Signal-to-Noise Ratio for Lightwave Systems Using Avalanche Photodiodes

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***Abstract*—We derive an expression for the signal-to-noise ratio (SNR) at the output of an avalanche photodiode (APD) receiver that includes the effects of photoinjected carriers, dark-generated carriers, and the receiver circuitry. We find that the dark current alters both the magnitude of the SNR and the value of the mean multiplication where the SNR achieves its maximum value.**

I. INTRODUCTION

AVALANCHE photodiodes (APD's) are often used in lightwave systems as detectors of optical signals because their multiplication property can enhance the signal-to-noise ratio (SNR) of a detected optical signal, even though APD's introduce excess noise [1]. In calculating the SNR, the signal is the mean multiplied electron current generated by the detected photons, while the noise arises from three sources: the randomness of the multiplication of the photoinjected carriers [2], the randomness of the multiplication of dark-generated carrier pairs [3], and the thermal noise in the receiver circuitry [4].

Photogenerated carriers injected at the edge of the APD depletion region have the advantage that each carrier experiences full multiplication. Volume dark noise, on the other hand, originates from the generation of electron-hole pairs at random locations *throughout* the depletion region [4]. This results in a decrease of the mean multiplication (relative to carriers injected at the edge) because most dark-generated carriers experience multiplication over a distance shorter than the full depletion width. It also results in an increased excess noise factor (again relative to carriers injected at the edge) that results from the added randomness imparted by the uncertainty in locations at which the dark-generated carrier pairs are born. Photogenerated carriers produced by light incident on the depletion region volume are also subject to these modified multiplication statistics. Explicit expressions for the mean multiplication and excess noise factor, when photoinjected and dark-generated carriers are both present, have recently been obtained [3].

In this paper we obtain the SNR at the output of a uniformly multiplying APD in the presence of photogenerated carriers, volume dark current, and receiver circuitry noise. Our formula differs from the expression that is conventionally used [5] in that it makes use of different excess noise factors for photogenerated and dark-generated carriers.

II. SIGNAL-TO-NOISE RATIO

The SNR of the electron current I_n at the output of an APD is given by

$$\mathrm{SNR}\,(I_n) = \langle I_n \rangle^2 / \mathrm{Var}(I_n) \tag{1}$$

where $\langle \cdot \rangle$ indicates averaging and $\mathrm{Var}(\cdot)$ represents the variance [5]. The three sources of noise described above are independent so that their variances can be added and (1) can be written as

$$\mathrm{SNR}(I_n) = \frac{\langle I_e \rangle^2 \langle M_e \rangle^2}{2q\langle I_e \rangle B \langle M_e \rangle^2 F_e + 2q \langle I_d \rangle B \langle M_d \rangle^2 F_d + \sigma_T^2} \tag{2}$$

where the terms in the denominator arise from multiplication of the photoinjected photocarriers, multiplication of the dark-generated carriers, and circuit noise, respectively. $\langle I_e \rangle$ is the primary photoinjected mean electron current before multiplication, $\langle I_d \rangle$ is the primary mean dark-generated current, $\langle M_e \rangle$ and $\langle M_d \rangle$ are the mean multiplications of the photoinjected and dark-generated carriers, respectively, F_e and F_d are their respective excess noise factors, q is the electronic charge, B is the electrical bandwidth of the receiver, and σ_T^2 is the variance of the circuit noise (which has zero mean).

The mean signal multiplication of the APD is given by [2], [6]

$$\langle M_e \rangle = \frac{1 - k}{e^{-\alpha w (1-k)} - k}, \qquad k \neq 1 \tag{3}$$

where α is the electron ionization rate, w is the width of the depletion region, and k is the hole-to-electron ionization-rates ratio. The photoinjected-carrier variance at the output of the device is proportional to $\langle M_e \rangle^2 F_e$, where F_e is the usual excess noise factor for electron multiplication [2], [6]

$$F_e = \left[k \langle M_e \rangle + (2 - 1/\langle M_e \rangle)(1 - k) \right]. \tag{4}$$

The mean multiplication of the dark-generated carriers is given by [3]

$$\langle M_d \rangle = \frac{1}{\alpha w} \frac{1 - e^{-\alpha w (1-k)}}{e^{-\alpha w (1-k)} - k}, \qquad k \neq 1 \tag{5a}$$

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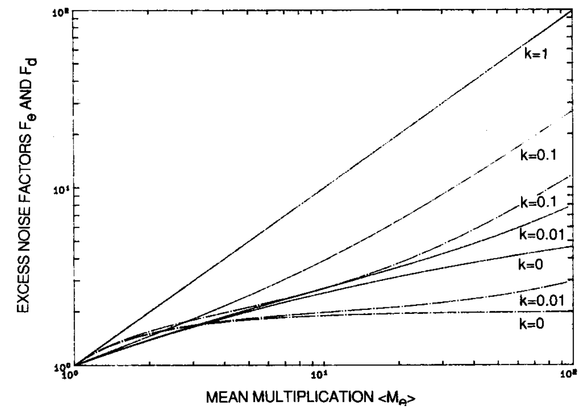

Fig. 1. Dependence of the excess noise factors F_e (dot-dash curves) and F_d (solid curves) on the mean multiplication $\langle M_e \rangle$ for $k = 0$, 10^{-2}, 10^{-1}, and 1. F_e and F_d are identical for $k = 1$.

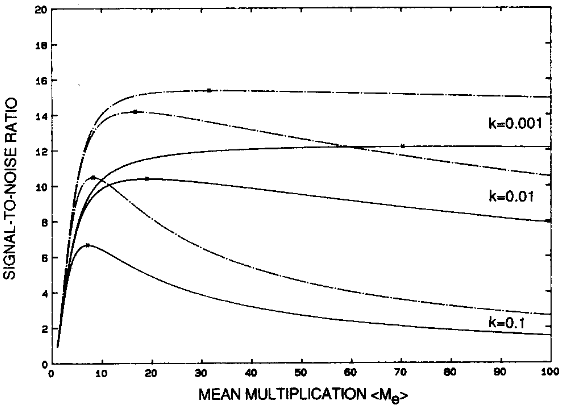

Fig. 2. Effect of dark noise on the SNR in the presence of photoinjected carriers and thermal circuit noise. The dot-dash curves represent the SNR in the absence of dark noise while the solid curves represent the SNR in the presence of dark noise. The parameter values for this plot are $\langle I_e \rangle$ = 10 nA, $\langle I_d \rangle$ = 20 nA, σ_T^2 = 100 nA^2, and B = 1 GHz. F_e, F_d, and $\langle M_d \rangle$ depend on $\langle M_e \rangle$ (see (5b) and Fig. 1). Curves are shown parametrically for $k = 10^{-3}$, 10^{-2}, and 10^{-1}. The peaks of the SNR curves are indicated by crosses.

which can be written in terms of $\langle M_e \rangle$ as

$$\langle M_d \rangle = \frac{1}{\alpha w}\left(\langle M_e \rangle - 1\right), \qquad k \neq 1. \tag{5b}$$

The dark-generated carrier variance is proportional to $\langle M_d \rangle^2 F_d$ where F_d is the dark-generated carrier excess noise factor [3]

$$F_d = \frac{\left(1 + \alpha w \langle M_d \rangle\right)\left(1 + k\alpha w \langle M_d \rangle\right)}{\langle M_d \rangle}. \tag{6}$$

F_d can alternatively be written in terms of $\langle M_e \rangle$ by using (5b). When $k = 1$, $\langle M_d \rangle = \langle M_e \rangle = 1/(1 - \alpha w)$, and $F_e = F_d = \langle M_e \rangle$ [3], [6].

In Fig. 1, we show the dependence of F_e (dot-dash curves) and F_d (solid curves) on the mean multiplication $\langle M_e \rangle$ for various values of k. F_d can be substantially greater than F_e for large values of $\langle M_e \rangle$, thereby causing the dark-current noise to dominate the photoinjected-carrier noise when the primary currents are comparable.

III. EFFECT OF $F_d \neq F_e$ ON THE SNR

When σ_T^2 dominates (i.e., when the multiplication-noise contributions are negligible in comparison with σ_T^2), the maximum value of the SNR occurs at $\langle M_e \rangle = \infty$, as is evident from (2). If all three sources of noise are present, but photoinjected-current noise dominates, our results reduce to the standard formula [5] which assumes that $F_d = F_e$. However, if the magnitude of the dark-current noise is of the same order as that of the photoinjected-current noise, the usual assumption equating F_d with F_e will not suffice and (2) must be used. This is shown in Fig. 2, where the dot-dash curves exclude dark current while the solid curves include it. The presence of dark noise reduces the magnitude of the SNR and shifts the position of its maximum to higher values of the mean multiplication when k is small. Conversely, the position of its maximum is shifted to lower values of the mean multiplication when k is large.

It is well known that the presence of thermal circuit noise can cause the SNR curves to exhibit a maximum at a specific value of the mean multiplication. In the absence of circuit noise, dark noise plays a similar role, as illustrated by the solid curves of Fig. 3. The standard formula [5], obtained by setting $F_d = F_e$ in (2), also shows such an effect (dot-dashed curves), but it provides incorrect values for both the magnitude and mean multiplication at which the SNR achieves its maximum value. This is because F_e and F_d do not grow in the same way with $\langle M_e \rangle$, as is evident in Fig. 1.

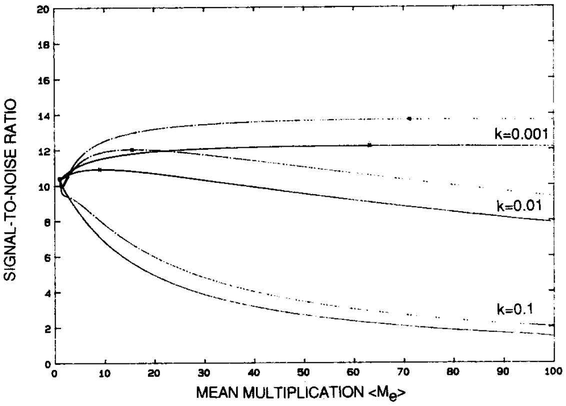

Fig. 3. Effect of dark noise when photoinjected carriers are present but thermal circuit noise is absent. The solid curves are the correct results obtained from (2); the dot-dash curves correspond to the usually used results obtained by setting $F_d = F_e$ in (2). The parameter values for this plot are $\langle I_e \rangle$ = 10 nA, $\langle I_d \rangle$ = 20 nA, $\sigma_T^2 = 0$, and B = 1 GHz. Curves are shown parametrically for $k = 10^{-3}$, 10^{-2}, and 10^{-1}. The peaks of the SNR curves are indicated by crosses.

The solid curves in Fig. 4 represent plots of (2) when all three contributions to the noise are present. The photoinjected current and dark current are of the same order of magnitude, $\langle I_d \rangle = 2\langle I_e \rangle$. Curves are shown parametrically for $k = 10^{-3}$, 10^{-2}, and 10^{-1}. For small values of $\langle M_e \rangle$, circuit noise dominates and the SNR grows as $\langle M_e \rangle^2$. As $\langle M_e \rangle$ increases, multiplication noise begins to contribute and the growth of the SNR slows and exhibits a peak. The dot-dash curves are the results obtained by setting $F_d = F_e$ in (2). It is apparent from the figure that use of $F_d = F_e$ gives an SNR error $\approx$ 10% for this particular set of parameter values. However, the calculated value of

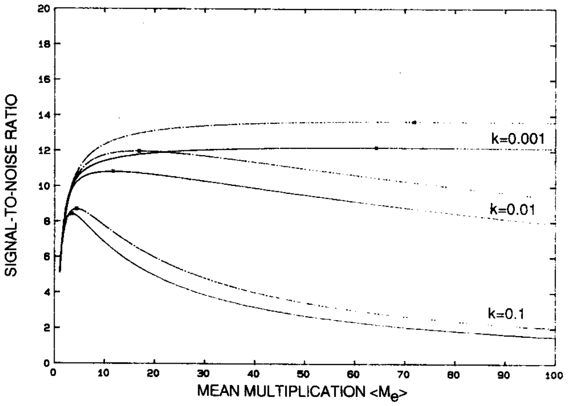

Fig. 4. SNR given by (2) (solid curves) in the presence of photoinjected, dark, and circuit noise. The dot-dash curves represent the SNR calculated assuming $F_d = F_e$. The parameter values for this plot are $\langle I_e \rangle = 10$ nA, $\langle I_d \rangle = 20$ nA, $\sigma_T^2 = 10$ nA^2, and $B = 1$ GHz. Curves are shown parametrically for $k = 10^{-3}$, 10^{-2}, and 10^{-1}. The peaks of the SNR curves are indicated by crosses.

the mean multiplication where the SNR achieves its maximum value can be more seriously in error. For example, the optimal value of the mean multiplication is 11.6 for $k = 0.01$, whereas using $F_d = F_e$ in (2) results in a value of 16.8. Using (2), the optimal value is always smaller than when assuming $F_d = F_e$.

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