Photocounting Receiver Performance for Detection of Multimode Laser or Scattered Radiation

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Abstract

Photocounting distributions, the decision threshold, and the probability of error are obtained for a binary optical communication system in which both the signal and background radiation are Gaussian processes. This would be the case, for example, when the radiation source is a nonlocked multimode laser or when the received radiation is scattered from a satellite. Dark current is considered. The results indicate that when possible, counting times should be adjusted to be large in comparison with the background radiation coherence time, and optical amplification before detection should be employed.

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I. Introduction

Consideration has been given by several authors to the application of photocounting detection of laser radiation to optical communications systems in the vacuum channel. In particular, both Lachs and Jankowich [1] and Helstrom [2] have treated maximum likelihood detection for a communications system in which the source is a coherent optical signal (ideal single-mode laser) and the noise is thermal background radiation. Similar results for the ideal single-mode laser in the presence of background radiation for which all fluctuations have been averaged (counting distributions for both the signal and the noise are then Poisson) have been given by Bar-David [3], Pratt [4], and Gagliardi and Karp [5].

In this paper, we present results for the maximum likelihood receiver detecting radiation which arises from a multimode laser with independently oscillating modes [6] or from a scattering source such as a rotating satellite [7]. In these cases, the electric field has a Gaussian probability distribution, while the intensity is exponentially distributed, as for a thermal or chaotic source [8]-[10]. For noise arising from independent noninterfering background radiation, with a correlation time considerably smaller than the detection interval, Poisson noise counts will be observed in the absence of signal. This case and the reverse, when the detection time is much shorter than the background radiation correlation time, are both treated.

The probability distributions of the signal intensity I_S and the background noise intensity I_B may be written [8]

$$P_S(I_S) = \frac{1}{\langle I_S \rangle} \exp\left(-\frac{I_S}{\langle I_S \rangle}\right) u(I_S) \tag{1a}$$

$$P_B(I_B) = \frac{1}{\langle I_B \rangle} \exp\left(-\frac{I_B}{\langle I_B \rangle}\right) u(I_B) \tag{1b}$$

where $u(x)$ is the unit step function, and $\langle I_S \rangle$ and $\langle I_B \rangle$ are the intensity means of the signal and the background noise, respectively. The radiation beam is passed through a binary ON-OFF gate; the signal beam is transmitted during each ON period and blocked during each OFF period. The photocounter will receive the signal by appropriate time synchronization. The probability p that the signal will be transmitted is therefore proportional to the total ON period, and the probability $1 - p$ that the signal will be blocked is proportional to the total OFF period. If, on the average, the total ON period and the total OFF period are equal, we have $p = 1 - p = 0.5$.

Because the background radiation enters the receiver from all directions rather than only parallel to the incoming laser beam, interference will be assumed to be averaged. Therefore, the total effective intensity I at the receiver is just the sum of the signal intensity I_S and the background noise intensity I_B,

$$I(t) = I_S(t) + I_B(t). \tag{2}$$

This result is also obtained if interference occurs, but is outside the bandwidth of the detector.

In the following, we consider two cases: 1) the detector counting time T is large compared to the coherence time τ_B of the background intensity fluctuation, and 2) T is short compared to τ_B. However, in both cases, the coherence time τ_S of the laser signal is taken to be large compared to both T and τ_B. This is a good approximation for a system using either a multimode laser or scattering from a target such as a satellite. If the Gaussian signal source is broadband, such that τ_S is small compared to both T and τ_B, then of course the signal fluctuations are averaged, giving rise to a Poisson distribution of counts, treated previously [1]-[5].

II. Chaotic Source and Stable Noise

The coherence time of background radiation τ_B is of the order of 10^{-12} seconds for a photodetector shielded with a narrow ($\cong$ 1 Å) interference filter. Thus for $T \gg \tau_B$, the photodetector cannot follow the background fluctuations and sees only the mean background intensity $\langle I_B \rangle$. In this case, the probability density function for I_B is simply a delta function,

$$P_B(I_B) = \delta(I_B - \langle I_B \rangle). \tag{3}$$

Since the laser signal and the background noise are independent, to obtain the probability density function $P(I)$ for I, we merely take the convolution integral of $P_S(I_S)$ and $P_B(I_B)$ to get [11]

$$\begin{aligned} P(I) &= \int_{-\infty}^{\infty} P_S(I-\lambda) P_B(\lambda)\, d\lambda \\ &= \frac{1}{\langle I_S \rangle} \exp\left(-\frac{I - \langle I_B \rangle}{\langle I_S \rangle}\right) u(I - \langle I_B \rangle). \end{aligned} \tag{4}$$

Since the coherence time of the laser signal τ_S is large compared to the counting time T, we can obtain the photoelectron counting distribution $p_{SB}(n, N_S, N_B)$ due to the signal and background noise within a time interval T by using Mandel's formula [12]. Thus,

$$\begin{aligned} p_{SB}(n, N_S, N_B) &= \frac{(\alpha T)^n}{n!} \int_0^{\infty} I^n \exp(-\alpha T I)\, p(I)\, dI \\ &= \frac{N_S^n}{(N_S+1)^{n+1}} \frac{\exp(N_B/N_S)}{n!} \Gamma\left[n+1, (N_S+1)\frac{N_B}{N_S}\right] \\ &= \frac{N_S^n \exp(-N_B)}{(N_S+1)^{n+1}} \sum_{k=0}^{n} \left[(N_S+1)\frac{N_B}{N_S}\right]^k \frac{1}{k!} \end{aligned} \tag{5}$$

where $N_B = \alpha T \langle I_B \rangle$ and $N_S = \alpha T \langle I_S \rangle$ are the mean counts due to background noise and signal, respectively, and α is the detector quantum efficiency. The quantity

$$\Gamma[n+1, x] = \int_x^{\infty} v^n \exp(-v)\, dv$$

is the incomplete Gamma function. The dark current, on the other hand, gives rise to a Poisson distribution for the photoelectron count,

$$p_D(n, N_D) = \frac{N_D^{\,n}}{n!} \exp(-N_D) \tag{6}$$

where N_D is the mean count due to the dark current noise.

Now, since the dark current noise is independent of both signal and background radiation, the probability distribution of the total photoelectron count $p_{SH}(n, N_S, N_B, N_D)$ is given by the convolution sum of $p_D(n, N_D)$ and $p_{SB}(n, N_S, N_B)$ [13]

$$p_{SH}(n, N_S, N_B, N_D) = \sum_{k=0}^{n} p_{SB}(n-k, N_S, N_B) p_D(k, N_D). \tag{7}$$

After some algebraic manipulation, one obtains finally

$$\begin{aligned} p_{SH}(n, N_S, N_B, N_D) &= \frac{N_S^n}{(N_S+1)^n} \frac{\exp(N_H/N_S)}{n!} \Gamma\left[n+1, (N_S+1)\frac{N_H}{N_S}\right] \\ &= \frac{N_S^n \exp(-N_H)}{(N_S+1)^{n+1}} \sum_{k=0}^{n} \left[(N_S+1)\frac{N_H}{N_S}\right]^k \frac{1}{k!} \\ &= p_{SH}(n, N_S, N_H) \end{aligned} \tag{8}$$

where $N_H = N_B + N_D$ is the total noise mean count. Equation (8) arises since the stable background noise alone yields a Poisson distribution for the photoelectron counts [8], and two independent Poisson distributions combine to form another Poisson distribution with a mean given by the sum of the two original means [13]. The addition of the dark current therefore serves only to increase the noise mean. The counting distribution $p_H(n, N_B, N_D)$ due to both background noise and dark current noise is therefore

$$p_H(n, N_B, N_D) = \frac{N_H^{\,n}}{n!} \exp(-N_H) = p_H(n, N_H). \tag{9}$$

This equation, along with (8), is plotted in Fig. 1 for the arbitrary value $N_S = 30$, with N_H as a parameter. Visual comparison of these curves will be found useful in physically understanding the likelihood detection which we will discuss in a later section.

III. Chaotic Source and Chaotic Noise

If, on the other hand, the background radiation entering the detector is narrow-band ($T \ll \tau_B$) and still noninterfering and independent of the signal radiation, then

$$P_B(I_B) = \frac{1}{\langle I_B \rangle} \exp\left(-\frac{I_B}{\langle I_B \rangle}\right) u(I_B). \tag{10}$$

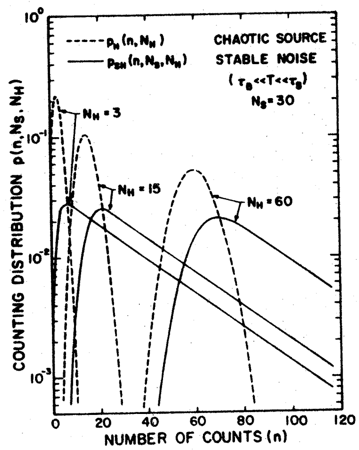

Fig. 1. The counting distributions $p_{SH}(n, N_S, N_H)$ arising from chaotic signal plus stable noise (solid curves) and $p_H(n, N_H)$ arising from stable noise alone (dashed curves) for the regime $\tau_B \ll T \ll \tau_S$. Plots are given for $N_H = 3, 15$, and 60, all with mean signal count $N_S = 30$.

The probability density function for the total effective intensity I for this case may therefore be written

$$P(I) = \int_{-\infty}^{\infty} P_B(I - \lambda) P_S(\lambda)\, d\lambda$$

$$= \frac{\exp(-I/\langle I_B \rangle)}{\langle I_S \rangle \langle I_B \rangle} \int_0^I \exp\left(\lambda\left(\frac{1}{\langle I_B \rangle} - \frac{1}{\langle I_S \rangle}\right)\right) d\lambda . \tag{11}$$

Generally, $\langle I_B \rangle \neq \langle I_S \rangle$, and (11) yields

$$P(I) = \frac{1}{\langle I_S \rangle - \langle I_B \rangle} (\exp(-I/\langle I_S \rangle) - \exp(-I/\langle I_B \rangle)) u(I), \quad \langle I_B \rangle \neq \langle I_S \rangle. \tag{12a}$$

For the special case where $\langle I_B \rangle = \langle I_S \rangle$, we obtain

$$P(I) = \frac{I}{\langle I_S \rangle^2} \exp(-I/\langle I_S \rangle) u(I), \quad \langle I_B \rangle = \langle I_S \rangle. \tag{12b}$$

Again, using Mandel's formula, one easily obtains the counting distribution $p_{SB}(n, N_S, N_B)$,

$$p_{SB}(n, N_S, N_B) = \frac{1}{N_S - N_B} \left[\left(\frac{N_S}{N_S + 1}\right)^{n+1} - \left(\frac{N_B}{N_B + 1}\right)^{n+1} \right], \quad N_B \neq N_S \tag{13a}$$

$$p_{SB}(n, N_S, N_B) = (n + 1) \frac{{N_S}^n}{(N_S + 1)^{n+1}}, \quad N_B = N_S. \tag{13b}$$

The dark current counting distribution is still taken to be Poisson. Following the same procedure as in the last section, therefore, we obtain

$$\begin{aligned} p_{SH}(n, N_S, N_B, N_D) &= \sum_{k=0}^{n} p_{SB}(n - k, N_S, N_B) p_D(k, N_D) \\ &= \frac{\exp(-N_D)}{N_S - N_B} \left\{ \left(\frac{N_S}{N_S + 1}\right)^{n+1} \cdot \sum_{k=0}^{n} \frac{1}{k!} \left[(N_S + 1) \frac{N_D}{N_S}\right]^k \right. \\ &\quad \left. - \left(\frac{N_B}{N_B + 1}\right)^{n+1} \cdot \sum_{k=0}^{n} \frac{1}{k!} \left[(N_B + 1) \frac{N_D}{N_B}\right]^k \right\}, \quad N_B \neq N_S \end{aligned} \tag{14a}$$

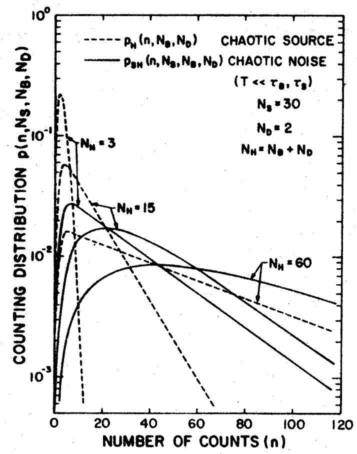

Fig. 2. The counting distributions $p_{SH}(n, N_S, N_B, N_D)$ arising from chaotic signal plus chaotic noise (solid curves) and $p_H(n, N_B, N_D)$ arising from noise alone (dashed curves) for the regime $T \ll \tau_B, \tau_S$. Plots are given for $N_H = 3$, 15, and 60, all with mean signal count $N_S = 30$ and mean dark current count $N_D = 2$.

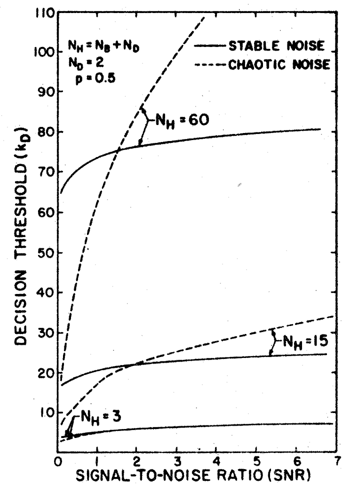

Fig. 3. Decision threshold k_D versus SNR for stable noise (solid curves) and for chaotic noise (dashed curves). Plots are given for $N_H = 3$, 15, and 60 with $p = 0.5$ and $N_D = 2$. Mean dark current count need only be specified for dashed curves.

and

$$p_{SH}(n, N_S, N_B, N_D) = \frac{N_B{}^n \exp(-N_D)}{(N_B + 1)^{n+2}}$$

$$\cdot \sum_{k=0}^{n} \left(\frac{N_B + 1}{N_B}\right)^k \frac{N_D{}^k}{k!} (n - k + 1), \quad N_B = N_S. \tag{14b}$$

The counting distribution which arises from background alone is geometric or Bose-Einstein [8]-[10],

$$p_B(n, N_B) = \frac{N_B{}^n}{(N_B + 1)^{n+1}} \tag{15}$$

while the counting distribution $p_H(n, N_B, N_D)$ due to background radiation plus dark current is given by

$$p_H(n, N_B, N_D) = \sum_{k=0}^{n} p_B(n - k, N_B) p_D(k, N_D)$$

$$= \frac{N_B{}^n \exp(-N_D)}{(N_B + 1)^{n+1}} \sum_{k=0}^{n} \left[(N_B + 1)\frac{N_D}{N_B}\right]^k \frac{1}{k!}. \tag{16}$$

This equation, along with (14), is plotted in Fig. 2. The mean signal count is once again taken to be $N_S = 30$, but now the dark current count N_D cannot be lumped with the background count N_B since both have different distributions. The mean dark current count is arbitrarily chosen as $N_D = 2$ for presentation in Fig. 2. Comparison with Fig. 1 shows that the counting distributions for chaotic noise, as well as for signal plus chaotic noise, are broader and overlap a good deal more than in the Poisson noise case. Furthermore, the most probable count occurs at lower count number, as expected for the increased fluctuations.

IV. Likelihood Detection

Using the likelihood detection criterion, a signal is judged to be present if [4]

$$\frac{p_{SH}(n, N_S, N_B, N_D)}{p_H(n, N_B, N_D)} \geqslant \frac{1 - p}{p} \tag{17}$$

where p is the a priori probability that a signal is present. The decision threshold k_D is defined as the smallest count number n for which (17) is satisfied. This quantity has been computed for both cases considered previously and is presented in Fig. 3. In this figure, k_D is plotted against the signal-to-noise ratio (SNR), where SNR $= N_S/N_H$, with N_H as a parameter. We have chosen $p = 0.5$ and $N_D = 2$. We note that $k_D \to 0$ as SNR $\to 0$, and $k_D \to \infty$ as SNR $\to \infty$. The latter may be seen by noting that as $N_S \to \infty$, $p_{SH} \to 0$ for all n, while $p_H \to 0$ only for $n \to \infty$. The likelihood threshold is observed to increase with SNR for all cases, but with a greater slope for the chaotic noise source. This arises from the broad nature of these counting distributions.

Using the likelihood criterion, a false detection occurs either when the signal is present but the count number falls below the threshold k_D, or when the signal is absent and

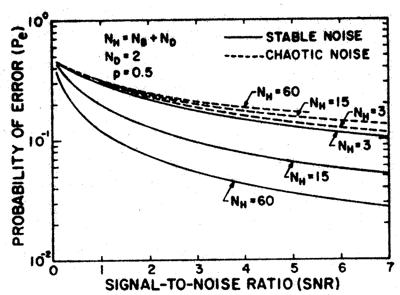

Fig. 4. Probability of error P_e versus SNR for stable noise (solid curves) and for chaotic noise (dashed curves). Plots are given for N_H = 3, 15, and 60, with p = 0.5 and N_D = 2.

the count number falls above k_D. Thus the probability of error is defined as [4]

$$P_e = p(1 - p_{SH}{}^B) + (1 - p)p_H{}^B \tag{18}$$

where

$$p_{SH}{}^B = \sum_{n=k_D}^{\infty} p_{SH}(n, N_S, N_B, N_D) \tag{19a}$$

and

$$p_H{}^B = \sum_{n=k_D}^{\infty} p_H(n, N_B, N_D). \tag{19b}$$

For the case $p = 0.5$ and $N_D = 2$, we present curves for P_e as a function of SNR for several values of N_H. The results for both stable and chaotic noise are presented in Fig. 4. The maximum value achieved by P_e is 0.5 for this binary system and occurs at SNR = 0, as can be seen from (18) and (19). This, of course, corresponds to complete lack of knowledge as to whether a signal is or is not present.

Clearly, the probability of error decreases with increasing SNR, and is, in all cases, lower for stable noise than for chaotic noise. What is unexpected, however, is the dependence of the probability of error on the mean noise count for a fixed value of SNR. For chaotic noise with a small and fixed dark current count, lower values of mean noise count (and, therefore, proportionately lower values of mean signal count as well, since the SNR is fixed) improve performance, while the reverse is true for stable noise. In the limit as SNR $\to \infty$, $P_e \to 0$ for all cases, which is expected; mathematically, this can be seen from (18) and (19). The curves therefore indicate that P_e depends strongly on the absolute signal level as well as on the SNR, in agreement with the results of previous authors [1], [2], [4]. By way of comparison, Lachs and Jankowich [1], Helstrom [2], and Pratt [4] all reported for their systems a decreasing probability of error with increasing signal for fixed SNR.

V. Conclusion

The photoelectron counting distributions due to signal plus noise and due to noise alone are calculated and plotted for counting times small in comparison with the chaotic source coherence time. Curves for both $T \gg \tau_B$ and $T \ll \tau_B$ are given. In the latter case, new results for the intensity distribution function $P(I)$, and for the counting distributions $p_{SB}(n, N_S, N_B)$, $p_{SH}(n, N_S, N_B, N_D)$, and $p_H(n, N_B, N_D)$ due to signal plus background noise, signal plus total noise, and due to noise alone are derived. Only noninterfering additive noise, such as might arise from the random incoming direction of the background radiation or from beats falling outside the detector bandwidth, has been considered.

The decision threshold k_D and the probability of error P_e have been evaluated and plotted for these cases, using some typical parameters. For increasing SNR, k_D increases without limit, while P_e always decreases asymptotically to zero.

It is apparent from the probability of error curves (Fig. 4) that it is preferable to operate in the regime $T \gg \tau_B$, whenever possible. That is, improved results are obtained for the receiver which does not detect irradiance fluctuations due to background radiation. This is the usual regime for broad-band background radiation. In this case, furthermore, the probability of error is lower for higher signal levels (at fixed SNR). Thus it may be profitable to provide optical amplification before photodetection, provided that this additional process does not itself introduce excessive noise.

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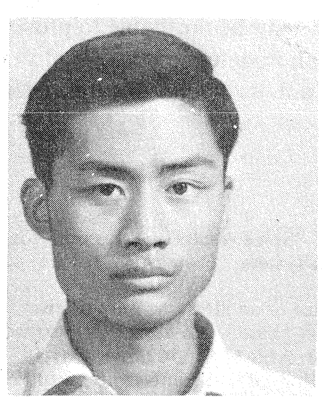

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