

Investigating Cellular Vibrations in the Cochlea Using the Continuous Wavelet Transform and the Short-Time Fourier Transform

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Abstract - The Continuous Wavelet Transform (*CWT*) and the Short-Time Fourier Transform (*STFT*) are used to analyze the time course of cellular motion in the guinea-pig inner ear. The velocity responses of individual auditory cells (outer hair cells and Hensen's cells) to amplitude-modulated (*AM*) acoustical signals display characteristics typical of nonlinear systems, such as harmonic generation. Nonlinear effects are particularly pronounced at the highest stimulus levels, where half-harmonic, and sometimes quarter-harmonic, components are also seen. Both the *CWT* and the *STFT* are found to be useful in analyzing these velocity responses.

Keywords - Continuous Wavelet Transform, Short-Time Fourier Transform, cochlea, cellular vibration, nonlinear dynamics, chaos.

I. INTRODUCTION

The velocity of vibration of cellular structures in the guinea-pig cochlea has been measured by using laser-heterodyne interferometry [1]. The Short-Time Fourier Transform (*STFT*) is a useful tool for following the time course of the frequency components generated in response to an amplitude-modulated (*AM*) stimulus. *STFT* analysis has revealed that the cellular vibrations are strongly nonlinear: they exhibit spectral components not only at the carrier frequency of the *AM* stimulus f_c , but at its higher harmonics, and often at its half-harmonics as well [2]-[4]. In this paper, we show that the Continuous Wavelet Transform (*CWT*) is similarly useful.

II. METHODS

The *CWT* and *STFT* of the measured velocity response were calculated. The *CWT* of a signal $x(t)$ is defined as

$$CWT_x^h(r, \tau) = \frac{1}{\sqrt{|r|}} \int_{-\infty}^{+\infty} x(t) h^* \left(\frac{t - \tau}{r} \right) dt, \quad (1)$$

with r as a scale variable, τ as a time variable, $x(t)$ as the velocity signal to be analyzed, $h(t)$ as the prototype wavelet basis function, and $*$ denoting complex conjugation. The fast-*CWT* algorithm of Jones and Baraniuk [5] was used to calculate a discrete approximation

of (1). The discrete-time prototype wavelet was obtained by sampling the continuous Morlet wavelet $h(t) = \exp(jct) \exp(-\alpha t^2/2)$ (with $c = 4750$ and $\alpha = 12207$) at 5000 Hz, the sampling rate of the original data set. For convenience in interpreting the *CWT*, we mapped the scale variable r to frequency f , using the mapping $f = K/r$, with $K = 756$.

The *STFT* of a signal $x(t)$ is defined as

$$STFT_x^g(f, \tau) = \int_{-\infty}^{\infty} x(t) g^*(t - \tau) \exp(-j2\pi ft) dt, \quad (2)$$

with f as a frequency variable, τ as a time variable, and $g(t)$ as a window function in time. The Gaussian window $g(t) = \exp(-\beta t^2/2)$ (with $\beta = 12207$) was chosen. A discrete approximation of the *STFT* was calculated by taking the Fast Fourier Transforms of windowed sections of the sampled velocity waveform [2]-[4]. The discrete-time window was obtained by sampling the Gaussian window at 5000 Hz.

III. RESULTS

Figure 1(a) shows the velocity waveform of a third-turn outer hair cell to an *AM* tone (modulation index = 100%; modulation frequency = 2.44 Hz) with a carrier frequency $f_c = 756$ Hz. This frequency lies at the characteristic frequency (CF) of the cell, i.e., at the acoustic frequency to which the cell responds maximally. The peak sound intensity of the stimulus was 104 dB re .0002 dynes/cm². The velocity waveform is seen to be highly irregular. The *CWT* magnitude of this waveform is shown in 3D format in Fig. 1(b) and in 2D contour format in Fig. 1(c). The *CWT* comprises frequency components at integer multiples of the quarter-harmonic frequency, i.e., at $jf_c/4$ with $j = 1, 2, \dots, 12$. At lower sound intensities (not shown) only multiples of the half-harmonic frequency appear, while at the lowest sound intensities only multiples of the carrier frequency are present. This pattern is indicative of a period-doubling route to chaos.

The *STFT* presented in Figs. 1(d) and 1(e) is similar to the *CWT*. It has been chosen to have the same time and frequency resolution as the *CWT* at the carrier frequency. The principal distinction between the two transforms lies in the manner in which the time and frequency resolutions trade off as a function of frequency. For the *STFT*, both the time and frequency resolutions are independent of frequency, whereas for the *CWT* the

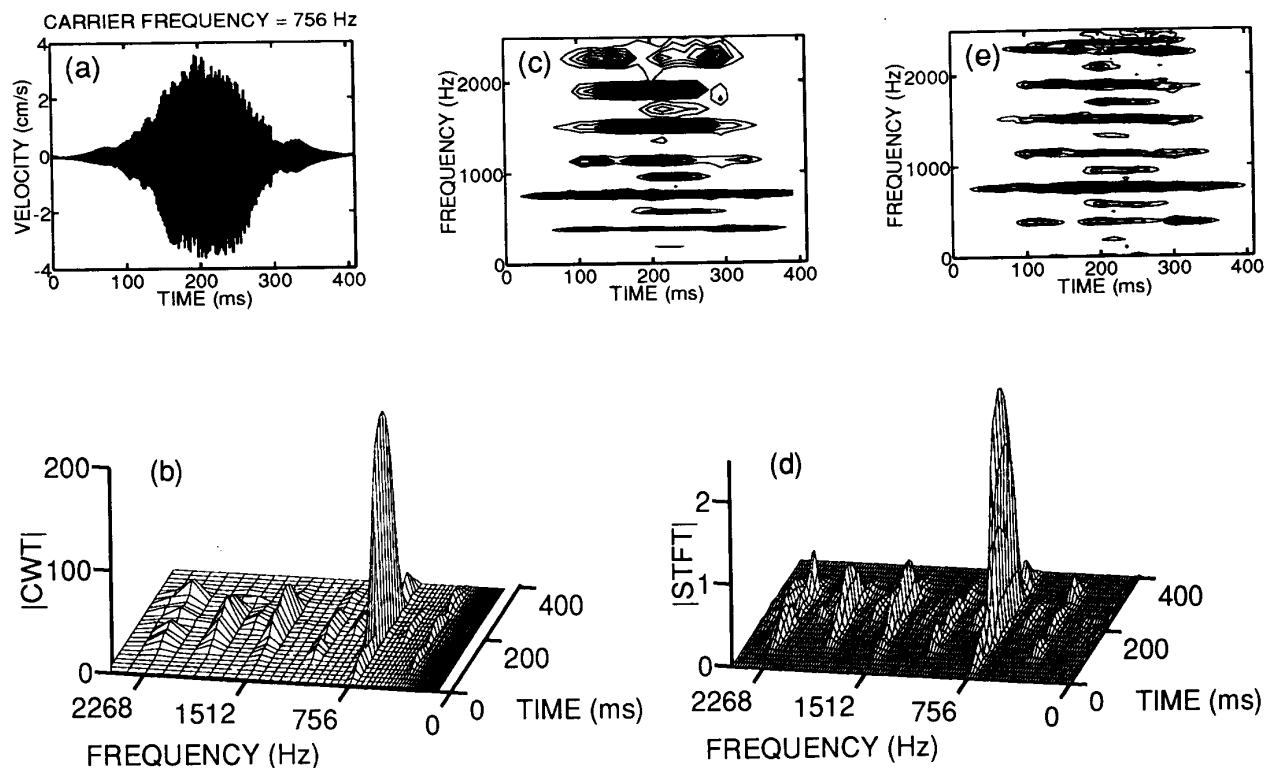


Figure 1: Velocity response of an outer hair cell in the third turn of a guinea-pig temporal-bone preparation to an *AM* stimulus with carrier frequency $f_c = 756$ Hz. (a) Time waveform of the response. (b) 3D plot of the *CWT* magnitude of the velocity response shown in (a). (c) Same *CWT* magnitude as shown in (b), but with 100 equally spaced contour lines joining points of constant magnitude. (d) 3D spectral plot of the *STFT* magnitude of the velocity response shown in (a). (e) Same *STFT* magnitude as shown in (d), but now plotted in 2D contour format, with 100 equally spaced contour lines.

frequency resolution improves (at the expense of the time resolution) as the frequency decreases. Thus, the *STFT* reveals a temporal switching pattern at $f_c/2$ whereas the *CWT* provides a better estimate of the frequency. Similarly, the *STFT* provides improved resolution of the successive quarter-harmonic frequencies near $3f_c$.

IV. ACKNOWLEDGMENTS

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