# Effect of Dead Space on Gain and Noise of Double-Carrier-Multiplication Avalanche Photodiodes

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***Abstract*—The effect of dead space on the statistics of the gain in a double-carrier-multiplication avalanche photodiode (APD) is determined using a recurrence method. The dead space is the minimum distance that a newly generated carrier must travel in order to acquire sufficient energy to become capable of causing an impact ionization. We derive recurrence equations for the first moment, second moment, and the probability distribution function of two random variables that are related, in a deterministic way, to the random gain of the APD. These equations are solved numerically to produce the mean gain and the excess noise factor. The presence of dead space reduces both the mean gain and the excess noise factor of the device. This may have a beneficial effect on the performance of the detector when used in optical receivers with photon noise and circuit noise.**

## I. Introduction

Avalanche photodiodes (APD's) with a variety of structures have been used as detectors in fiber-optic communication systems. The performance of such systems is strongly dependent on the mean gain and the random fluctuations of the APD, measured by the excess noise factor. A fundamental assumption implicit in most models of noise of conventional APD's [1]–[9] is that the probability of impact ionization by a carrier is independent of its ionization history, so that the ionization rate is the same at all times, including the instant following the carrier's own generation. From a physical point of view, however, a newly generated carrier must travel some distance in order to build up sufficient energy to enable it to initiate an ionization [10], [11]. To take this effect into account, a model is formulated in which the ionization probability of a carrier is set to zero for a certain distance, called the dead space, immediately following its generation. More realistically, the ionization probability would be expected to increase gradually, beginning from zero and reaching a steady-state value after some distance called the "sick space."

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Using a sick-space ionization model, LaViolette [12], and LaViolette and Stapelbroek [13] developed a numerical technique for evaluating the mean, excess noise factor, and probability distribution function for the gain. Saleh, Hayat, and Teich [14] treated the dead-space model using the theory of age-dependent branching processes. They obtained analytical expressions for the mean gain and the excess noise factor, and determined numerically the mean and standard deviation of the impulse response function of the APD as a function of time following a photoexcitation. Their theory is also applicable to the sick-space model. Both of these theories have been limited to APD's with only single-carrier multiplication (SCM).

The problem is more complex in the case of double-carrier multiplication (DCM). Okuto and Crowell [10], [11] calculated numerically the mean gain for the DCM dead-space model. Marsland [15] attempted to extend the classical McIntyre theory [1] for the mean gain and excess noise factor for a conventional APD to an APD with dead space. However, the formulation is incomplete; it takes into account the dead space associated only with the initial carrier pair and does not extend it to the subsequent carriers. Consequently, Marsland's results for the special case of SCM disagree with the complete theory [14].

In this paper, we develop a theory for the gain statistics of a DCM APD assuming a sick-space model. Recurrence equations are derived for the probability distribution function, and the first and second moments, of the numbers of electrons and holes. These random variables are related to the random gain in a deterministic way. As a special case, the dead-space model is applied to these recurrence equations and numerical solutions are obtained for the mean gain and the excess noise factor.

Dead space in APD's tends to reduce both the mean gain and the excess noise factor in a manner which has a beneficial effect on the performance of the optical receiver. The magnitude of this effect depends on the relative magnitudes of the photon noise, which depends on the mean photon flux, and the noise in the receiver circuit, which depends on the design of the preamplifier.

## II. Model

Under consideration is a single-carrier injection double-carrier multiplication APD. An electron injected at

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$x = 0$ travels in the $x$ direction with a fixed velocity $v_e$ under the effect of the electric field. After a random distance $X_e$ (called the electron lifespan) an impact ionization occurs. Upon ionization, an electron–hole pair is generated, so that the original electron is replaced by two electrons and a hole. The two electrons behave in a statistically identical and independent manner. The hole, on the other hand, travels in the $-x$ direction with a fixed velocity $v_h$ and ionizes after traveling a random distance $X_h$ (called the hole lifespan), resulting in two holes and an electron. The electrons and holes repeat the process as they travel through the multiplication region, and so on. The multiplication region extends from $x = 0$ to $x = W$. When an electron reaches the right edge of the multiplication region its role ends. Similarly, a hole ceases to ionize when it reaches the left edge. If the process terminates, it does so when all possible carriers have reached the edges of the multiplication region.

The random variables $X_e$ and $X_h$ are assumed to be statistically independent and have probability density functions $h_e(x_e)$ and $h_h(x_h)$, respectively. In the dead-space model, for example,

$$h_e(x_e) = \begin{cases} 0, & x_e < d_e \\ \alpha e^{-\alpha(x_e - d_e)}, & x_e \geq d_e \end{cases} \tag{1a}$$

and

$$h_h(x_h) = \begin{cases} 0, & x_h < d_h \\ \beta e^{-\beta(x_h - d_h)}, & x_h \geq d_h \end{cases} \tag{1b}$$

where $d_e$ and $d_h$ are the electron and hole dead spaces, respectively, and $\alpha$ and $\beta$ are the ionization rates for electrons and holes that have traveled beyond the dead space, respectively.

The total number of electron–hole pairs generated within the device, including the original electron which initiated the entire multiplication process, is a random number $G$ that constitutes the gain of the device. Our objective is to determine the statistics of $G$.

## III. Statistics of the Gain

Our approach is based on writing recurrence equations for the totoal number of carriers generated by a single carrier at an arbitrary location within the multiplication region. These type of recurrence equations arise naturally in branching processes [16], in which the occurrence of an event independently replicates a statistically identical process of event generation. In fact, a similar approach underlies the original theory advanced by McIntyre [1] for conventional APD's in the absence of dead space.

### A. Recurrence Equations

Consider an electron and a hole at location $x$. Assume that the electron is responsible for the production of a random sum $Z(x)$ of electrons and holes, including the initiating electron itself. Similarly, $Y(x)$ is the random number of all electrons and holes produced by the hole and its offsprings, including the hole itself. Thus

$$M(x) = \tfrac{1}{2}(Z(x) + Y(x)) \tag{2}$$

is the total number of carrier pairs generated as a result of the original carrier pair at location $x$, including the original pair. The theory developed by McIntyre [1] is based on developing a recurrence equation for the mean $m(x) = \langle M(x) \rangle$, where brackets denote an ensemble average. In a device in which there is a single electron–hole pair at $x = 0$, the gain

$$G = M(0) = \tfrac{1}{2}(Z(0) + Y(0)).$$

Clearly, $Y(0) = 1$ since a hole at $x = 0$ does not travel into the device and cannot ionize, so that

$$G = \tfrac{1}{2}(Z(0) + 1). \tag{3}$$

Once the statistical properties of $Z(x)$ and $Y(x)$ are determined, the statistics of $M(x)$ and $G$ can be inferred from (2) and (3), respectively.

We now proceed to develop recurrence equations for the random variables $Z(x)$ and $Y(x)$. Consider an electron at location $x$ and examine the events that occur as a result of this electron, the pairs it generates, and their offsprings. If the first ionization occurs at location $x < \xi < W$, we have two electrons and a hole at location $\xi$. Let $Z_1(\xi)$, $Z_2(\xi)$, and $Y(\xi)$ denote the total number of carriers produced by the first electron, the second electron, and the hole, respectively. Since each of these carriers acts independently, the random variables $Z_1(\xi)$, $Z_2(\xi)$, and $Y(\xi)$ are statistically independent. Clearly, $Z_1(\xi)$ and $Z_2(\xi)$ are identically distributed. Conditioning on the first ionization occurring at location $\xi$, the total number of carriers produced by the original electron is

$$Z(x|\xi) = Z_1(\xi) + Z_2(\xi) + Y(\xi). \tag{4}$$

The conditioning can be removed by averaging over all possible $\xi$ in the interval $x < \xi < W$. Thus if we assume that the original electron ionizes before reaching the edge at $x = W$, then

$$\begin{aligned}\langle Z(x) \rangle &= \int_x^W \langle Z(x)|\xi) \rangle \, h_e(\xi - x)\, d\xi \\ &= \int_x^W \langle [Z_1(\xi) + Z_2(\xi) + Y(\xi)] \rangle \, h_e(\xi - x)\, d\xi.\end{aligned} \tag{5}$$

It is also possible, however, that the original electron will not ionize at all (i.e., $Z(x) = 1$). The probability of this event is $1 - H_e(W - x)$, where

$$H_e(x) = \int_{-\infty}^{x} h_e(\xi)\, d\xi \tag{6}$$

is the distribution function of the electron lifespan random variable $X_e$.

Similarly, for a single hole at $x$

$$Y(x|\xi) = Y_1(\xi) + Y_2(\xi) + Z(\xi). \tag{7}$$

If we assume that the original hole ionizes before reaching the edge at $x = 0$ then by averaging over all possible $\xi$ in the interval $0 < \xi < x$ we obtain

$$\langle Y(x)\rangle = \int_0^x \langle Y(x|\xi)\rangle\, h_h(x - \xi)\, d\xi = \int_0^x \langle [Y_1(\xi) + Y_2(\xi) + Z(\xi)]\rangle\, h_h(x - \xi)\, d\xi. \tag{8}$$

The probability that the holes does not ionize at all (i.e., $Y(x) = 1$) is $1 - H_h(x)$, where

$$H_h(x) = \int_{-\infty}^{x} h_h(\xi)\, d\xi \tag{9}$$

is the distribution function for the hole lifespan random variable $X_h$.

### B. Mean Gain

In the light of the discussion in Section III-A, we take the ensemble average of $Z(x)$ and $Y(x)$ and obtain

$$z(x) = [1 - H_e(W - x)] + \int_x^W [2z(\xi) + y(\xi)]h_e(\xi - x)\, d\xi \tag{10}$$

and

$$y(x) = [1 - H_h(x)] + \int_0^x [2y(\xi) + z(\xi)]h_h(x - \xi)\, d\xi \tag{11}$$

where $z(x) = \langle Z(x)\rangle$ and $y(x) = \langle Y(x)\rangle$ are the means of $Z(x)$ and $Y(x)$, respectively. Equations (10) and (11) are valid only for $0 < x < W$. At $x = W$

$$z(W) = 1 \tag{12a}$$

and at $x = 0$

$$y(0) = 1. \tag{12b}$$

The coupled integral equations (10) and (11) are the basic equations from which the mean gain $\langle G\rangle$ will be determined by taking the ensemble average of both sides of (3). Thus the mean multiplication

$$m(x) = \tfrac{1}{2}(z(x) + y(x)) \tag{13a}$$

and in particular, the mean gain is

$$\langle G\rangle = \tfrac{1}{2}(z(0) + 1). \tag{13b}$$

In our earlier work [14], which dealt with the SCM case, we obtained a recurrence integral equation that is a special case of (10) with $h_h(x) = 0$, $H_h(x) = 0$, and $y(x) = 1$.

The dead-space model can be applied by inserting the expressions for the functions $h_e(x)$ and $h_h(x)$, as given by (1), into (10) and (11)

$$z(x) = [1 - (1 - e^{-\alpha(W-x-d_e)})u(W - x - d_e)] + \int_x^W [2z(\xi) + y(\xi)]\alpha e^{-\alpha(\xi - x - d_e)} \cdot u(\xi - x - d_e)\, d\xi \tag{14}$$

and

$$y(x) = [1 - (1 - e^{-\beta(x-d_h)})u(x - d_h)] + \int_0^x [2y(\xi) + z(\xi)]\beta e^{-\beta(x-\xi-d_h)} \cdot u(x - \xi - d_h)\, d\xi \tag{15}$$

where $u(x) = 1$ for $x \geq 0$, and 0 otherwise.

### C. Excess Noise Factor

The excess noise factor can be determined by developing similar expressions for the mean squared values of $Z(x)$ and $Y(x)$. We start by squaring both sides of (4) to obtain

$$Z^2(x|\xi) = (Z_1(\xi) + Z_2(\xi) + Y(\xi))^2. \tag{16}$$

If we assume that the original electron ionizes before reaching the edge at $x = W$ then the conditioning can be removed by averaging over $\xi$ in the interval $x < \xi < W$

$$\langle Z^2(x)\rangle = \int_x^W \langle Z^2(x|\xi)\rangle\, h_e(\xi - x)\, d\xi = \int_x^W \langle [Z_1(\xi) + Z_2(\xi) + Y(\xi)]^2\rangle\, h_e(\xi - x)\, d\xi. \tag{17}$$

Thus the ensemble average of $Z^2(x)$ can be written as

$$z_2(x) = [1 - H_e(W - x)] + \int_x^W [2z_2(\xi) + y_2(\xi) + 4z(\xi)y(\xi) + 2z^2(\xi)]h_e(\xi - x)\, d\xi \tag{18}$$

where $z_2(x) = \langle Z^2(x)\rangle$, and $y_2(x) = \langle Y^2(x)\rangle$ are the second moments of $Z(x)$ and $Y(x)$, respectively. Similarly, we start with (7) to obtain a similar expression

$$y_2(x) = [1 - H_h(x)] + \int_0^x [2y_2(\xi) + z_2(\xi) + 4z(\xi)y(\xi) + 2y^2(\xi)]h_h(x - \xi)\, d\xi. \tag{19}$$

At $x = W$

$$z_2(W) = 1 \tag{20a}$$

and at $x = 0$

$$y_2(0) = 1. \tag{20b}$$

These recurrence equations for the second moments require knowledge of the first moments $z(x)$ and $y(x)$, which must be computed separately by the use of (10) and (11).

The excess noise factor

$$F = \frac{\langle G^2 \rangle}{\langle G \rangle^2}$$

is related to $z(0)$ and $z_2(0)$ by

$$F = \frac{\langle M^2(0) \rangle}{(\langle M(0) \rangle)^2} = \frac{z_2(0) + 2z(0) + 1}{(z(0) + 1)^2}. \tag{21}$$

Our earlier results for the SCM case are reproduced from (18), (19), and (21) by setting $h_h(x) = 0$, $H_h(x) = 0$, and $y(x) = 1$.

In general, recurrence equations for yet higher moments can be generated in a similar manner. These recurrence equations involve lower moments as well. So one can systematically solve for the higher order moments by using the previously calculated lower moments in the recurrence equations.

For the dead-space model, (18) and (19) yield

$$z_2(x) = [1 - (1 - e^{-\alpha(W-x-d_e)})u(W - x - d_e)] + \int_x^W [2z_2(\xi) + y_2(\xi) + 4z(\xi)y(\xi) + 2z^2(\xi)]\alpha e^{-\alpha(\xi - x - d_e)}u(\xi - x - d_e)\, d\xi \tag{22}$$

and

$$y_2(x) = [1 - (1 - e^{-\beta(x-d_h)})u(x - d_h)] + \int_0^x [2y_2(\xi) + z_2(\xi) + 4z(\xi)y(\xi) + 2y^2(\xi)]\beta e^{-\beta(x - \xi - d_h)}u(x - \xi - d_h)\, d\xi. \tag{23}$$

### D. Probability-Distribution Function of the Gain

We now proceed to determine the probability distribution function (pdf) of the gain. Let $P_Z(k, x)$, $k = 1, 2, 3, \cdots$, denote the pdf of $Z(x)$, i.e., $P_Z(k, x) = \Pr\{Z(x) = k\}$. Similarly, $P_Y(k, x)$ is the pdf of $Y(x)$. The random variable $Z(x|\xi) = Z_1(\xi) + Z_2(\xi) + Y(\xi)$ defined in (4) has the conditional pdf $P_{Z|\xi}(k, x|\xi) = \Pr\{Z(x|\xi) = k\}$. Since the random variables $Z_1(\xi)$, $Z_2(\xi)$, and $Y(\xi)$ are statistically independent, $P_{Z|\xi}(k, x|\xi)$ is the discrete convolution of the pdf's of $Z_1(\xi)$, $Z_2(\xi)$, and $Y(\xi)$ evaluated at $k$. Furthermore, since $Z_1(\xi)$ and $Z_2(\xi)$ are identically distributed, they have identical pdf's $P_Z(k, x)$, so that

$$P_{Z|\xi}(k, x|\xi) = \sum_{j=1}^{k-1} \sum_{i=1}^{j-1} P_Z(j - i, \xi) P_Z(i, \xi) P_Y(k - j, \xi). \tag{24}$$

The conditioning can be removed by averaging over all $\xi$ in the interval $x < \xi < W$ to obtain an integral equation relating $P_Z(k, x)$ and $P_Y(k, x)$. For $k > 1$

$$P_Z(k, x) = \int_x^W h_e(\xi - x) P_{Z|\xi}(k, x|\xi)\, d\xi = \int_x^W h_e(\xi - x) \left( \sum_{j=1}^{k-1} \sum_{i=1}^{j-1} P_Z(j - i, \xi) \cdot P_Z(i, \xi) P_Y(k - j, \xi) \right) d\xi. \tag{25a}$$

For $k = 1$

$$P_Z(1, x) = 1 - H_e(W - x). \tag{25b}$$

Similarly

$$P_Y(k, x) = \int_0^x h_h(x - \xi) \left( \sum_{j=1}^{k-1} \sum_{i=1}^{j-1} P_Y(j - i, \xi) \cdot P_Y(i, \xi) P_Z(k - j, \xi) \right) d\xi \tag{26a}$$

and

$$P_Y(1, x) = 1 - H_h(x). \tag{26b}$$

The two coupled integral equations (25) and (26) can be solved, in principle, to determine the pdf's $P_Z(k, x)$ and $P_Y(k, x)$. The pdf of the gain $P_G(k) = \Pr\{G = k\}$ can then be found using (3) to yield $P_G(k) = P_Z(2k - 1, 0)$.

If only electrons are allowed to ionize, as it is the case with an SCM APD, $P_Y(k, x) = 1$ for $k = 1$, and 0 otherwise. In this case, (25) becomes equivalent to the result obtained in our earlier work [14].

The complexity of (25) and (26) arises in part from the appearance of summations of the discrete-convolution type under the integrals; nonetheless, these summations can be transformed into multiplications with the aid of generating functions. Let $F_Z(s, x)$ and $F_Y(s, x)$ be the generating functions of $Z(x)$ and $Y(x)$, respectively, i.e.,

$$F_Z(s, x) = \sum_{k=0}^{\infty} P_Z(k, x) s^k, \qquad |s| \le 1 \tag{27}$$

and

$$F_Y(s, x) = \sum_{k=0}^{\infty} P_Y(k, x) s^k, \qquad |s| \le 1. \tag{28}$$

Using (27) and (28) in (25) and (26), respectively, gives

$$F_Z(s, x) = s[1 - H_e(W - x)] + \int_x^W h_e(\xi - x) \cdot [F_Z(s, \xi)]^2 F_Y(s, \xi)\, d\xi, \qquad |s| < 1 \tag{29a}$$

$$F_Z(1, x) = 1 \tag{29b}$$

$$F_Y(s, x) = s[1 - H_h(x)] + \int_0^x h_h(x - \xi) \cdot [F_Y(s, \xi)]^2 F_Z(s, \xi)\, d\xi, \qquad |s| < 1 \tag{30a}$$

and

$$F_Y(1, x) = 1. \tag{30b}$$

The foregoing coupled nonlinear integral equations can be solved numerically. Once the generating functions are obtained, the pdf's can be determined by the use of the relation

$$P(k, x) = \frac{1}{k!} \left( \frac{\partial^k F(s, x)}{\partial s^k} \right)_{s=0}$$

with the proper subscripts on $P$ and $F$. Furthermore, these integral equations can be used to derive the recurrence equations for the first and second moments $z(x)$, $y(x)$, $z_2(x)$, and $y_2(x)$ ((10), (11), (18), and (19)). This may be accomplished by use of the standard relations

$$z(x) = \left(\frac{\partial}{\partial s} F_Z(s, x)\right)_{s=1}$$

and

$$z_2(x) = z(x) + \left(\frac{\partial^2}{\partial s^2} F_Z(s, x)\right)_{s=1}.$$

## IV. Results

We have used the recurrence equations obtained in Sections III-B and III-C to determine the mean gain and excess noise factor of a DCM APD with ionization rates $\alpha$ and $\beta$ and dead-space distances $d_e$ and $d_h$. The numerical computations are carried out as follows: i) $z(x)$ and $y(x)$ are set to zero everywhere in the interval $0 \leq x \leq W$, with the two exceptions $z(W) = y(0) = 1$, in accordance with (12). ii) Equation (15) is discretized, using a suitable mesh size, and then used to generate estimates of $y(x)$ in the interval $0 \leq x \leq W$. iii) Using this estimate of $y(x)$ in the discrete version of (14), an estimate of $z(x)$ is generated in the interval $0 \leq x \leq W$. iv) An improved estimate of $y(x)$ is obtained by substituting in the discrete version of (15) the previously calculated estimate of $z(x)$. v) Steps iii) and iv) are repeated until convergence is achieved. The mean multiplication $m(x)$ is computed by the use of (13a).

The validity of the numerical results has been verified by comparison with known expressions in the special case of no dead space and with the SCM theory [14].

The effect of dead space on $m(x)$ is shown in Fig. 1 for different values of the hole-to-electron ionization ratio $k = \beta/\alpha$. As expected, dead space reduces $m(x)$ everywhere in the interval $0 \leq x \leq W$. In particular, it reduces the mean gain $\langle G \rangle = m(0)$. This effect is more significant for larger $k$, as can be deduced from Fig. 2 which shows the dependence of $\langle G \rangle$ on $k$ for different values of the ratio $d_e/W = d_h/W$. The dead-space effect is substantially greater here than in the results of Marsland [15] which fail to account for the dead space encountered by secondary carriers.

In a similar manner, the second moments $z_2(x)$ and $y_2(x)$ have been computed numerically using discrete version of (22) and (23). With the aid of (21), the excess noise factor $F$ has been computed. The effect of dead space on $F$ is illustrated in Fig. 3. In this figure, $F$ is plotted as a function of $\langle G \rangle$ with the ratio $d_e/W = d_h/W$ as a parameter and for different values of $k$. It is seen that for fixed values of $k$ and $\langle G \rangle$, an increase in the ratio $d_e/W = d_h/W$ causes a reduction in $F$. In the special case where $k = 0$, $F$ initially increases with $\langle G \rangle$, reaches a maximum, and then decays monotonically to an asymptotic value. This decrease in $F$ can be understood in the context of our earlier work [14].

Since the presence of dead space is responsible for a reduction of both the mean gain and the excess noise factor, it is not clear whether it is of advantage to the overall performance of the detector. To assess this effect consider a communication system receiving a photon flux $\phi$ (photons per second). Assuming Poisson photon statistics, the signal-to-noise ratio (SNR) of the total charge accumulation in the detection circuit in a time interval $T$ is given by [17]

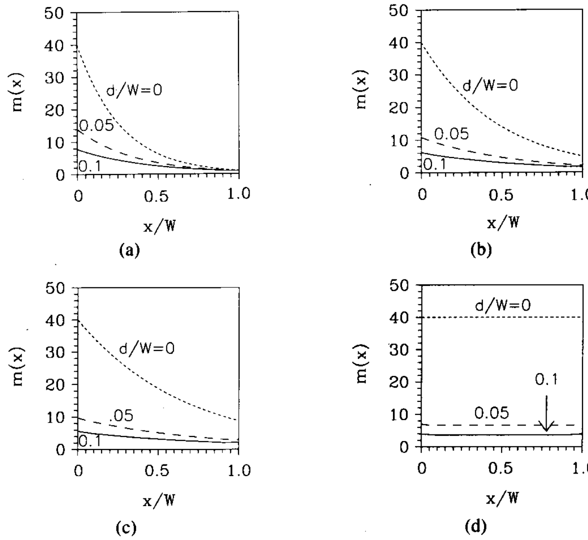


Fig. 1. Mean number of electron-hole pairs produced by an initial electron-hole pair at location $x$ in the multiplication region of width $W$. The value of $\alpha W$ is selected so that the mean gain $\langle G \rangle = m(0) = 40$ in the absence of dead space. The quantity $d/W = d_e/W = d_h/W$ assumes values 0, 0.05, and 0.1 in the four cases: (a) $k = 0$ ($\alpha W = 3.689$); (b) $k = 0.1$ ($\alpha W = 2.333$); (c) $k = 0.2$ ($\alpha W = 1.893$); (d) $k = 1$ ($\alpha W = 0.975$).

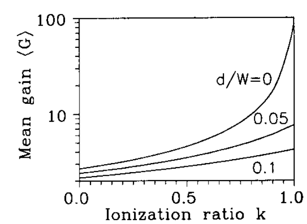


Fig. 2. Mean gean $\langle G \rangle = m(0)$ as a function of the hole-to-electron ionization rates ratio $k$ for $d/W = d_e/W = d_h/W = 0$, 0.05, and 0.1.

$$\text{SNR} = \frac{\phi T \langle G \rangle^2}{\langle G \rangle^2 F + \dfrac{\sigma^2}{\phi T}} \tag{31}$$

where $\sigma = i/eT$, is the rms current, and $e$ is the electron charge. Thus $\phi T$ is the mean number of photons collected and $\sigma$ is the rms circuit noise charge flow in the time interval $T$ (units of number of electrons). The quantum efficiency of the APD is assumed to be unity. Since the SNR for an ideal photon-noise-limited receiver ($\sigma = 0$, $F = 1$) is $\phi T$, the performance factor

$$P = \frac{\langle G \rangle^2}{\langle G \rangle^2 F + \dfrac{\sigma^2}{\phi T}} \tag{32}$$

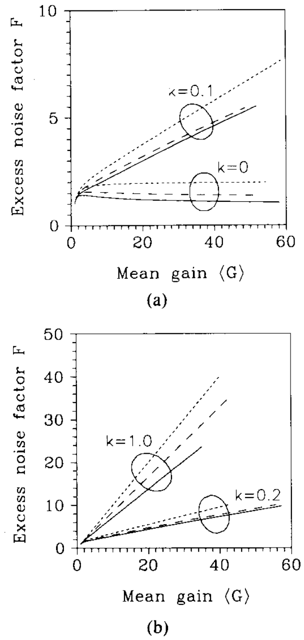


Fig. 3. Excess noise factor $F$ as a function of the mean gain $\langle G \rangle = m(0)$ for $d_e/W = d_h/W = 0$ (dotted), 0.05 (dashed), and 0.1 (solid). Four values of the ionization ratio are shown: (a) $k = 0$, $k = 0.1$; (b) $k = 0.2$, $k = 1$.

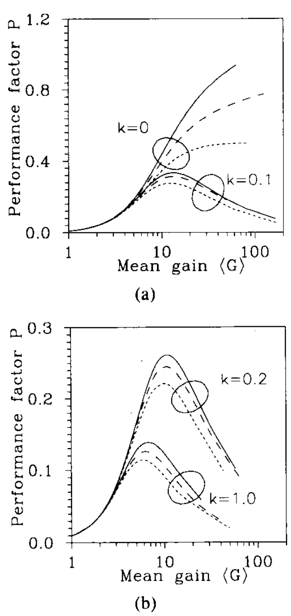


Fig. 4. The performance factor $P$, which governs the signal-to-noise ratio of an optical receiver that uses an APD, as a function of the mean gain $\langle G \rangle$. The circuit-noise to photon-noise ratio $\sigma^2/\phi T = 100$. Values of $d_e/W = d_h/W$ are taken to be 0, 0.05, and 0.1 (corresponding, respectively, to the dotted, dashed, and solid curves) and used for four values of the ionization ratio: (a) $k = 0$, $k = 0.1$; (b) $k = 0.2$, $k = 1$.

represents the SNR reduction caused by the combination of gain fluctuations and circuit noise. Thus the importance of the role played by dead space is governed by the ratio $\sigma^2/\phi T$. If $\sigma^2/\phi T << \langle G \rangle^2 F$, then $P \propto 1/F$, so that the performance is enhanced by the presence of dead space. On the other hand, if $\sigma^2/\phi T >> \langle G \rangle^2 F$, then $P \propto \langle G \rangle^2$, so that dead space has a performance degradation effect. However, the mean gain can usually be increased by simply increasing the applied voltage to the device. The beneficial effort of dead space on $P$ as a function of $\langle G \rangle$ is depicted in Fig. 4 for different values of ratio $d_e/W = d_h/W$ and for a fixed value of the circuit-noise to photon-noise parameter $\sigma^2/\phi T = 100$.

## V. Conclusion

The effect of dead space on the mean gain and noise properties of double-carrier multiplication APD's has been studied using recurrence relations in the form of coupled integral equations.

We found that dead space reduces the mean gain since it results in fewer ionizations. The reduction was found to be relatively greater as the hole-to-electron ionization ratio $k$ approached 1 since the growth rate of the branching process is reduced by the inhibiting effect of dead space.

We have also shown that dead space causes a lower excess noise factor since it introduces some orderliness in the random ionization process. In as much as real devices have intrinsic dead space built into them, the results here may therefore elucidate some enigmatic results in the literature [18]. Under certain conditions, the dead space has a beneficial effect on the performance of optical receivers. It may therefore be advantageous to select materials for which the dead space is enhanced, without jeopardizing other parameters such as large $\alpha$ and small $k$.

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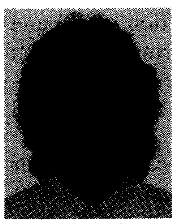

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