From the slope of the curve in Fig. 1 the Raman gain constant is calculated at $5 \cdot 1 \times 10^{-12}$ cm/W. This is lower than published values of $6 \cdot 0 \times 10^{-12}$ calculated at this wavelength[2] which, in addition to experimental error, is thought to be due to random walk-on and walk-off of the pulses arising from small variations in the zero dispersion wavelength down the fibre. This has not yet been confirmed experimentally but it will be the subject of further study in the near future.

The effect of saturation is also evident from Fig. 1 where the departure from linearity in the characteristic increases for an increasing signal level. From Fig. 3 it is estimated that a gain compression of -3 dB is obtained at an output signal level in excess of $+25$ dBm.

Using the pulsed pumping technique described here for a soliton system operating at say a 10% duty cycle in a dispersion shifted fibre of length 100 km, the loss (0·2 dB/km) would be equalised by Raman gain for a mean pump power of 40 mW. This compares favourably with a pump power of 20–30 mW typical of the requirement for a line amplifier based on the 'special' erbium fibre that is needed to achieve this gain.

For amplification at $1 \cdot 56\,\mu$m the pump wavelength would be the same as that for erbium fibre amplifiers but the diode lasers used in the Raman based schemes need to be mode-locked with short external cavities to fix the required bit-rate.

Conclusion: Synchronous pumping of a Raman-fibre amplifier has been experimentally demonstrated for the first time. In a practical span length using soliton transmission the mean power for amplification obtained in this way is comparable to that required in erbium fibre amplifiers.

The immediate applications for this technique would be in high bit-rate single hop communication systems. It is also envisaged that a repeatered system could be feasible by arranging for adjacent amplifier pumps to be time referenced by synchronous pumping using the transmitted pump power from the previous amplifier.

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References

1 BYRON, K. C.: 'The status of Raman amplification in current and future systems'. IEE colloquium on 'Non-linear effects in fibre communications, London, 20th Nov. 1990

2 STOLEN, R. H., and IPPEN, E. P.: 'Raman gain in glass optical waveguides', *Appl. Phys. Lett.*, 1973, **22**, p. 276

BIT-ERROR RATE FOR A LIGHTWAVE COMMUNICATION SYSTEM INCORPORATING AN ERBIUM-DOPED FIBRE AMPLIFIER

Indexing terms: Optical communication, Optoelectronics

The photon-number distribution at the output of an erbium-doped fibre amplifier (EDFA) with coherent light at the input is shown to obey the noncentral-negative-binomial (NNB) distribution. The use of this distribution in a binary on/off keying system gives rise to a lower bit-error rate (BER) than the Gaussian distribution, because the tails of these distributions differ (even for large mean input photon numbers).

Introduction: Erbium-doped fibre amplifiers (EDFA) are finding increasing use in optoelectronic systems. There are a number of theoretical formulations of the laser amplification process that are useful for dealing with amplifiers of different configuration.[1,2] In the present paper we study the evolution of the photon statistics of a coherent light beam as it passes through an EDFA and the BER of a lightwave system incorporating such an amplifier. Each of the photons in our EDFA configuration can be viewed as initiating its own BDI process. In Section 2, we obtain the relationship between the input and output probability distributions (and probability generating functions) for a BDI process with coherent light at its input. In Section 3, the bit-error rate (BER) of a binary on/off keying system employing an EDFA optical preamplifier is calculated using the probability distributions obtained in Section 2.

Theory: Following the early work of Shimoda *et al.*,[3] the amplifier's photons are treated as a population governed by a simple BDI process, with birth and death representing stimulated emission and absorption, respectively, and with immigration representing spontaneous emission. Let t denote either the time of traversal or the depth of penetration of a medium characterised by a birth rate per particle $a(t)$, a death rate per particle $b(t)$ and an immigration rate $c(t)$ that is independent of the current population size. By taking the limit of a difference equation that accounts for these three processes, it is readily shown[3] that the probability distribution $P(n, t)$, with n particles present at time (or depth) t, satisfies the equation

$$\frac{dP(n, t)}{dt} = -[(a + b)n + c]P(n, t) + [a(n - 1) + c] \times P(n - 1, t) + b(n + 1)P(n + 1, t) \quad (1)$$

Saturation is assumed to be absent. The probability generating function (PGF), given the initial condition $P(n, 0) = \delta(1)$ (which means that there is only one photon at the input of the amplifier), satisfies[4]

$$G_{BDI}(s, t) = \sum_n P(n, t)s^n = \left[\frac{1 + (g - \langle n_{th}\rangle)(s - 1)}{1 - \langle n_{th}\rangle(s - 1)}\right] \times [1 - \langle n_{th}\rangle(s - 1)]^{-M} \quad (2)$$

where $g(t) = \exp\{\int_0^t [a(\tau) - b(\tau)]\, d\tau\}$ represents the overall gain of the amplifier, $M = c(t)/a(t)$ represents the number of degrees of freedom or modes and $\langle n_{th}\rangle = g(t) \int_0^t a(\tau)\, d\tau$ represents the mean spontaneous-emission population $\langle n_0\rangle$ divided by the number of modes M. The length L of the EDFA specifies the time t (which is equal to the ratio of the amplifier length to the speed of light). Eqn. 2 can be written as the product $G_{BDI}(s) = G_{BD}(s)G_I(s)$, where

$$G_{BD}(s) = [1 + (g - \langle n_{th}\rangle)(s - 1)]/[1 - \langle n_{th}\rangle(s - 1)]$$

and

$$G_I(s) = [1 - (\langle n_0\rangle/M)(s - 1)]^{-M}$$

with $\langle n_0\rangle = M\langle n_{th}\rangle$.

When the initial population of the BDI process is, instead of one photon, a random number of photons N governed by the distribution $P(n, 0) = P_{in}(N)$, with PGF $G_{in}(s)$, the output PGF can be written in terms of the input PGF as[4] $G_{out}(s) = [G_{in}(G_{BD})]G_I$. Coherent input light to the EDFA exhibits Poisson photon counting statistics. Using the expressions for

$G_{BD}(s)$ and $G_I(s)$ provided above, together with $G_{in}(s) = \exp[\langle N \rangle(s-1)]$ for a Poisson of mean $\langle N \rangle$, we obtain a PGF at the output of the EDFA given by

$$G_{out}(s) = \exp\left[\frac{g\langle N \rangle(s-1)}{1 - \langle n_{th} \rangle(s-1)}\right]\left[1 - \frac{\langle n_0 \rangle}{M}(s-1)\right]^{-M} \quad (3)$$

This is recognised as the PGF of the noncentral-negative-binomial (NNB) distribution[5]

$$P(n) = \frac{\langle n_{th} \rangle^n}{(1 + \langle n_{th} \rangle)^{n+M}} \exp\left(-\frac{g\langle N \rangle}{1 + \langle n_{th} \rangle}\right) \times L_n^{(M-1)}\left[-\frac{g\langle N \rangle}{\langle n_{th} \rangle(1 + \langle n_{th} \rangle)}\right] \quad (4)$$

where $L_n^{(M-1)}$ is the generalised Laguerre polynomial $L_n^{(M-1)}(-x) = \sum_{(k=0,n)} x^k(n+M-1)!/[(k+M-1)!(n-k)!k!]$. This distribution can alternatively be derived from a wave point of view by considering an interfering superposition of coherent and thermal light.[5] The mean and variance of this distribution are $\langle n \rangle = g\langle N \rangle + M\langle n_{th} \rangle$ and $\mathrm{Var}(n) = g\langle N \rangle + 2g\langle N \rangle\langle n_{th} \rangle + M\langle n_{th} \rangle(1 + \langle n_{th} \rangle)$, respectively, which are in accordance with the usually used expressions.[2,6]

Bit-error rate: Consider a binary on/off keying (OOK) system and define P_0 and P_1 as the *a priori* probabilities that a '0' and a '1' are transmitted, respectively. The probability of error, or the bit error rate (BER), is then given by[7] $BER = P_0 \sum_{(n=D,\infty)} P_N(n) + P_1 \sum_{(n=0,D)} P_{SN}(n)$, with D representing the optimal detection threshold and $P_N(n)$ and $P_{SN}(n)$ the photon-number distribution of the output of the system when a '0' (the output is noise alone) and a '1' (the output is signal-plus-noise) is transmitted, respectively.

We now evaluate the performance of an OOK system using an EDFA in the absence of extraneous background light, and with a receiver of unity quantum efficiency and negligible dark and electronic (thermal) noise. It is assumed that all of the photons are collected in each bit and that there is no inter-symbol interference. $P_{SN}(n)$ is then the NNB distribution represented in eqn. 4. Taking $\langle N \rangle = 0$ in eqn. 4 gives $P_N(n) = \langle n_{th} \rangle^n/(1 + \langle n_{th} \rangle)^{n+M}$, which is the well-known negative-binomial (NB) distribution, a special case of the NNB. Substituting these two results into the equation for the BER allows us to calculate the BER for various values of $\langle N \rangle$. The results are shown by the solid curve in Fig. 1.

The Gaussian distribution is often used to calculate the BER for lightwave systems[6] using EDFAs.[8] For comparison with the results using the NNB, we calculate the BER using Gaussian distributions for $P_N(n)$ and $P_{SN}(n)$, with the same means and variances as used in the NNB calculation. The results are shown as the broken curve in Fig. 1. Use of the Gaussian distribution overestimates the BER. The reason for the difference is that, as shown in Fig. 2, the tails of the NNBs differ from those of the corresponding Gaussian distributions. In particular, they behave differently even in the limit of large $\langle N \rangle$, as shown in the Appendix. Note further that the NNB curves cross at a higher photon number than do the Gaussian curves so that the optimal value of D differs for these two cases.

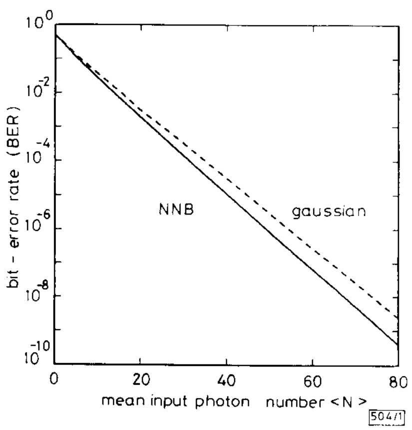

Fig. 1 *BER for a binary OOK system using an EDFA modelled by NNB and Gaussian statistics. Convenient parameters were chosen to generate the curves: $g = 158$, $M = 1$, $\langle n \rangle = 157$ and $P_0 = P_1 = 0.5$. $M = 1$ implies that the spontaneous emission is filtered so that only those photons emitted into the same mode as the stimulated photons are detected*

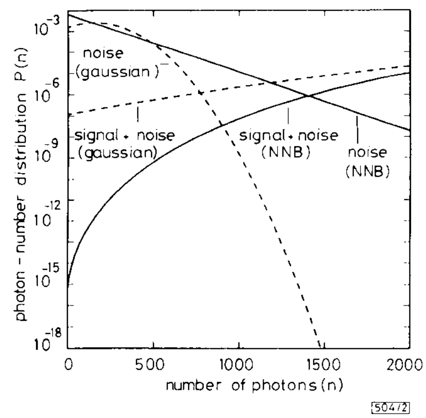

Fig. 2 *Photon-number distributions for $P_N(n)$ and $P_{SN}(n)$. The solid curves represent NNB distributions whereas the dashed curves represent Gaussian distributions with the same means and variances as the NNBs. The parameters used to generate the curves are $g = 158$, $M = 1$, $\langle n \rangle = 157$ and $\bar{N} = 0$ and 30*

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References

1 SHEPHERD, T. J., and JAKEMAN, E.: 'Statistical analysis of the amplified spontaneous emission of DFB semiconductor laser amplifiers', *J. Opt. Soc. Amer. B*, 1987, **4**, pp. 1860–1869

2 GOLDSTEIN, E. L., and TEICH, M. C.: 'Noise in resonant optical amplifiers of general resonator configuration', *IEEE J. Quantum Electron.*, 1989, **25**, pp. 2289–2296

3 SHIMODA, K., TAKAHASHI, G., and TOWNES, C. H.: 'Fluctuations in amplification of quanta with application to maser amplifiers', *J. Phys. Soc. Jpn.*, 1957, **12**, pp. 686–700

4 PARZEN, E.: 'Stochastic processes' (Holden-Day, San Francisco, 1962)

5 SALEH, B. E. A.: 'Photoelectron statistics' (Springer-Verlag, New York, 1978)

6 YAMAMOTO, Y.: 'Noise and error rate performance of semiconductor laser amplifiers in PCM-IM optical transmission system', *IEEE J. Quantum Electron.*, 1980, **16**, pp. 1073–1081

7 SCHWARTZ, M.: 'Information transmission, modulation, and noise' (McGraw-Hill, New York, 1990), 4th edn.

8 GILES, C. R., DESURVIRE, E., ZYSKIND, J. L., and SIMPSON, J. R.: 'Noise performance of erbium-doped fiber amplifier pumped at 1·49 μm, and application to signal preamplification at 1·8 Gbits/s', *IEEE Photon. Technol. Lett.*, 1989, **1**, pp. 367–369

Appendix: From the equation for the BER we know that it depends on the right tail of $P_N(n)$ and on the left tail of $P_{SN}(n)$. If $P_N(n)$ is the NB distribution, as $n \to \infty$,

$$P_N(n) = \langle n_{th} \rangle^n/(1 + \langle n_{th} \rangle)^{n+M} \propto [(1 + \langle n_{th} \rangle)/\langle n_{th} \rangle]^{-n}$$

However, for a Gaussian with the same mean and variance as the NB, as $n \rightarrow \infty$,

$$P_N(n) = \exp\,[-(n - \langle n_{th}\rangle)^2/(2\langle n_{th}\rangle^2)]/[\sqrt{(2\pi)}\langle n_{th}\rangle]$$
$$\propto \exp\,(-n^2)$$

Thus, the right tails of the two distributions differ in form, as is readily seen in Fig. 2.

As is also evident in Fig. 2, as $n \rightarrow 0$ the left tail of the NNB distribution decays more rapidly than the left tail of the equivalent Gaussian. Indeed for $n = 0$, $P_{SN}(0)$ for the NNB is

$$P_{SN}(0) = \exp\,[-g\langle N\rangle/(1 + \langle n_{th}\rangle)]/(1 + \langle n_{th}\rangle)^M$$
$$\approx \exp\,(-3b\langle N\rangle)[\exp\,(-b\langle N\rangle)/\langle n_{th}\rangle^M]$$

with $b = g/(4\langle n_{th}\rangle)$, whereas for a Gaussian with the same mean $g\langle N\rangle$ and variance $2g\langle N\rangle\langle n_{th}\rangle$ as the NNB (assuming $\langle N\rangle$ very large),

$$P_{SN}(0) = [\langle N\rangle^{-1/2}/(4\pi g\langle n_{th}\rangle)^{1/2}]\,\exp\,[-g\langle N\rangle/(4\langle n_{th}\rangle)]$$
$$\approx \langle N\rangle^{-1/2}[\exp\,(-b\langle N\rangle)/(4\pi g\langle n_{th}\rangle)^{1/2}].$$

Thus, even as $\langle N\rangle$ becomes very large, the left tail of the NNB cannot be approximated by a Gaussian.

UNIFORM CNR DESIGN RULES FOR COHERENT SUBCARRIER MULTIPLEXED SYSTEM WITH MULTIOCTAVE FREQUENCY ALLOCATION

Indexing terms: Multiplexers and multiplexing, Phase modulation

For a CSCM system with multioctave configuration, the CNR difference among channels is significant and needs to be taken into consideration. Here we take 'equal optimal CNR of the first and central channels' as a criterion, then we derive a design rule to reduce CNR difference significantly. The example shows that it can be lessened from 7 to 1·5 dB with lower received signal power to achieve the same CNR requirement.

Introduction and system description: For a coherent subcarrier multiplexed (CSCM) system as shown in Fig. 1, the degree of nonuniform CNR in a multioctave configuration may be large as reported in Reference 1. Usually in a multioctave system,

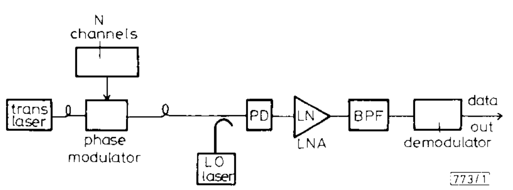

Fig. 1 *System block diagram of CSCM*

the second-order intermodulation (IMD_2) contaminates the first channel mostly, but the third-order intermodulation (IMD_3) contaminates the central channel mostly.[1] In this letter, we first obtain the optimal CNR expressions of both the first and the central channel and use the criterion 'equal optimal CNR of the first and central channels' to equalise the CNR performance approximately. Then we can obtain the phase modulation (PM) index in terms of channel spacing, the total number of channels, and the 'octave-number' with corresponding received power to meet the CNR requirement of the first channel (worst case in a multioctave system). The optical CSCM system consists of N equispaced channels with signal bandwidth B and channel separation Δf. To reduce IMD_2, we locate the frequency of the ith channel ($i = 1, \ldots, N$) at $f_i = (i-1)\,\Delta f + F_{min}\,\Delta f + \Delta f/2$, where F_{min} is an integer. The offset frequency $\Delta f/2$ is employed to let IMD_2 degrade the channel signal least. The power spectra of IMD_2 and IMD_3 can be taken as the convolution of the power spectrum of each channel[2] and their magnitudes are determined by the signal power level and the phase modulation (PM) index.

Derivation of CNR and optimum PM index: The CNR for large local oscillator power which suppresses the thermal noise can be derived by using the first-order approximations of $J_0(\beta)$ and $J_1(\beta)$ as[1]

$$CNR = (4qB/(RP_S\beta^2) + h_2 K_2\beta^2/4 + h_3 K_3\beta^4/16)^{-1} \qquad (1)$$

where q is the electron charge, R is the photodiode responsivity, P_S is the received signal power, and β is the PM index (assumed equal for all channels). h_2 and h_3, related to the power spectra of IMD_2 and IMD_3 in the neighbourhood of the signal band, are the fractions of the power within the passband of the bandpass filter. Under the condition of equal channel spacing, the value of h_2 for the ideal rectangular signal spectrum can be expressed as $h_2 = (3 - \Delta f/B)^2/8$ for $\Delta f < 3B$, and $h_2 = 0$ for $\Delta f \geq 3B$. The value of h_3 for the ideal rectangular signal spectrum is 2/3. $K_2(i)$ and $K_3(i)$ represent the numbers of IMD_2s and IMD_3s contaminating the ith channel, respectively. They can be expressed for a multioctave configuration as in References 3 and 4, case (A) $(1 < X \leq 2)$: $K_2(i) = N(1 - 1/X) - i + 1$ for $1 \leq i \leq N - F_{min}$, $K_2(i) = 0$ for $N - F_{min} + 1 \leq i \leq F_{min} + 1$ and $K_2(i) = (i - N/X - 1)/2$ for $F_{min} + 2 \leq i \leq N$; case (B) $(2 < X)$: $K_2(i) = N(1 - 1/X) - i + 1$ for $1 \leq i \leq F_{min} + 1$, $K_2(i) = [N(2 - 3/X) - i + 1]/2$ for $F_{min} + 2 \leq i \leq N - F_{min}$ and $K_2(i) = (i - N/X - 1)/2$ for $N - F_{min} + 1 \leq i \leq N$. For both cases (A) and (B), $K_3(i) = i(N - i + 1)/2 + [(N-3)^2 - 5]/4$, where $X \equiv N/F_{min}$ (the 'octave-number') and $X = 2, 3, \ldots$, etc. represent the two-, three-, ..., octave configuration.

Here we define the CNR difference, $\Delta(i, j)$, between channels i and j from eqn. 1 as

$$\Delta(i, j) = 10\log_{10}\frac{CNR(i)}{CNR(j)}$$
$$= 10\log_{10}\frac{1 + [h_2 K_2(j) + h_3 K_3(j)\beta^2/4]/[16qB/(RP_S\beta^4)]}{1 + [h_2 K_2(i) + h_3 K_3(i)\beta^2/4]/[16qB/(RP_S\beta^4)]} \qquad (2)$$

We can obtain the optimal PM index that maximises the CNR of the ith channel as $\beta_{opt} = \langle 0{\cdot}5\{-y + \sqrt{[y^2 + (64/3h_3 K_3(i)CNR)]}\}\rangle^{1/2}$, where $y = [8h_2 K_2(i)]/[3h_3 K_3(i)]$.[1] The corresponding receiver sensitivity is $P_S = (4qB/R)/[\beta_{opt}^2/CNR - h_2 K_2(i)\beta_{opt}^4/4 - h_3 K_3(i)\beta_{opt}^6/16]$. Then we can express this maximum obtainable CNR for this channel in terms of β_{opt} as $CNR_{opt} = [h_2 K_2(i)\beta_{opt}^2/2 + 3h_3 K_3(i)\beta_{opt}^4/16]^{-1}$.

The design rules: We usually have the same received power P_S and unified system PM index β_{sys} for all the channels in the CSCM system with total N_{sys} channels. We also need to keep the CNR of all channels above a specific value. Therefore, we may choose the appropriate P_S, β_{sys} and Δf to meet the requirement. Here, we are concerned about 'uniform CNR for all channels' and will solve this problem as follows: first, we take channels 1 and $N_{sys}/2$ as the worst channels under the consideration of IMD_2 and IMD_3, respectively. We apply 'equal optimal CNR of channels 1 and $N_{sys}/2$' as a criterion, that is, $CNR_{opt}(1) = CNR_{opt}(N_{sys}/2)$, to obtain the corresponding $\beta_{sys} = 2(3 - \Delta f/B)[K_2(1) - K_2(N_{sys}/2)/N_{sys}(N_{sys} - 2)]^{0{\cdot}5}$. Then we may simplify it as $\beta_{sys} = 2(3 - \Delta f/B)(1 - 1/X/N_{sys} - 2)^{0{\cdot}5}$ (case (A)), and $\beta_{sys} = (3 - \Delta f/B)[(1 + 2/X)/N_{sys} - 2)]^{0{\cdot}5} + 0{\cdot}016$ (case (B)). Hence, the unified system PM index can be obtained from the given total channel number and octave-number; together with the specified channel spacing Δf to achieve the required CNR value with the corresponding P_S.