Noise in Resonant Optical Amplifiers of General Resonator Configuration

EVAN L. GOLDSTEIN AND MALVIN C. TEICH, FELLOW, IEEE

Abstract—**The population-statistical description of a linear optical amplifier, extended to include both a resonant optical cavity and explicit input and output photon fluxes, is shown to yield three useful noise relationships. These apply to amplifiers of quite general resonator structure, including those employing distributed feedback and distributed Bragg reflection. Despite the treatment's plainly particulate character, it provides an expression for the variance of the time-integrated output-photon-detection process in which terms associated with interferometrically generated beat noise are prominent. For systems whose photodetector electrical bandwidth B is much smaller than the amplifier's optical bandwidth $\Delta\nu$, as is almost invariably the case for semiconductor amplifiers in lightwave-communication systems, the time-integrated photocurrent variance can be written in a simple, physically intuitive form. Two quotients are of central importance: $(\Delta\nu/B)$ and the ratio $(\langle n_S \rangle / \langle n_{\text{ASE}} \rangle)$ of mean amplified-signal power to mean amplified-spontaneous-emission power. In the usual case of interest for optical amplifiers in communication systems, where $\langle n_S \rangle \gg \langle n_{\text{ASE}} \rangle$, the signal-to-noise ratio of the integrated photocurrent, on which the error probability in an on-off-keyed system depends, is $\approx \frac{1}{2}(\Delta\nu/B)(\langle n_S \rangle / \langle n_{\text{ASE}} \rangle)$.**

I. Introduction

THE photon-statistical treatment of noise in inverted-population amplifiers has its origins in the efforts of Shimoda, Takahasi, and Townes [1] to apply simple branching-process ideas from cosmic-ray counting and population biology to fluctuations in the internal photon population of a maser. Their work resulted in a set of quite general expressions for the moments and distribution of the maser's internal photon number. More than 20 years later, Yamamoto [2] recognized that the variance expression obtained from this particle treatment could be rewritten as a simple, physically-transparent equation containing contributions from both beat noise and shot noise. The resulting well-known expression for the variance [3] has served as a point of departure for a good deal of the subsequent work on the subject.

Although treatments of this nature have been extended to resonant Fabry–Perot-type optical amplifiers [2], [4], [5], the photon statistics in their usual form apply properly only to amplifiers of traveling-wave structure. There are two reasons for this. First, the treatment describes only the amplifier's internal photon population, which is viewed as evolving stochastically from some initial photon distribution at the time origin. There is no provision for a continuous input-photon flux representing a signal incident upon a resonant structure, nor is there any explicit *output*-photon flux. While this is not troublesome for traveling-wave-amplifier descriptions, where the output is taken as the internal photon population delivered through the structure after a single-pass transit time, difficulties arise when the treatment is extended to resonant structures, where the output process is not readily viewed as the offspring of an initial distribution after a fixed time interval. In addition, for power gains greater than unity (i.e., amplification), the model's internal population must be described by a process whose mean grows exponentially in time; hence, when the model is applied to such devices, its time variable must be interpreted as a well-defined propagation time for a wave traversing an effectively single-pass structure.

Furthermore, a shortcoming for both traveling-wave and resonant amplifier treatments is that, since the standard photon-statistical description treats only the internal photon-number evolution, it yields no temporal information about the output-photon point process. In particular, it does not reveal the output correlation properties. Hence, one obtains little direct information about the higher moments of the photocurrent that would be measured by a real, bandwidth-limited photodetector placed at the amplifier output.

In spite of these obstacles to a theoretical treatment of their noise, resonant optical amplifiers command increasing interest. In addition to the familiar Fabry–Perot devices, amplification has recently been demonstrated in novel resonant structures. For example, optical amplifiers incorporating distributed-feedback (DFB) resonators have recently been shown to be capable of rapidly tunable, high-resolution, wavelength-selective amplification [6]–[9]. However, little is known about their noise properties [10]. Thus, a noise theory that proceeds directly from the statistics of the *output* photon number, rather than the internal photon population, and that applies in general to resonant optical amplifiers, would be welcome.

Recently, significant progress toward this end has been made by introducing new variables into the formal

Manuscript received March 31, 1989. This work was supported in part by the Center for Telecommunications Research and the Joint Services Electronics Program at Columbia University.

E. L. Goldstein is with Bellcore, Morristown, NJ 07960.

M. C. Teich is with the Center for Telecommunications Research, Department of Electrical Engineering, Columbia University, New York, NY 10027.

IEEE Log Number 8930608.

0018-9197/89/1100-2289$01.00 © 1989 IEEE

branching-process treatment [11], [12], and by reinterpreting old variables along lines first suggested in the paper of Shimoda *et al.* [1]. This has generated the beginnings of a fully photon-statistical theory of amplification noise in resonant optical structures. One virtue of this treatment is that all statistical quantities of interest, including both the distributions and the correlations of the detector's time-integrated photocurrent, are obtained directly from the photon statistics themselves, without recourse to phenomenological modifications aimed at accounting for the finite RF bandwidth of the photodetector. As before, the formal results are fairly abstract. However, we show in this paper that some simple physics lurks within them and that, if this is recognized, one can draw several useful inferences. By examining the resonant amplifier's output photon-counting process in much the same spirit as that applied by Yamamoto to the internal-population branching process of a single-pass device, we show that the resonant amplifier's output statistics can be recast in a simple, physically-transparent form. In particular, the physics responsible for the form of the variance expression is clarified revealing that, as with the internal population number, the prominent output-noise terms take the form of a combination of interferometric beat noise and shot noise.

We first review the state of theoretical knowledge about the internal-photon statistics (Section II) and the output fluctuations (Section III) of resonant optical amplifiers, pointing out, enroute, this treatment's relationship to earlier work. We then establish a physical interpretation (Section IV) that illuminates the mechanism responsible for the photocurrent variance generated by a detector placed at the amplifier output. Finally (Section V), we use the physical insight so obtained to generate three relationships governing the noise in resonant optical amplifiers. Applications to DFB amplifiers in lightwave-communication systems are pointed out.

II. Resonant-Cavity Population Statistics

Following the early work of Shimoda *et al.* [1], the amplifier's photons are treated as a population governed by a simple birth–death–immigration process, with birth and death representing stimulated emission and absorption, respectively, and with immigrants supplied by a spontaneous-emission process. The birth rate aN and death rate bN then depend on the population size N, whereas the immigration rate c does not. We cast the discussion entirely in the language of population statistics, whose underpinnings in the quantum statistics of an electromagnetic-field mode interacting with a two-level atomic system have received attention elsewhere in the literature [13]–[16]. The amplifier is assumed linear in the sense that light incident upon it is coupled to a collection of nonsaturable atoms.

By taking the limit of a difference equation it is easy to show that the probability $P(N; t)$ of finding N individuals in the population at time t satisfies the set of forward Kolmogorov equations

$$\frac{d}{dt} P(N; t) = -[(a+b)N + c] P(N; t) + [a(N-1) + c] P(N-1; t) + b(N+1) P(N+1; t) \qquad (1)$$

whose initial condition is provided by the initial distribution $P(N; 0)$. The solutions of (1) for arbitrary time and for an arbitrarily-distributed initial photon population $N_0 \equiv N(t = 0)$, though well known [1], are complex. Even the population's first two central moments are unwieldy:

$$\langle N \rangle = \langle N_0 \rangle e^{(a-b)t} + \frac{c}{a-b}[e^{(a-b)t} - 1], \qquad (2)$$

$$\mathrm{var}(N) = \left[\frac{a+b}{a-b}\langle N_0 \rangle + \frac{ab + c(a-b)}{(a-b)^2} + \mathrm{var}(N_0)\right] e^{2(a-b)t} - \left[\frac{a+b}{a-b}\langle N_0 \rangle + \frac{c(a+b)}{(a-b)^2}\right] \cdot e^{(a-b)t} + \frac{bc}{(a-b)^2}. \qquad (3)$$

These have simple physical interpretations, however. Equation (2) says that after a time t, the amplifier's initial population N_0 has on average experienced a gain of $e^{(a-b)t}$, with its spontaneously-emitted population similarly amplified. Equation (3), a cornerstone of recent optical-amplifier-noise discussions, has been shown by Yamamoto [2] to represent beat noise, shot noise, and other fluctuations that are usually negligible.

Although (2) and (3) hold for arbitrary (but constant) a, b, and c, it is evident that gains larger than one occur only if the stimulated-emission coefficient a exceeds the loss coefficient b. In this case, however, N is nonstationary and has a mean that grows exponentially in time. Thus, the treatment applies naturally only to single-pass amplifiers possessing a well-defined wave-propagation time t.

Imposing a resonant cavity on the population is in fact a simple matter, however, and it results in considerable simplification of the expressions provided above. One need only reinterpret several variables [1], [11], [12]. Since the above discussion assumes no particular structure for the amplifier, (1)–(3) actually continue to apply. However, the existence of a stationary equilibrium cavity population N for large t requires that the formal rate coefficients satisfy $b > a$, which in turn forces one to view the device's gain a little differently than before. This is in any case unavoidable since, for $b > a$, it is known that the equilibrium population properties are independent of the initial population N_0. The equilibrium distribution $P_E(N)$ is then of simple negative-binomial form [11]:

$$P_E(N) = \begin{cases} \binom{N + (c/a) - 1}{N} (a/b)^N (1 - a/b)^{c/a}, \\ \quad N \geq 0 \\ 0, \quad N < 0. \end{cases} \tag{4}$$

Imposing a cavity and allowing $b > a$ (and therefore losing the statistical dependence on N_0) has a second consequence: the quantity representing the amplifier's input has vanished. This may be remedied [1] simply by reinterpreting the description's immigration parameter c. For the results to this point, c represents only spontaneous emission and is set equal to a. One now lets it also represent the photon flux incident on the amplifier, by virtue of which signal photons enter the cavity at a constant rate c_i per unit time. Thus, the entire description obtained so far remains intact, with the immigration rate given by $c = c_i + a$. In particular, the first two moments of the cavity population are still given by (2) and (3). However, for $b > a$, all the exponentials in (2) and (3)—which yield the dominant terms in the standard treatment due to Yamamoto [2], [4], [5]—decay away with time. Only a few terms then remain, and the moments of the equilibrium population become simple indeed:

$$\langle N \rangle = \frac{a + c_i}{b - a} \tag{5}$$

$$\text{var}(N) = \frac{b(a + c_i)}{(b - a)^2}. \tag{6}$$

At equilibrium, the cavity thus contains an average of $c_i/(b - a)$ daughter photons from the amplified-signal process and $a/(b - a)$ due to spontaneous emission. The quantity $(b - a)^{-1}$, which is proportional to the quality factor of the loaded cavity, will be seen to determine the crucial correlation properties.

III. Output-Process Statistics

A. Moments

Despite the simplicity of the resulting expressions, the reinterpretation just described illuminates only the cavity-population statistics. The statistics of central concern, namely those of the amplifier's output photon number, have not been dealt with. Indeed, the description as it stands contains no output flux at all. Again, this may be remedied [11], [17] by reinterpreting a branching-process variable. Whereas b had previously represented only internal losses, one now includes in it losses through the amplifier's coupling ports. Thus, one sets $b = b_u + b_i + b_o$, representing the processes by which photons are lost internally at a rate $b_u N$, are coupled out the device's input port at a rate $b_i N$, and are coupled to the output port at a rate $b_o N$. This results in increased damping but requires no formal changes in the expressions obtained so far.

We now have the process of central concern, namely the counting process defined by the coupling of cavity photons to the amplifier's output port. Its statistics are needed. The distribution $P^{\text{count}}(n; T)$ is defined as the probability of obtaining n photoelectron counts at the detector within an integration time T when the cavity population is in the steady state. Its associated factorial-moment-generating function $Q^{\text{count}}(s; T)$, defined by

$$Q^{\text{count}}(s; T) = \sum_{n=0}^{\infty} P^{\text{count}}(n; T)(1 - s)^n, \tag{7}$$

has been derived in [11], [12] and is

$$\begin{aligned} Q^{\text{count}}&(s; T) \\ &= \exp[\gamma(1 + c_i/a)] \cdot \{\cosh y(s) \\ &\quad + [y(s)/2\gamma + \gamma/2y(s)] \sinh y(s)\}^{-(1 + c_i/a)}, \end{aligned} \tag{8}$$

where

$$\gamma = (b - a)T/2 \tag{9}$$

$$y(s) = \sqrt{\gamma^2 + ab_o sT^2}. \tag{10}$$

Inverting (8) presents difficult integrals, so it does not readily yield an expression for $P^{\text{count}}(n; T)$ that is valid for arbitrary T. However, by differentiating (8) and setting $s = 0$ one obtains the factorial moments of the counting distribution, and from these the central moments follow. We need only the first two. The mean

$$\langle n \rangle = \frac{b_o(a + c_i)}{b - a} T, \tag{11}$$

could have been readily guessed; by comparison with (5), it is evident that the mean photocount registered in time T is simply $b_o T$ times the mean equilibrium cavity population. However, the photocount variance is far less transparent:

$$\begin{aligned} \text{var}(n) = \frac{2b_o^2 a(a + c_i)}{(b - a)^4} &[(b - a)T \\ &+ e^{-(b-a)T} - 1] + \frac{b_o(a + c_i)}{b - a} T. \end{aligned} \tag{12}$$

B. Correlations

To reveal the physics lurking in (12), we need the two-time correlation properties of the cavity photon population N and the output photon number n, which are known, and which we express in autocovariance form. For the equilibrium cavity population, the usual normalized autocovariance is [11]

$$\frac{\langle N(t_1) N(t_2) \rangle - \langle N \rangle^2}{\langle N \rangle^2} = \left(\frac{a}{a + c_i} + \frac{1}{\langle N \rangle} \right) e^{-(b-a)|t_2 - t_1|}, \tag{13}$$

where t_1 and t_2 are arbitrary times. For the output counting process, a corresponding quantity, the normalized integrated photoelectron-number autocovariance, may be constructed by considering the covariance of the number n of counts in $(t_1, t_1 + T)$ and the number n' of counts in the nonoverlapping interval $(t_2, t_2 + T)$. This function, defined by

$$\frac{\langle n(t_1)\, n(t_2)\rangle - \langle n\rangle^2}{\langle n\rangle^2} = \frac{1}{\langle n\rangle^2} \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} (n - \langle n\rangle)(n' - \langle n\rangle) \cdot P^{\text{count}}(n, n'; t_1, t_2, T), \tag{14}$$

where $P^{\text{count}}(n, n'; t_1, t_2, T)$ is the joint distribution of n and n', can be written [11] as

$$\frac{\langle n(t_1)\, n(t_2)\rangle - \langle n\rangle^2}{\langle n\rangle^2} = \frac{a}{a + c_i} \left(\frac{\sinh\,[(b - a)\,T/2]}{(b - a)\,T/2}\right)^2 e^{-(b-a)|t_2 - t_1|}. \tag{15}$$

The assumption of nonoverlapping counting intervals requires that $|t_2 - t_1| \geq T$, with equality occurring in the case of contiguous intervals.

Optical fluctuations in the amplifier and its output occur over the time scales represented in the autocovariance functions of (13) and (15). According to (13), the two-point correlation properties of the internal photon number N are determined by the function $e^{-(b-a)|t_2 - t_1|}$. Thus, the population fluctuations may be characterized by a correlation time

$$\tau_c \equiv \frac{1}{b - a}, \tag{16}$$

whose inverse measures the amplifier's bandwidth

$$\Delta\nu \equiv \tau_c^{-1} = (b - a). \tag{17}$$

These two equations express the reasonable assertion that if the birth rate aN and death rate bN are nearly equal, then the cavity population exhibits correlations over long time intervals. If the integration time T is small compared to the cavity correlation time $(b - a)^{-1}$, (15) shows that fluctuations in the photocount n follow those of N. However, for integration times $T \gg (b - a)^{-1}$, it is easy to show from (15) that the integrated count covariance satisfies

$$\frac{\langle n(t_1)\, n(t_2)\rangle - \langle n\rangle^2}{\langle n\rangle^2} \leq \left[\frac{1}{(b - a)\,T}\right]^2, \tag{18}$$

which implies that the integrated photocount correlations vanish for such large T.

IV. Physical Interpretation

A. Limiting Forms of Amplification Noise

The physical mechanisms responsible for the photocount-variance expression of (12) are revealed by a little algebra, if one considers two limiting cases. We examine the noise behavior of amplifiers followed by optical detectors that are, in turn, broadband and narrowband with respect to the amplifier's bandwidth. For detectors whose bandwidth $B \equiv (1/T)$ is much larger than the amplifier bandwidth $(b - a)$, the photocount variance, given by (12), may be expanded as a Taylor series in T, yielding

$$\operatorname{var}(n) \approx \frac{b_o^2 a(a + c_i)}{(b - a)^2}\, T^2 + \frac{b_o(a + c_i)}{(b - a)}\, T, \qquad B \gg (b - a), \tag{19}$$

where we have discarded terms above second order. On the other hand, for detectors of bandwidth $B \ll (b - a)$, the exponential in (12) becomes negligible, so that

$$\operatorname{var}(n) \approx \frac{2 b_o^2 a(a + c_i)}{(b - a)^3}\, T + \frac{b_o(a + c_i)}{(b - a)}\, T, \qquad B \ll (b - a). \tag{20}$$

Some order is introduced by observing that the cavity population consists of amplified-signal photons entering the cavity at rate c_i and amplified-spontaneous-emission photons generated at rate a, each undergoing a cavity buildup determined by $(b - a)^{-1}$, as indicated by (5). In an interval T, on the average, a fraction $b_o T$ of each subpopulation is coupled to the output and detected, possibly with interaction. Thus, from (11), the mean photocount can be expressed as the sum $\langle n\rangle = \langle n_S\rangle + \langle n_{\text{ASE}}\rangle$, where

$$\langle n_S\rangle = \frac{b_o c_i}{b - a}\, T \tag{21}$$

is the average number of amplified signal photons and

$$\langle n_{\text{ASE}}\rangle = \frac{b_o a}{b - a}\, T \tag{22}$$

is the average number of amplified spontaneous-emission photons detected in time T. This generates a total integrated photocurrent $i = h\nu R n/T$ in a detector of responsivity R, with a rectangular impulse response of duration T, for photons of energy $h\nu$. Hence, making use of (21) and (22) in (19) and (20), the variance of the integrated photocurrent may be written simply as

$$\operatorname{var}(i) \approx \begin{cases} \left(\frac{h\nu R}{T}\right)^2 \left\{ \langle n_{\text{ASE}} \rangle^2 + \langle n_S \rangle \langle n_{\text{ASE}} \rangle + \langle n_S \rangle + \langle n_{\text{ASE}} \rangle \right\}, & B \gg (b-a), \\ 2\left(\frac{h\nu R}{T}\right)^2 \left\{ \left(\frac{B}{b-a}\right) \langle n_{\text{ASE}} \rangle^2 + \left(\frac{B}{b-a}\right) \cdot \langle n_S \rangle \langle n_{\text{ASE}} \rangle + \frac{\langle n_S \rangle}{2} + \frac{\langle n_{\text{ASE}} \rangle}{2} \right\}, & B \ll (b-a). \end{cases} \tag{23}$$

B. Limiting Forms of Interferometric Noise

To shed light on the variance expression of (23), we compare it with the photocurrent fluctuations that result when the fields emitted by two identical but independent single-mode semiconductor lasers are allowed to fall on the surface of a square-law detector. Optical phase fluctuations are then interferometrically converted to intensity, and hence to photocurrent fluctuations. We represent the lasers' emitted electric fields $\boldsymbol{E}_{1,2}(t)$ in the usual way as $\boldsymbol{E}_{1,2}(t) = \boldsymbol{A}_{1,2} \exp i(\omega_0 t + \phi_{1,2}(t))$, where ω_0 is the center optical angular frequency, common to both lasers, $\boldsymbol{A}_{1,2}$ are the respective polarization vectors, and $\phi_{1,2}(t)$ are independent Wiener–Levy random walks representing the respective optical phases [18]. We assume the lasers to have identical linewidth $\Delta\nu'$; hence, the $\phi_{1,2}(t)$ are related through their structure functions $\langle [\phi_{1,2}(t_2) - \phi_{1,2}(t_1)]^2 \rangle = 2\pi\Delta\nu' |t_2 - t_1|$ where t_1 and t_2 are arbitrary times.

A lengthy argument via the fourth-order coherence functions of the electric fields [19] shows that the resulting photocurrent's two-sided power spectral density is given by the Lorentzian

$$S_i(\nu) = \frac{2}{\pi} R^2 P_1 P_2 \left(\frac{\Delta\nu'}{(\Delta\nu')^2 + \nu^2} \right), \tag{24}$$

where $P_{1,2}$ are the optical powers transported by the interfering beams, whose polarization states we assume to be matched. R is again the responsivity. Taking the integrating detector, for computational simplicity, to have a rectangular RF passband of $(0, B')$ Hz implies that the variance of the detector current is

$$\operatorname{var}(i) = \frac{2}{\pi} \int_{-B'}^{B'} \left(\frac{R^2 P_1 P_2 \Delta\nu'}{(\Delta\nu')^2 + \nu^2} \right) d\nu, \tag{25}$$

which integrates to

$$\operatorname{var}(i) = \frac{4}{\pi} R^2 P_1 P_2 \arctan(B'/\Delta\nu'). \tag{26}$$

If noise of this sort arises from two processes, taken to be independent, and representing both signal–spontaneous and spontaneous–spontaneous beating, then the photocurrent variance is

$$\operatorname{var}(i) = \frac{4}{\pi} R^2 \left[P_{\text{ASE}}^2 \arctan(B'/\Delta\nu'_{\text{sp-sp}}) + P_S P_{\text{ASE}} \arctan(B'/\Delta\nu'_{\text{s-sp}}) \right], \tag{27}$$

where P_{ASE} and P_S are the mean amplified-spontaneous-emission (ASE) power and mean signal power, respectively, and where $\Delta\nu'_{\text{sp-sp}}$ and $\Delta\nu'_{\text{s-sp}}$ are the widths of the spontaneous–spontaneous and signal–spontaneous beat-noise spectra. Our concern is with the limiting forms of (27) for large and small detector bandwidths B'. Taking limits, and using the relations $P_{S,\text{ASE}} = h\nu(\langle n_{S,\text{ASE}} \rangle / T)$ between mean optical powers and mean counts, gives

$$\operatorname{var}(i) \approx \begin{cases} 2\left(\frac{h\nu R}{T}\right)^2 \left\{ \langle n_{\text{ASE}} \rangle^2 + \langle n_S \rangle \langle n_{\text{ASE}} \rangle \right\}, & B' \gg \max(\Delta\nu_{\text{s-sp}}, \Delta\nu_{\text{sp-sp}}), \\ \frac{4}{\pi}\left(\frac{h\nu R}{T}\right)^2 \left\{ \left(\frac{B'}{\Delta\nu_{\text{sp-sp}}}\right) \langle n_{\text{ASE}} \rangle^2 + \left(\frac{B'}{\Delta\nu_{\text{s-sp}}}\right) \cdot \langle n_S \rangle \langle n_{\text{ASE}} \rangle \right\}, & B' \ll \min(\Delta\nu_{\text{s-sp}}, \Delta\nu_{\text{sp-sp}}). \end{cases} \tag{28}$$

C. Comparison of Amplification Noise With Interferometric Noise

It is now easy to appreciate the physics inherent in the resonant amplifier's output photocount variance, given by (12). In the limits of broad and narrow detector RF bandwidth, (12) takes the form of (23); thus, in either limit the photocount variance may be written as a sum of four terms. The first two of these, which depend on $\langle n_{\text{ASE}} \rangle^2$ and $\langle n_S \rangle \langle n_{\text{ASE}} \rangle$, are of precisely the same form as the beat-noise variance expressions of (28), obtained by examining the interference of two uncorrelated, phase-fluctuating single-mode waves. In fact, the expressions are identical up to constant factors that result from the simple bandwidth measures, different for (23) and (28), that were chosen to facilitate the integrations. Furthermore, the second two variance terms of (23) are also easy to appreciate. For arbitrary integration times T, these may be written as

$$2e\left(e\frac{\langle n_S \rangle}{T} + e\frac{\langle n_{\text{ASE}} \rangle}{T} \right) \frac{1}{2T}, \tag{29}$$

where e is the electronic charge. This is just the familiar expression for the variance due to shot noise.

Thus, the photon-statistical model of a linear optical amplifier, extended by adding a resonator and explicit input and output signals, provides a noise description that is in fact transparent. The integrated photocount variance contains four terms representing, respectively: spontaneous–spontaneous beat noise, signal–spontaneous beat noise, amplified-signal shot noise, and amplified-spontaneous-emission shot noise. The beat-noise terms have their origin in the nonlinear interaction between the signal and the ASE fields. The shot-noise terms arise from the detector's linear filtering of the fluctuations residing individually in the signal and ASE output point processes. Remarkably, a modified, but simple, branching-process characterization captures both.

V. Applications

Having resolved the variance of i into a sum of contributions from four physical processes, as shown in (23), it is easy to identify the conditions under which each contribution becomes important. For example, shot noise generally dominates for very short integration times T, as may be seen from (23) by inspection, setting $\max(\langle n_S \rangle, \langle n_{ASE} \rangle) < 1$, or from (12) by taking the small-T limit. However, our concern is to describe systems that now show practical promise. Thus, we restrict our attention to semiconductor amplifiers and, in particular, to their applications in optical communications systems. Intensity-modulated non-return-to-zero (NRZ) binary signaling at 1 Gb/s is assumed. Our comments hold for resonant devices of rather general structure, including those with Fabry–Perot, distributed-feedback (DFB), and distributed-Bragg-reflector (DBR) resonator configurations. Operation below the lasing threshold is assumed.

For this broad class of device, the short-counting-time limit mentioned a moment ago is of no interest. This is because the RF bandwidth B of a minimum-error-rate detector is on the order of the bit rate; thus, to amplify the significant frequency components within even the information bandwidth of the signal requires $\Delta\nu \geq B$ or equivalently, from (17), $T \geq (b - a)^{-1}$. Moreover, one is actually concerned principally with much longer counting times. The optical bandwidths $\Delta\nu$ of current research devices, even DFB's and DBR's operated near threshold [6]–[9], are rarely smaller than 10 GHz, with Fabry–Perot-type devices typically being far broader. We therefore confine ourselves to amplifier-detector combinations satisfying $\Delta\nu >> B$ or, equivalently, $T >> (b - a)^{-1}$. From (11) and (23), the first two moments of i for such systems are

$$\langle i \rangle \approx h\nu R \frac{b_o(a + c_i)}{(b - a)} = h\nu RB(\langle n_S \rangle + \langle n_{ASE} \rangle), \tag{30}$$

$$\operatorname{var}(i) \approx 2(h\nu RB)^2 \left\{ \left(\frac{B}{\Delta\nu}\right) \langle n_{ASE} \rangle^2 + \left(\frac{B}{\Delta\nu}\right) \cdot \langle n_S \rangle \langle n_{ASE} \rangle + \frac{\langle n_S \rangle}{2} + \frac{\langle n_{ASE} \rangle}{2} \right\}. \tag{31}$$

This variance expression, written explicitly in terms of the first moments of the two counting distributions, permits three broad observations.

First, by comparing the first pair of terms in (31) with the second pair, it follows that the ratio of beat-noise power to shot-noise power in the photocount, or equivalently in the photocurrent, is given by

$$\frac{\operatorname{var}(i)_{\text{beat}}}{\operatorname{var}(i)_{\text{shot}}} \approx 2\left(\frac{B}{\Delta\nu}\right) \langle n_{ASE} \rangle = \left(\frac{2}{h\nu\Delta\nu}\right) P_{ASE}, \tag{32}$$

where P_{ASE} is the average amplified-spontaneous-emission optical power. Thus, the fraction of noise power residing in terms of interferometric origin is proportional to the *mean* ASE power divided by the amplifier's optical bandwidth. For InGaAsP DFB amplifiers biased close to threshold, one typically finds that $P_{ASE} \approx -15$ dBm and $\Delta\nu \approx 10$ GHz [9]. Thus, one expects beat noise to dominate shot noise by about four orders of magnitude. In traveling-wave amplifiers, by comparison, beat noise has similarly been found to dominate, typically by 2–3 orders of magnitude [20]. We consequently confine our attention to beat noise.

Second, by comparing the first and second terms of (31), it is apparent that the ratio of signal–spontaneous to spontaneous–spontaneous beat noise is simply equal to the first-moment ratio

$$\frac{\operatorname{var}(i)_{\text{S-SP}}}{\operatorname{var}(i)_{\text{SP-SP}}} \approx (P_S/P_{ASE}) = (c_i/a). \tag{33}$$

The powers appearing in (33) are readily measured: P_{ASE} by blocking the signal, and P_S with the help of a lock-in amplifier [9]. As will be seen momentarily, large photocurrent signal-to-noise ratios require that the quotient c_i/a be large. Thus, when the system is configured to produce small error rates, the photocurrent variance will be dominated by signal–spontaneous beat noise. This, too, is true of traveling-wave and Fabry–Perot amplifiers [20].

Finally, defining the signal-to-noise ratio of the output photocurrent as $\langle i_S \rangle^2/\operatorname{var}(i)$, it follows from (30) and (31) that

$$\frac{\langle i_S \rangle^2}{\operatorname{var}(i)} \approx \frac{1}{2}\left[\frac{\Delta\nu}{B}\right]\left[\frac{(\langle n_S \rangle/\langle n_{ASE} \rangle)^2}{1 + (\langle n_S \rangle/\langle n_{ASE} \rangle)}\right], \tag{34}$$

where we have neglected the shot-noise terms. In general, the error rate in a digital system cannot be inferred from ratios of this form; rather, full information about the photocount distribution is required. And while the full negative-binomial distribution of the cavity population is known [(4)], that of the photocounts is available only in the form of a generating function [(8)] that resists inversion. In such cases, one is often tempted to resort to Gaussian approximations; however, it is known both from theory [19] and from experiment [21] that interferometric beat noise in semiconductor-laser devices cannot be relied upon to be Gaussian. Indeed, if the detector bandwidth greatly exceeds the noise bandwidth, beat-noise distributions tend to be bimodal [21].

However, in the limit of practical interest, where $T \gg (b - a)^{-1}$, good approximations are available. If the detector's integration time T and the amplifier's optical bandwidth $\Delta\nu$ are sufficiently large, one can divide T into a number of subintervals, each of duration ΔT satisfying $\Delta T \Delta\nu \gg 1$. Then, from (13), measurements of the cavity population made at any times in two distinct subintervals are uncorrelated. Moreover, from (15) and (18), the photocounts accumulated in any two distinct subintervals are uncorrelated and, on physical grounds, independent. Thus, the integrated photocurrent generated by a detector of bandwidth $1/T$ is represented by a sum process whose fluctuations are approximately Gaussian. In this case, the moment ratio of (34) indeed suffices to infer the error rate in an on-off-keyed system: $\langle i_S \rangle^2/\text{var}(i) \approx 150$ is needed to achieve an error rate of 10^{-9}. Thus, for $(\Delta\nu/B) \approx 10$, as is typical for Gb/s signals traversing DFB amplifiers, low error rates are attainable only if the second factor of (34) is made large. For such systems, the signal-to-noise ratio becomes simply

$$\frac{\langle i_S \rangle^2}{\text{var}(i)} \approx \frac{1}{2}\left(\frac{\Delta\nu}{B}\right)\left(\frac{\langle n_S \rangle}{\langle n_{\text{ASE}} \rangle}\right). \tag{35}$$

For on-off signals falling on a detector whose noise contribution is small compared with the amplification noise described here, the error probability P_e approximately equals one-half the conditional probability of error given a mark. Thus, $P_e \approx (1/4)\,\text{erfc}\,(\langle i_S \rangle/\sqrt{8\,\text{var}(i)})$, with $\langle i_S \rangle/\sqrt{\text{var}(i)}$ given by (35).

Clearly, the signal-to-noise ratio in the regime of interest is a product of two factors. The second factor, $\langle n_S \rangle/\langle n_{\text{ASE}} \rangle$, although expressed as an easily measured first-moment ratio, in fact represents the ratio of output-signal power to output signal-spontaneous beat-noise power; the latter, we have seen, is the dominant source of fluctuations in a low-noise system. The first factor, given by the bandwidth ratio $\Delta\nu/B$, is simply the reciprocal of the fraction of this beat-noise power that falls within the bandwidth of the photodetector. Thus, the signal-to-noise ratio of the integrated photocurrent simply equals the quotient of mean signal power divided by the portion of signal-spontaneous beat-noise power that falls within the receiver bandwidth. This result, which holds for resonant amplifiers of quite general resonator configuration, agrees, as a special case, with the conclusions obtained in [2], [5], [20] for devices of Fabry–Perot structure.

VI. Conclusion

By reinterpreting key variables, a simple branching-process treatment of inverted-population amplifiers may be modified to include both a resonator and explicit input and output photon streams, thus exposing the previously inaccessible process representing the output photocounts. The resulting description yields physical insight when its photocurrent fluctuations are compared with those that result from the mixing of two uncorrelated, phase-fluctuating, single-mode waves. The physical insight, in turn, enables one to apply approximations suitable for lightwave-communication systems, so that simple closed-form expressions result. While the analysis makes explicit appeal only to particle properties of the radiation, it generates the same noise expressions as those produced by fluctuations of interferometric origin. Indeed, a birth–death–immigration process with a Poisson initial population, in a structure of traveling-wave configuration, also has this behavior [2].

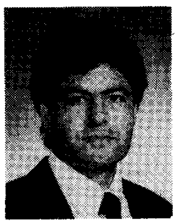
Acknowledgment

This work has been improved by conversations with E. Arthurs, C. A. Brackett, A. Descloux, S. J. Golestani, M. S. Goodman, H. Kobrinski, M. Tur, and M. P. Vecchi, and by the encouragement of C. A. Brackett and P. W. Shumate.

References

[1] K. Shimoda, H. Takahasi, and C. H. Townes, "Fluctuations in amplification of quanta with application to maser amplifiers," *J. Phys. Soc. Japan*, vol. 12, pp. 686-700, 1957.
[2] Y. Yamamoto, "Noise and error-rate performance of semiconductor laser amplifiers in PCM-IM optical transmission systems," *IEEE J. Quantum Electron.*, vol. QE-16, pp. 1073-1081, 1980.
[3] Equation (5) of [2], cf. [5, eq. 2].
[4] Y. Yamamoto, "Characteristics of AlGaAs Fabry-Perot cavity type laser amplifiers," *IEEE J. Quantum Electron.*, vol. QE-16, pp. 1047-1052, 1980.
[5] T. Mukai, Y. Yamamoto, and T. Kimura, "S/N and error-rate performance in AlGaAs semiconductor laser preamplifier and linear repeater systems," *IEEE J. Quantum Electron.*, vol. QE-18, pp. 1560-1568, 1982.
[6] H. Kawaguchi, K. Magari, K. Oe, Y. Noguchi, Y. Nakano, and G. Motosugi, "Optical frequency-selective amplification in a distributed feedback type semiconductor laser amplifier," *Appl. Phys. Lett.*, vol. 50, pp. 66-67, 1987.
[7] T. Numai, S. Murata, and I. Mito, "1.5 μm tunable wavelength filter with wide tuning range and high constant gain using a phase-controlled distributed feedback laser diode," *Appl. Phys. Lett.*, vol. 53, pp. 1168-1169, 1988.
[8] E. L. Goldstein, H. Kobrinski, M. P. Vecchi, and R. M. Bulley, "Nanosecond wavelength selection with 10-dB net gain in a subthreshold DFB laser amplifier," in *Proc. 11th IEEE Int. Semiconductor Laser Conf.*, IEEE LEOS, 1988, pp. 14-15.
[9] H. Kobrinski, M. P. Vecchi, E. L. Goldstein, and R. M. Bulley, "Wavelength selection with nanosecond switching times using distributed-feedback laser amplifiers," *Electron. Lett.*, vol. 24, pp. 969-970, 1988.
[10] T. Makino and J. Glinski, "Transfer matrix analysis of the amplified spontaneous emission of DFB semiconductor laser amplifiers," *IEEE J. Quantum Electron.*, vol. 24, pp. 1507-1518, 1988.
[11] T. J. Shepherd and E. Jakeman, "Statistical analysis of an incoherently coupled, steady-state optical amplifier," *J. Opt. Soc. Amer. B*, vol. 4, pp. 1860-1869, 1987.
[12] S. K. Srinivasan, *Point Process Models of Cavity Radiation and Detection.* Oxford, England: Oxford Univ., 1988, pp. 52-69.
[13] W. H. Louisell, *Radiation and Noise in Quantum Electronics.* New York: McGraw-Hill, 1964.
[14] M. Sargent, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics.* New York: Addison-Wesley, 1974.

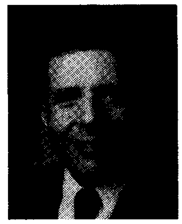
[15] S. Friberg and L. Mandel, "Photon statistics of the linear optical amplifier," in *Coherence and Quantum Optics V*, L. Mandel and E. Wolf, Eds. New York: Plenum, 1984.
[16] R. Loudon and T. J. Shepherd, "Properties of the optical quantum amplifier," *Optica Acta*, vol. 31, pp. 1243-1269, 1984.
[17] E. Jakeman and T. J. Shepherd, "Population statistics and the counting process," *J. Phys. A*, vol. 17, pp. L745-L750, 1984.
[18] C. H. Henry, "Theory of the phase noise and power spectrum of a single-mode injection laser," *IEEE J. Quantum Electron.*, vol. QE-19, pp. 1391-1397, 1983.
[19] J. A. Armstrong, "Theory of interferometric analysis of laser phase noise," *J. Opt. Soc. Amer.*, vol. 56, pp. 1024-1031, 1966.
[20] T. Mukai, Y. Yamamoto, and T. Kimura, "Optical amplification by semiconductor lasers," in *Semiconductors and Semimetals*, vol. 22, part E, R. K. Willardson and A. C. Beer, Eds. New York: Academic, 1985.
[21] M. Tur and E. L. Goldstein, "Measurement of the probability distribution of DFB laser phase-induced intensity noise," in *Proc. CLEO*, IEEE LEOS, 1989, paper FP5.



Evan L. Goldstein was born in Seattle, WA, on October 20, 1952. He received the B.A. degree from Antioch College, Yellow Springs, OH, and the M.A. and M.Phil. degrees from Columbia University, New York, in 1975, 1977, and 1981, respectively, all in philosophy. He then received the B.S., M.S., and Ph.D. degrees in electrical engineering from Columbia University in 1985, 1986, and 1989, respectively.

In 1986, he joined the Optical Networks Research Division, Bellcore, Morristown, NJ, where he is a Member of Technical Staff. His recent research has concerned the tuning properties of distributed-feedback laser amplifiers and the noise properties of semiconductor lasers and resonant amplifiers coupled to guided-wave optical systems.

Dr. Goldstein is a member of the Optical Society of America, the IEEE Lasers and Electro-Optics Society, and the American Physical Society.



Malvin C. Teich (S'62-M'66-SM'72-F'89) was born in New York City. He received the S.B. degree in physics from the Massachusetts Institute of Technology, Cambridge, MA, in 1961, the M.S. degree in electrical engineering from Stanford University, Stanford, CA, in 1962, and the Ph.D. degree in quantum electronics from Cornell University, Ithaca, NY, in 1966.

In 1966, he joined the M.I.T. Lincoln Laboratory, Lexington, MA, where he was engaged in work on coherent infrared detection. In 1967, he became a member of the faculty with the Department of Electrical Engineering, Columbia University, New York, where he is now teaching and pursuing his research interests in the areas of quantum optics, optical and infrared detection, and sensory perception. He served as Chairman of the Department from 1978 to 1980. He is also a member of the faculty with the Department of Applied Physics, and a member of the Columbia Radiation Laboratory, the Center for Telecommunications Research, and the Fowler Memorial Laboratory at the Columbia College of Physicians and Surgeons. He has authored or coauthored some 150 technical publications and holds one patent.

Dr. Teich was the recipient of the IEEE Browder J. Thompson Memorial Prize Award for his paper "Infrared heterodyne detection" in 1969 and he received a Citation Classic Award of the Institute for Scientific Information for this work in 1981. He received a Guggenheim Fellowship in 1973. He is a Fellow of the American Physical Society, the Optical Society of America, and the American Association for the Advancement of Science. He is a member of Sigma Xi, the Acoustical Society of America, and the Association for Research in Otolaryngology. He served as a member of the Editorial Advisory Panel for the journal *Optics Letters* from 1977 to 1979 and is currently a member of the Board of Editors of the journal *Quantum Optics*.