delta z/c$ , due to a single photon entering this region, is  $\pi_{st}N_2(z)A\Delta z$ . Similarly, the probability of a photon being absorbed is  $\pi_{ab}N_1(z)A\Delta z$ .

Defining the quantities

$$\gamma_{st}(z) = \pi_{st}AN_2(z), \quad (14)$$

$$\gamma_{ab}(z) = \pi_{ab}AN_1(z), \quad (15)$$

we see that the gain coefficient is  $\gamma_{st}(z) - \gamma_{ab}(z)$ . We therefore obtain the differential-difference equation for the photon-number distribution in a TWA at position  $z$  (at time  $t = z/c$ ):

$$\begin{aligned} \frac{\partial P(n, z)}{\partial z} = & -n[\gamma_{st}(z) + \gamma_{ab}(z)]P(n, z) \\ & + (n-1)\gamma_{st}(z)P(n-1, z) \\ & + (n+1)\gamma_{ab}(z)P(n+1, z) \end{aligned} \quad (16)$$

which is recognized as the forward Kolmogorov equation for a linear BD process given in (2).

Thus, in accordance with (4)–(6) and (10), we have

$$G_{AS}(s) = \frac{1 + (g - \langle n_{amp} \rangle)(s-1)}{1 - \langle n_{amp} \rangle(s-1)}, \quad (17)$$

$$P_{AS}(n) = \begin{cases} \frac{1 - g + \langle n_{amp} \rangle}{1 + \langle n_{amp} \rangle}, & n = 0 \\ \frac{g \langle n_{amp} \rangle^{n-1}}{(1 + \langle n_{amp} \rangle)^{n+1}}, & n > 0 \end{cases} \quad (18)$$

with

$$g = \exp \left[ \int_0^L [\gamma_{st}(z) - \gamma_{ab}(z)] dz \right] \quad (19)$$

and

$$\langle n_{amp} \rangle = g \int_0^L \frac{\gamma_{st}(z)}{g} dz. \quad (20)$$

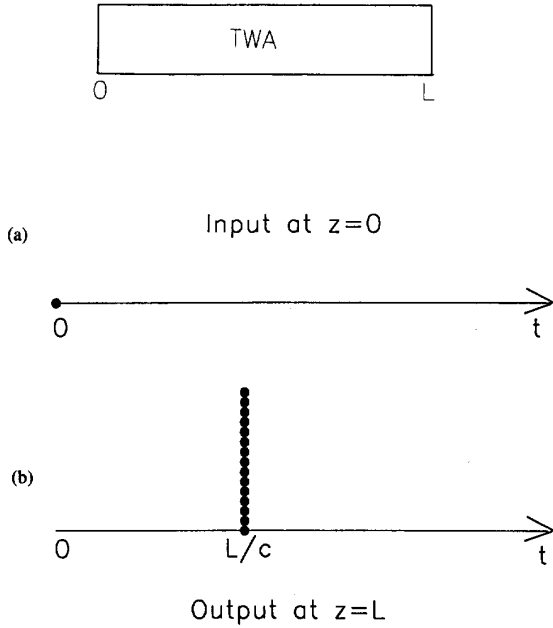


Fig. 1. For a TWA of length  $L$ , with a single photon at its input at  $t = 0$ , the AS appears at the output end only at  $t = L/c$ , and the population of photons obeys a BD counting distribution.

In the special case where  $\gamma_{st}(z)$  and  $\gamma_{ab}(z)$  are given by the constants  $\gamma_{st}$  and  $\gamma_{ab}$ , respectively, these formulas become

$$g = \exp [(\gamma_{st} - \gamma_{ab})L], \quad (21)$$

and

$$\langle n_{amp} \rangle = \frac{\gamma_{st}}{\gamma_{st} - \gamma_{ab}} (g - 1), \quad (22)$$

in accordance with (7) and (8). The mean and variance of this distribution are then

$$\overline{n_{AS}} = g \quad (23)$$

and

$$\text{Var}_{AS}(n) = g + 2g\langle n_{amp} \rangle - g^2, \quad (24)$$

respectively. Equation (23) indicates that  $g$  is the mean number of output photons when the input photon number is unity;  $g$  therefore represents the gain of the amplifier. Equation (24) shows that  $\langle n_{amp} \rangle$  characterizes the variability of the birth-death amplification process; therefore, we define it as the *amplification-noise parameter* of the amplifier.

In short, amplification in a TWA may be mathematically characterized by a BD process  $G_{AS}(s)$ , with amplifier gain  $g$  and amplification-noise parameter  $\langle n_{amp} \rangle$ .

### B. Output Point Process for an Arbitrary Sequence of Input Photons

Knowledge of the single-photon response of a TWA enables us to determine the output for an arbitrary sequence

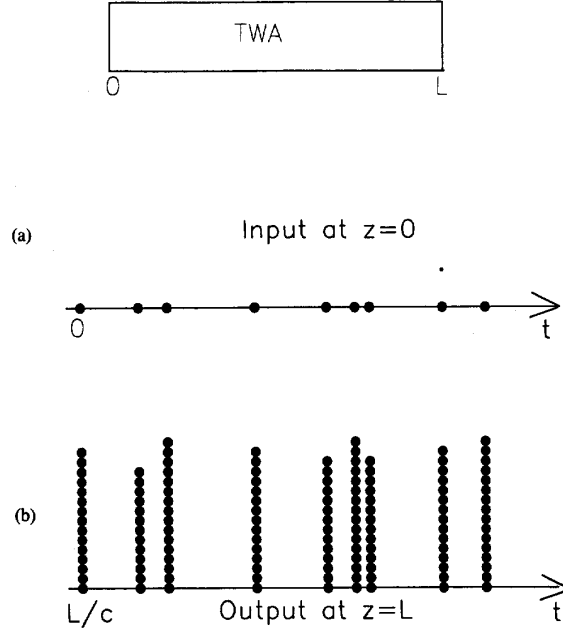


Fig. 2. (a) For coherent light at the input to the TWA, the photons obey a Poisson point process. (b) The photon point process of the AS at the output  $z = L$  is a marked-Poisson (MP) point process.

of photons at the input. Each of the input photons can be viewed as giving rise to its own BD processes, with time delay  $L/c$ , at the output of the amplifier. The output point process is therefore the union of all these.

Input photons associated with coherent light, for example, are characterized by a Poisson process with rate  $\lambda_s$ . With laser light at the input to the TWA, therefore, the AS photons can be modeled as a marked-Poisson (MP) point process, as illustrated in Fig. 2. The marks, representing the random number of photons after amplification, obey the BD counting distribution characterized by the mark-PGF  $G_{AS}(s)$  defined in (17).

### C. Photon-Counting Statistics for a Poisson Number of Photons at the Input

The statistical properties of the AS, such as its photon-counting statistics, inter-event-time statistics, and correlation function can all be determined from the MP point process associated with amplified coherent light.

In the counting time  $T$ , there are a Poisson number of primary photons, each of which independently generates a BD-distributed number of amplified photons. Thus the total number of photons counted during  $T$  is the cascade of these two random variables. The PGF of the total number of counted photons  $G_{ASc}$  is, therefore [20], [30],

$$G_{ASc}(s) = G_P(G_{AS}(s)), \quad (25)$$

where  $G_P(s)$  is the PGF of a Poisson distribution with mean input photon number  $\langle n_s \rangle = \lambda_s T$ , i.e.,

$$G_P(s) = \exp [\langle n_s \rangle (s - 1)]. \quad (26)$$

Using (17) and (26), we obtain

$$G_{ASc}(s) = \exp \left[ \frac{g \langle n_s \rangle (s-1)}{1 - \langle n_{amp} \rangle (s-1)} \right]. \quad (27)$$

From (9) and (27) we obtain the AS photon-number distribution

$$P_{ASc}(n) = \left[ \frac{\langle n_{amp} \rangle}{(1 + \langle n_{amp} \rangle)} \right]^n \exp \left[ -\frac{g \langle n_s \rangle}{1 + \langle n_{amp} \rangle} \right] \cdot L_n^{(-1)} \left[ -\frac{g \langle n_s \rangle}{\langle n_{amp} \rangle (1 + \langle n_{amp} \rangle)} \right], \quad (28)$$

where

$$L_n^{(-1)}(-x) = \begin{cases} \sum_{k=0}^n x^k \frac{kn!}{n(n-k)!(k!)^2}, & n > 0 \\ 1, & n = 0. \end{cases}$$

This is a special case of the noncentral-negative-binomial (NNB) distribution (see Section V) with  $M = 0$ . Its mean and variance are

$$\overline{n_{ASc}} = g \langle n_s \rangle \quad (29)$$

and

$$\text{Var}_{ASc}(n) = g \langle n_s \rangle + 2g \langle n_s \rangle \langle n_{amp} \rangle, \quad (30)$$

respectively, while its Fano factor is

$$F_{ASc} \equiv \frac{\text{Var}_{ASc}(n)}{\overline{n_{ASc}}} = 1 + 2 \langle n_{amp} \rangle. \quad (31)$$

Equation (31) illustrates that an amplified coherent signal is noisier than the coherent light at the input, whose PND is a Poisson distribution with a Fano factor of 1. Even without spontaneous emission, the price of birth-death amplification is the introduction of multiplicative noise resulting from the randomness of the process. Equation (30) shows that the AS noise consists of two terms, the first of which is often referred to as signal shot noise while the second is called amplification noise.

#### IV. AMPLIFIED SPONTANEOUS EMISSION IN A TRAVELING-WAVE AMPLIFIER

##### A. Output Point Process

We now calculate the output point process for a TWA with spontaneous emission occurring uniformly within it, when there are no signal photons at the input. We assume that each of the atoms in the upper level in the amplifier has the same probability density  $p_{sp}$  ( $s^{-1}$ ) of producing a photon spontaneously into a single mode. The probability density of spontaneous emission into  $M'$  modes is defined as  $P_{sp}$  so that [1]

$$P_{sp} = M' p_{sp} \quad (32)$$

and

$$M' = P_{sp}/p_{sp}. \quad (33)$$

Because they are independent emissions, the photons spontaneously emitted from the small volume surrounding

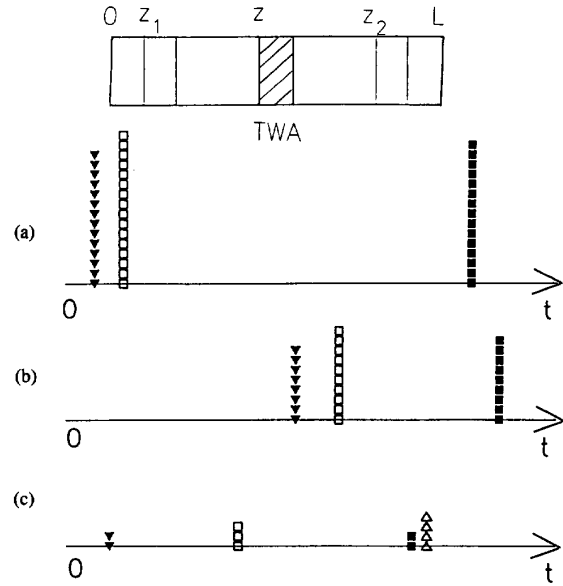


Fig. 3. The MP point processes at the amplifier output, resulting from spontaneous emissions in regions surrounding  $z_1$ ,  $z$ , and  $z_2$  respectively, are shown in (a)–(c). The different symbols represent different frequencies (or polarizations or directions) at which the photons are spontaneously emitted. The spontaneous emissions take the form of a Poisson point process. The average height of the mark for a particular position represents the gain of the BD process. Spontaneously emitted photons originating from  $z_1$  therefore experience a larger mean gain than those emitted from  $z$  or  $z_2$ .

position  $z$  form a Poisson process with rate  $\lambda(z)$ . This spontaneously emitted photon stream can be considered as an equivalent input signal to a sub-amplifier extending from position  $z$  to the output end of the TWA at  $z = L$ . In accordance with the results in (17)–(24), the output of the amplifier with this effective input may be described as a MP point process, as illustrated in Fig. 3, with the mark-PGF

$$G_{AS}(s, z) = \frac{1 + [g(z) - \langle n_{amp}(z) \rangle] (s-1)}{1 - \langle n_{amp}(z) \rangle (s-1)}, \quad (34)$$

where

$$g(z) = \exp \left[ \int_z^L [\gamma_{st}(z') - \gamma_{ab}(z')] dz' \right] \quad (35)$$

and

$$\langle n_{amp}(z) \rangle = g(z) \int_z^L \frac{\gamma_{st}(z')}{g(z')} dz'. \quad (36)$$

It is clear that the further the position  $z$  is from the amplifier output, the more the spontaneous emission photons are amplified on average (see Fig. 3). Also, the higher the density  $N_2(z)$ , the higher the rate  $\lambda(z)$ .

The ASE emerging from the entire TWA can therefore be modeled as a point process formed from the union of all the MP sub-processes produced by the photons spontaneously emitted from small volumes at different position  $z$  along the amplifier. Since the spontaneous emissions

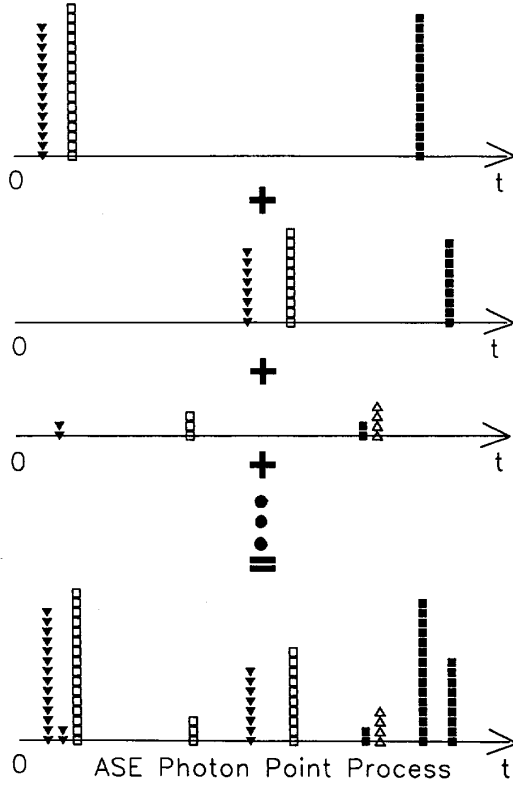


Fig. 4. The total ASE photon point process at the output of a TWA is a MP point process that is the union of the constituent MP processes generated from the photons spontaneously emitted from different positions  $z$  along the amplifier.

from the different atoms are independent, and the amplifier is assumed to be operating in the linear (unsaturated) region, the overall ASE point process retains the form of an MP point process [30], as shown in Fig. 4. The overall rate is

$$\lambda_{ASE} = \int_0^L P_{sp} AN_2(z) dz, \quad (37)$$

and the mark-PGF is

$$G_{ASE}(s) = \frac{\int_0^L G_{AS}(s, z) P_{sp} AN_2(z) dz}{\int_0^L P_{sp} AN_2(z) dz}.$$

Using the results following (34), after some algebra we obtain

$$\begin{aligned} G_{ASE}(s) &= 1 - \frac{P_{sp}}{\lambda_{ASE} \pi_{st}} \ln [1 - \langle n_{amp} \rangle (s - 1)] \\ &= 1 - \frac{M'}{\lambda_{ASE} \tau_c} \ln [1 - \langle n_{amp} \rangle (s - 1)], \end{aligned} \quad (38)$$

where  $\langle n_{amp} \rangle$  is the amplification-noise parameter defined by (20),  $\tau_c$  is the duration of a photon wavepacket (i.e., the coherence time [1]), and  $p_{sp} = \pi_{st}/\tau_c$ .

In short, the ASE at the output of a TWA is represented, like the AS, by an MP point process but now with a mark-PGF  $G_{ASE}(s)$  given by (38). This is illustrated in Fig. 4.

### B. Photon-Counting Statistics

With the ASE point process in hand, we can now examine its photon-counting statistics. Using (25), (26), and (38), we obtain the PGF of the ASE photon-counting distribution

$$\begin{aligned} G_{ASEc}(s) &= \exp \{ \lambda_{ASE} T (G_{ASE}(s) - 1) \} \\ &= \exp \left\{ \lambda_{ASE} T \frac{-M'}{\lambda_{ASE} \tau_c} \ln [1 - \langle n_{amp} \rangle (s - 1)] \right\} \\ &= [1 - \langle n_{amp} \rangle (s - 1)]^{-M}, \end{aligned} \quad (39)$$

with  $M = M' T / \tau_c$ . Equation (39) is the PGF of a negative-binomial (NB) distribution with  $M$  degrees of freedom. Indeed, this negative-binomial distribution may be viewed as a Poisson-driven logarithmic distribution, as was established in the BDI literature long ago [31], [32]. Using (39) and (9), we explicitly write the ASE PND as

$$P_{ASEc}(n) = \binom{n + M - 1}{n} \frac{\langle n_{amp} \rangle^n}{(1 + \langle n_{amp} \rangle)^{n+M}} \quad (40)$$

which, as will be explained in Section V, is also a special case of the NNB distribution. The mean and variance of the ASE photon-number are

$$\overline{n_{ASEc}} = M \langle n_{amp} \rangle \quad (41)$$

and

$$\text{Var}_{ASEc}(n) = M \langle n_{amp} \rangle (1 + \langle n_{amp} \rangle), \quad (42)$$

respectively, and the Fano factor is

$$F_{ASEc} = 1 + \langle n_{amp} \rangle. \quad (43)$$

The number of degrees of freedom, more generally, is given by

$$M \approx M' \frac{T}{\tau_c} \frac{A}{A_c} \frac{2}{1 + \mathcal{P}^2} \quad (44)$$

where  $M' = P_{sp}/p_{sp}$ , as defined earlier. The quantity  $T/\tau_c$  is the ratio of the counting time to the coherence time,  $A/A_c$  is the ratio of the detection area to the coherence area, and  $\mathcal{P}$  is the degree of polarization. This expression for  $M$  is valid for  $T/\tau_c \gg 1$  and  $A/A_c \gg 1$  [33]. For  $T \ll \tau_c$ ,  $A \ll A_c$ , and linear polarization,  $M \approx M'$ . The number of modes  $M$  is therefore decreased by narrowing the optical filter bandwidth at the output of the amplifier (which increases the coherence time  $\tau_c$  and thereby decreases  $M$ ), by using a short integration time  $T$ , by using a small detection area  $A$ , and by using linearly polarized light ( $\mathcal{P} = 1$ ).

### V. OVERALL OUTPUT PHOTON POINT PROCESS FOR A TRAVELING-WAVE AMPLIFIER

Armed with knowledge of the AS and ASE point processes, we now examine the statistics of the photon events at the output of a TWA with various kinds of input.

#### A. No Input Light

Without input light, the TWA output is simply the ASE point process. As described in the preceding section, it is therefore characterized by a MP point process (Fig. 4) with a rate  $\lambda_{ASE}$  given by (37) and a mark-PGF  $G_{ASE}(s)$  given in (38); its photon-counting distribution is the NB distribution given in (40).

#### B. Single Input Photon

With a single photon entering the TWA at time  $t = 0$ , the overall output is the union of the AS and ASE point processes. The overall point process therefore consists of (1) a cluster of photons appearing at  $L/c$  with the PGF of a BD process  $G_{AS}(s)$  defined in (17), and (2) the ASE MP point process with rate  $\lambda_{ASE}$  given by (37) and mark-PGF  $G_{ASE}(s)$  given in (38).

The total output photon-counting statistics, when the counting time  $T$  covers the time point  $L/c$ , is represented by the PGF

$$G_{out}^{(1)}(s) = G_{AS}(s) \cdot G_{ASE}(s) \\ = \left[ \frac{1 + (g - \langle n_{amp} \rangle)(s - 1)}{1 - \langle n_{amp} \rangle(s - 1)} \right] \\ \cdot [1 - \langle n_{amp} \rangle(s - 1)]^{-M}, \quad (45)$$

which is the PGF of the BDI process [26], [29].

#### C. Coherent Input Light

With coherent light at the amplifier input, the total TWA output consists of two parts. The first part is, in accordance with Section III, the AS MP point process with primary rate  $\lambda_s$  and mark-PGF  $G_{AS}(s)$ . The second part is, in accordance with Section IV, the ASE MP point process with rate  $\lambda_{ASE}$  and mark-PGF  $G_{ASE}(s)$ . Since the ASE is independent of the AS, the union of these two MP point processes is also a MP point process (see Fig. 5) with an effective rate

$$\lambda_{out} = \lambda_s + \lambda_{ASE}, \quad (46)$$

and with a mark-PGF

$$G_{out}(s) = \frac{\lambda_s G_{AS}(s) + \lambda_{ASE} G_{ASE}(s)}{\lambda_{out}}. \quad (47)$$

There are two convenient ways of determining the photon-counting statistics of the total output of a TWA with coherent input light. First, since the ASE and AS are independent, the PGFs multiply, i.e.,

$$G_{out}(s) = G_{AS}(s) G_{ASE}(s). \quad (48)$$

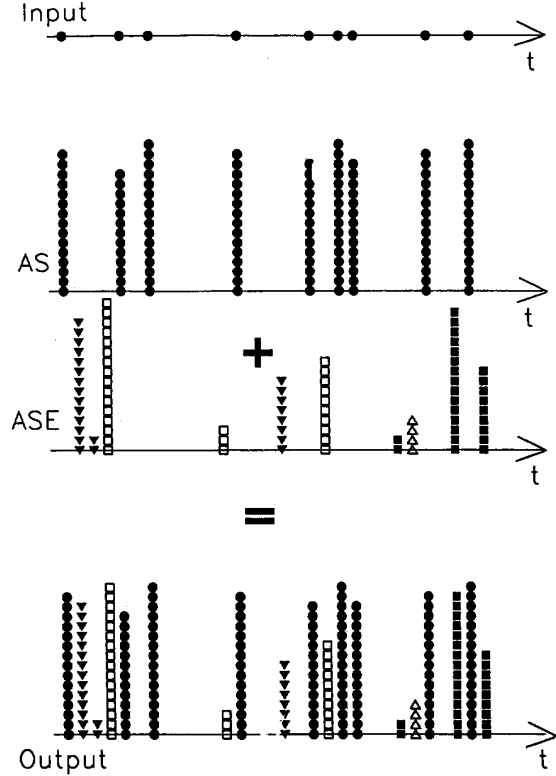


Fig. 5. The total output photon point process of a TWA with coherent input light is an MP process consisting of the union of the AS MP process and the ASE MP process.

Using (27) and (39), we therefore have

$$G_{out}(s) = \exp \left[ \frac{g \langle n_s \rangle (s - 1)}{1 - \langle n_{amp} \rangle (s - 1)} \right] \\ \cdot [1 - \langle n_{amp} \rangle (s - 1)]^{-M}. \quad (49)$$

Alternatively, since the total output of the amplifier itself is a MP point process, according to the approach used to derive (27) and (39), we have

$$G_{out}(s) = \exp [\lambda_{out} T (G_{out}(s) - 1)]. \quad (50)$$

Using (46) and (47) in (50), we obtain

$$G_{out}(s) = \exp [\lambda_s T (G_{AS}(s) - 1)] \\ \cdot \exp [\lambda_{ASE} T (G_{ASE}(s) - 1)] \quad (51)$$

which, with the help of (17) and (38), again yields (49).

Equation (49) is the PGF for the generalized Laguerre polynomial distribution, also known as the noncentral-negative-binomial (NNB) distribution [26], [28], [33]

$$P_{out}(n) = \frac{\langle n_{amp} \rangle^n}{(1 + \langle n_{amp} \rangle)^{n+M}} \exp \left[ -\frac{g \langle n_s \rangle}{1 + \langle n_{amp} \rangle} \right] \\ \cdot L_n^{(M-1)} \left[ -\frac{g \langle n_s \rangle}{\langle n_{amp} \rangle (1 + \langle n_{amp} \rangle)} \right], \quad (52)$$

where  $L_n^{(M-1)}$  is the generalized Laguerre polynomial

$$L_n^{(M-1)}(-x) = \sum_{k=0}^n x^k \frac{(n+M-1)!}{(k+M-1)!(n-k)!k!}.$$

It is of interest to note that  $P_{Asc}(n)$  defined in (28) is a special case of the NNB distribution  $P_{outc}(n)$  with  $M=0$ .  $P_{ASEc}(n)$ , defined in (40), is another special case of the NNB distribution  $P_{outc}(n)$  with  $\langle n_s \rangle = 0$ . All three of the PNDs,  $P_{Asc}(n)$ ,  $P_{ASEc}(n)$ , and  $P_{outc}(n)$ , are, therefore, NNB distributions. The mean and variance of the total output photon number of a TWA with coherent input light are

$$\overline{n_{outc}} = g \langle n_s \rangle + M \langle n_{amp} \rangle \quad (53)$$

and

$$\text{Var}_{outc}(n) = g \langle n_s \rangle + 2g \langle n_s \rangle \langle n_{amp} \rangle + M \langle n_{amp} \rangle (1 + \langle n_{amp} \rangle), \quad (54)$$

respectively, which accord with the expressions generally used [2], [23], [34], [35].

From (54) it is apparent that the total output photon-number noise of a TWA with coherent input light consists of three terms, generally designated signal shot noise, amplification noise, and ASE noise, respectively. The signal shot noise (first term) arises from the presence of the input light. The ASE noise (third term) is due to the spontaneous emission arising inside the amplifier. The amplification noise (second term) is a gain noise associated with the randomness of the amplification process of the TWA, and is unrelated to spontaneous emission.

The semiclassical analysis of optical amplifier noise gives rise to the same results as those given in (52)–(54) [36]. In that theory, however, the amplification noise (the second term on the right-hand side of (54)) is interpreted as signal-spontaneous beat noise rather than gain noise. The semiclassical theory imparts this interpretation because it treats the TWA as a purely deterministic amplifier plus an independent additive interfering ASE source of noise.

## VI. AMPLIFIER PERFORMANCE

### A. Probability of Error

Consider a binary on-off keying (OOK) direct-detection photon-counting lightwave communication system; the photon-number statistics approach is adequate for this configuration. If  $P_1$  is the probability of mistaking “1” for “0”, and  $P_0$  is the probability of mistaking “0” for “1”, and if the “1” and “0” bits are equally likely to be transmitted, then the probability of error is [37]

$$\text{PE} = \frac{1}{2} (P_0 + P_1), \quad (55)$$

with

$$P_0 = \sum_{n=D}^{\infty} P_N(n), \quad (56)$$

$$P_1 = \sum_{n=0}^{D-1} P_{SN}(n), \quad (57)$$

so that

$$\text{PE} = \frac{1}{2} \sum_{n=D}^{\infty} P_N(n) + \frac{1}{2} \sum_{n=0}^{D-1} P_{SN}(n). \quad (58)$$

The quantity  $D$  represents the detection threshold count, and  $P_N(n)$  and  $P_{SN}(n)$  represent the PND's at the output of the system when a “0” (the output is noise alone) and a “1” (the output is signal-plus-noise) is transmitted, respectively. Since “0” and “1” are equally likely to be transmitted, the optimal threshold  $D$  resulting in the minimum PE is set at the point for which  $P_N(D) = P_{SN}(D)$ , which is also known as the maximum likelihood threshold [37].

We now evaluate the performance of such a system incorporating a TWA, in the absence of extraneous background light and with a receiver of unity quantum efficiency and negligible dark and electronic (thermal) noise. For simplicity, it is assumed that all of the signal photons are collected in each bit and that there is no intersymbol interference.  $P_{SN}(n)$  is then the NNB counting distribution represented in (52) and  $P_N(n)$  is the NB distribution represented in (40). Substituting these two distributions into (58) allows us to calculate the PE for various values of  $\langle n_s \rangle$ . Because of the form of these distributions, (58) has no closed form solution; thus numerical methods are used to calculate it on a computer, and the results are shown as the solid curves in Figs. 6–8 for various values of  $g$ ,  $M$ , and  $\langle n_{amp} \rangle$ . The results presented in these figures show that: (1) increasing the mode parameter  $M$  increases the PE of the system (Figs. 6 and 7); (2) increasing the amplification-noise parameter  $\langle n_{amp} \rangle$  also increases the PE (Figs. 8d–8f); (3) when  $g$  is small ( $< 10$ ) increasing the gain  $g$  increases the PE of the system; however, when  $g$  is very large ( $> 100$ ), increasing the gain  $g$  has virtually no effect on PE (Figs. 8a–8c).

### B. Gaussian Approximation

Although using the NNB distributions provides an exact solution for the PE, the numerical calculation is complex and time consuming and difficult to carry out for large values of  $n$ . Therefore, simpler approximate forms for the PE are desired. Gaussian distributions, with the same means and variances as those of the NNB and NB, are often used to calculate the PE for lightwave systems using TWAs [23], [38]. In this case, (58) becomes

$$\text{PE} \approx \frac{1}{2} \left[ 1 - \frac{1}{2} \text{erf} \left( \frac{|D - \mu_1|}{\sqrt{2} \sigma_1} \right) - \frac{1}{2} \text{erf} \left( \frac{|D - \mu_0|}{\sqrt{2} \sigma_0} \right) \right], \quad (59)$$

where  $\mu_1$ ,  $\sigma_1$  and  $\mu_0$ ,  $\sigma_0$  are the means and standard deviations of the output when a “1” and a “0” bit is transmitted, respectively;

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (60)$$

$$6 - 2^{ND} \text{ to } 5$$



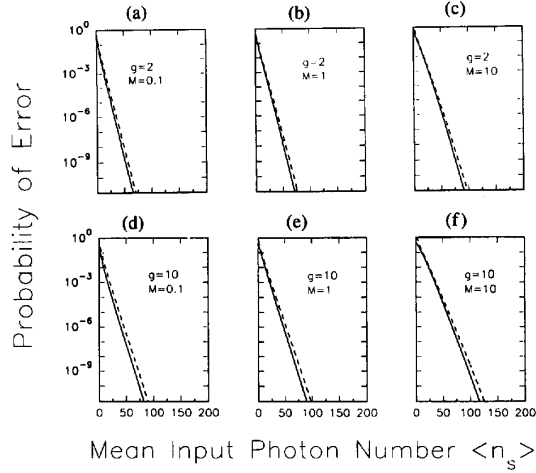


Fig. 6. Probability of error (PE) for a binary OOK direct-detection system using a TWA modeled by NNB (solid curves) and Gaussian (dashed curves) photon-number statistics. In all cases  $\langle n_{amp} \rangle = g - 1$ .  $M$  takes the values 0.1, 1, and 10 as indicated. For (a)–(c)  $g = 2$ , whereas for (d)–(f)  $g = 10$ .

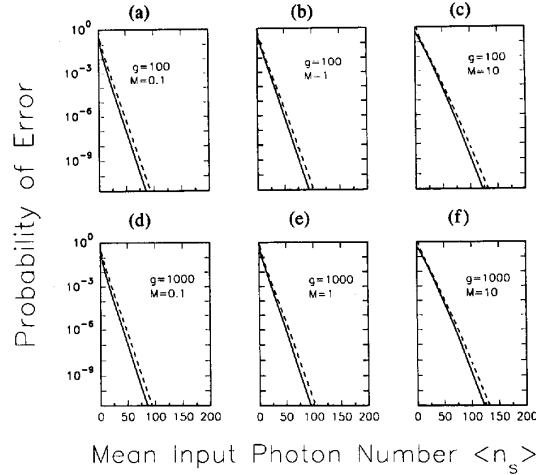


Fig. 7. PE as shown in Fig. 7, except that the gain is now higher; for (a)–(c)  $g = 100$  and for (d)–(f)  $g = 1000$ .

is the error function; and  $D$  is the optimal threshold set at the count for which  $P_{SN}(D) = P_N(D)$ .

Even using the Gaussian approximation, it is difficult to calculate the appropriate value of  $D$ . Thus further approximation is useful. Under the conditions

$$\begin{cases} Q = \frac{|\mu_1 - \mu_0|}{\sigma_1 + \sigma_0} \\ \sigma_1 \approx \sigma_0, \end{cases} \quad (61)$$

(59) can be approximated by [37], [39]

$$PE \approx \frac{1}{\sqrt{2\pi}} \frac{\exp(-Q^2/2)}{Q}. \quad (62)$$

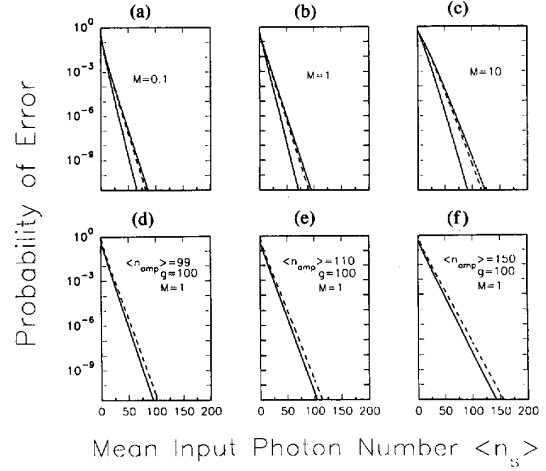


Fig. 8. (a)–(c) Influence of the amplifier's gain on the PE. Each figure has 4 curves representing the PE calculated from NNB distributions with  $g = 2$  (solid curves),  $g = 10$  (long-dash curves),  $g = 100$  (short-dash curves), and  $g = 1000$  (dotted curves). For all figures  $\langle n_{amp} \rangle = g - 1$  and  $M =$  (a) 0.1, (b) 1, and (c) 10. The short-dash and dotted curves are so close to each other as to be indistinguishable. (d)–(f) Influence of the amplification-noise parameter  $\langle n_{amp} \rangle$  on the PE. Each figure shows the PE calculated using NNB (solid curves) and Gaussian (dashed curves) statistics, with  $\langle n_{amp} \rangle =$  (d) 99, (e) 110, and (f) 150. For all figures  $g = 100$  and  $M = 1$ .

This formula is often used for calculating the PE of an OOK direct-detection system using a TWA [35], [38]. Using (62) we calculate the approximate PE of the system with the same parameters as those used to calculate the exact PE (solid curves) in Figs. 6, 7, and 8(d–f); the results are shown as the dashed curves in Figs. 6, 7, and 8(d–f). Comparing the two sets of curves, it is seen that the Gaussian approximation generally overestimates the PE, but with the parameters used, it is unexpectedly close to the exact PE (<1 dB difference). However, although the Gaussian and exact results are close, the optimal threshold predicted by the Gaussian approximation is quite different from that predicted by the NNB [26], [40].

### C. Discussion

Care must be exercised when using (62) to calculate the PE of an OOK direct-detection system using a TWA. First, (59) must provide a proper approximation for (58), since the PE depends essentially on the tails of  $P_{SN}(n)$  and  $P_N(n)$ . Even though  $P_{SN}(n)$  and  $P_N(n)$  can be approximated by Gaussian distributions under certain conditions (e.g., a high input photon rate), this does not mean that their tails can also be approximated by Gaussian tails. We have shown earlier that the tails of the NNB distributions do not approach those of Gaussian distributions, even when the input photon rate increases without bound [26]. Secondly, even if (59) does provide a proper approximation to (58), we must make sure that the condition in (61) is fulfilled in order to be able to use (62).

In the special case in which the quantum limit is to be calculated, with a PE =  $10^{-9}$  for example, only misses

are possible and the probability  $P_1$  of mistaking "1" for "0" is equal to the probability of detecting zero photons, i.e.,  $P_1 = P_{\text{Asc}}(0) = \exp(-\langle n_s \rangle)$ . When bit "0" is transmitted, there are no photons; the receiver decides correctly that bit "0" has been transmitted, so that  $P_0 = 0$ . The PE is then, using equation (55),

$$\text{PE} = \frac{1}{2} \exp(-\langle n_s \rangle) = \frac{1}{2} \exp(-2\langle n_a \rangle), \quad (63)$$

where

$$\langle n_a \rangle = \frac{1}{2} \langle n_s \rangle \quad (64)$$

is the overall mean number of input photons per bit. For  $\text{PE} = 10^{-9}$ , (63) gives  $\langle n_a \rangle \approx 10$  photons per bit.

However, using (62), we instead obtain a value  $\approx 38$  photons per bit [38]. The discrepancy arises because (1) at the quantum limit the input light is so weak that the Gaussian is not a proper approximation for  $P_{\text{SN}}(n)$ ; and (2) the  $\text{ASE} = 0$  so that (61) is not satisfied. Therefore (62) does not provide a good approximation in this case. The result is analogous to that obtained when calculating the quantum limit for an OOK direct-detection system using coherent light in the absence of an optical amplifier (Poisson PND). Using the exact Poisson form of  $P_{\text{SN}}(n)$  gives the well-known "quantum limit" of 10 photons/bit, but using the Gaussian approximation, (62) gives a value of 18 photons/bit [1].

## VII. CONCLUSION

We have shown that, with coherent light presented at the input of a TWA, the output photons obey a marked-Poisson point process which gives rise to the noncentral-negative-binomial (NNB) photon-number distribution. The amplification-noise parameter  $\langle n_{\text{amp}} \rangle$  represents gain-fluctuation noise arising from the random nature of the amplification process.

The results obtained here are also applicable to a cascade of optical amplifiers, even in the presence of intervening loss, provided that the mode parameter  $M$  is the same for all amplifiers [27], [41].

The probability of error (PE) for a binary on-off keying direct-detection photon-counting communication system employing a TWA has been calculated using the noncentral-negative-binomial distribution. The results show that reducing the mode parameter  $M$  (by suitably filtering the amplifier output), and reducing the amplification-noise parameter  $\langle n_{\text{amp}} \rangle$ , can significantly improve the performance of the system; however, altering the gain  $g$  of the amplifier when it is sufficiently large ( $> 100$ ) has virtually no influence on system performance.

Furthermore, use of the Gaussian approximation for the PE has been compared with the exact results for the PE. They are unexpectedly close, although the optimal count threshold predicted by the Gaussian approximation is very different from that predicted by the exact NNB distribution.

The results obtained here enable the degradation of system performance arising from intersymbol interference to be calculated. It will be useful to extend this approach to

include nonlinearity in the amplifier as well and we are currently examining this problem [17].

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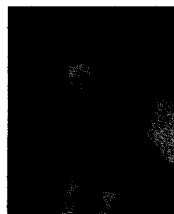


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