

Fractal Behavior of the Electrocardiogram: Distinguishing Heart-Failure and Normal Patients Using Wavelet Analysis

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Abstract—Heart-failure patients can be identified by computing certain statistics of the sequence of heartbeats, such as the Allan factor (AF) and its generalization, the wavelet Allan factor (WAF). These measures successfully determine whether a given patient suffers from heart failure or not. They succeed because they are jointly sensitive to both short-term, and long-term fractal, characteristics of the heartbeat time series.

I. INTRODUCTION

Statistical analysis of the sequence of human heartbeats can provide information about the health of the heart. We previously demonstrated that a variety of measures show *statistically* significant differences for normal and heart-failure patients [1]. We also showed that the Allan factor (AF) is a particularly valuable index for diagnosing heart failure because it provides a *clinically* significant measure for the presence (or absence) of heart failure in individual patients [2]. We consider the use of a generalized version of the AF, called the wavelet Allan factor (WAF) [3]. Like the AF, the WAF is useful for identifying patients that suffer from heart failure, or do not, because it is jointly responsive to both short-term, and long-term fractal, characteristics of the heartbeat time series. The WAF is potentially superior to the AF because it is resistant to nonstationarities in the heartbeat time series.

II. METHODS

The statistical behavior of the electrocardiogram can be studied by collapsing the complex waveform of an individual heartbeat (QRS-complex) into a single number, the time of occurrence of the contraction (R-phase). In mathematical terms, the heartbeat sequence is then represented as an unmarked point process. This simplification permits us to make use of the substantial methodology that has been developed for stochastic point processes.

Though the characteristics of a point process are often studied via the sequence of intervals between successive events, in some cases it is advantageous to examine the sequence of event numbers (counts) $\{N_k\}$ observed in successive counting times T [1]–[3].

The Allan factor $A(T)$, which is such a count-based measure, is defined as the ratio of the event-number Allan variance to twice the mean count:

$$A(T) = \frac{E\{[N_{k+1}(T) - N_k(T)]^2\}}{2E\{N_k(T)\}}, \quad (1)$$

where the expectation E is taken over the set of samples k . The Allan variance, in turn, is a measure of the dispersion of event numbers in sequential counting windows; it was first introduced in connection with the stability of atomic-based clocks [4]. The AF is useful for determining the degree of event clustering (or anticlustering) in a point process [relative to the benchmark homogeneous Poisson point process for which $A(T) = 1 \forall T$]. For a fractal stochastic point process such as the heartbeat, at sufficiently large counting times the AF increases as a fractional-power-law function of T [2],[3].

Equation (1) is computed in terms of numbers of events in Haar-function counting windows of duration T . A wavelet-based version of the Allan factor is generated by replacing these windows by wavelet and scaling functions, denoted $\psi(t)$ and $\phi(t)$ respectively, in an arbitrary wavelet basis. Wavelet and scaling *coefficients*, $d[T, k]$ and $c[T, k]$ respectively, can then be constructed from the signal point process $dN(t)$ [3]:

$$d[T, k] = \frac{1}{T^{1/2}} \int_{-\infty}^{+\infty} [\psi(u/T - k)]^* dN(u), \quad (2)$$

$$c[T, k] = \frac{1}{T^{1/2}} \int_{-\infty}^{+\infty} [\phi(u/T - k)]^* dN(u), \quad (3)$$

where $*$ denotes complex conjugation. The wavelet Allan factor $A_w(T)$ is then defined as [3],[5]

$$A_w(T) = T^{1/2} \frac{E\{[d[T, k]]^2\}}{E\{c[T, k]\}}, \quad (4)$$

where the expectation E is again over k . It is readily shown that the WAF computed in the Haar wavelet basis reduces to the AF [3].

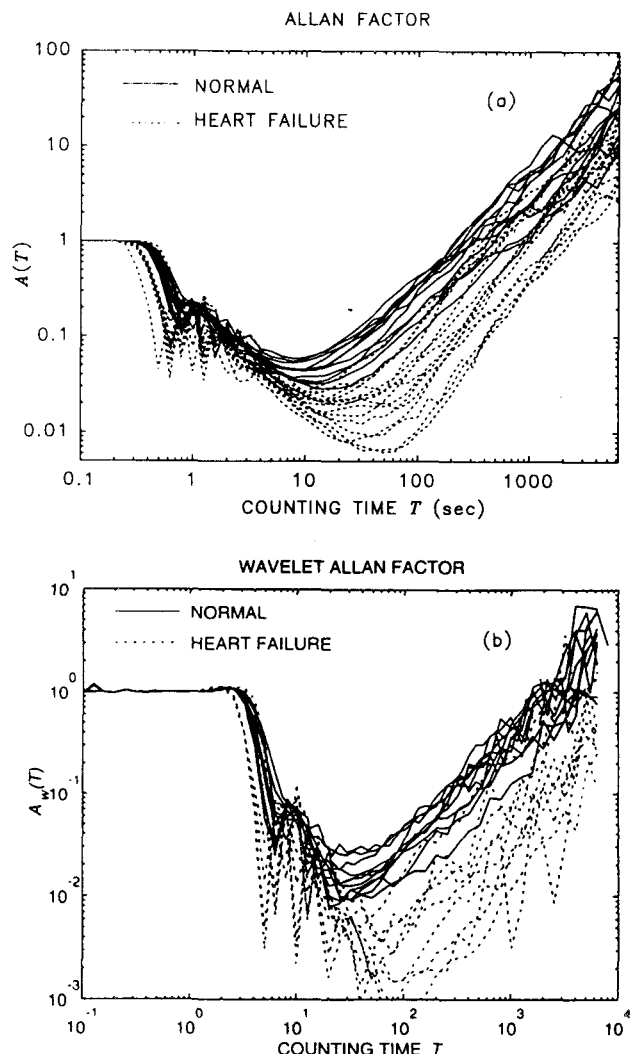


Fig. 1. (a) Allan factors for all 27 data sets. In the vicinity of $T = 10$ s, the magnitude of the AF falls into different ranges for heart-failure and normal patients, providing a measure with 100% sensitivity and 100% specificity. (b) Wavelet Allan factors for all 27 data sets. In the vicinity of $T = 50$ s, the magnitude of the WAF falls into different ranges for heart-failure and normal patients, providing a more general measure with 100% sensitivity and 100% specificity.

III. RESULTS

The AF and WAF were computed for 12 electrocardiogram records from normal patients and 15 records from heart-failure patients. Each recording was made with a Holter monitor over a period of some 20 hours.

The AFs and WAFs are illustrated in Figs. 1(a) and 1(b), respectively. For the particular data sets that are shown, it is apparent that threshold values for $A(T = 10 \text{ s})$ and $A_w(T = 50 \text{ s})$ can be chosen such that all heart-failure and normal patients can be properly identified. The underlying reason for the

success of the AF [2] and the WAF in achieving 100% sensitivity and 100% specificity is as follows. The smaller interevent-interval variances for the heart-failure patients result in their curves reaching lower levels than the normal curves at moderate values of T . At larger values of T , the heart-failure curves rise more aggressively because their fractal exponents are generally larger (they have a relatively greater proportion of low-frequency spectral components). The combination results in two sets of curves, one for the normals and another for the heart-failures. Both sets exhibit dips, but their shapes are sufficiently different that there is a critical counting

time T_c^{AF} (T_c^{WAF}) at which the ranges of the A (A_w) values for normals and heart-failures do not overlap.

The WAF, like the AF, therefore, provides a clinically significant measure for ascertaining the presence or absence of heart failure in individual patients. It will be useful to carry out further studies, using other wavelet bases and different data sets, to confirm and extend these results.

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