Heart Rate Variability

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Colleagues:

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- **Robert Turcott, Stanford Medical School**
- **Markus Feurstein, Wirtschaftsuniversität Wien**
- **Stefan Thurner, Allgemeines Krankenhaus Wien**

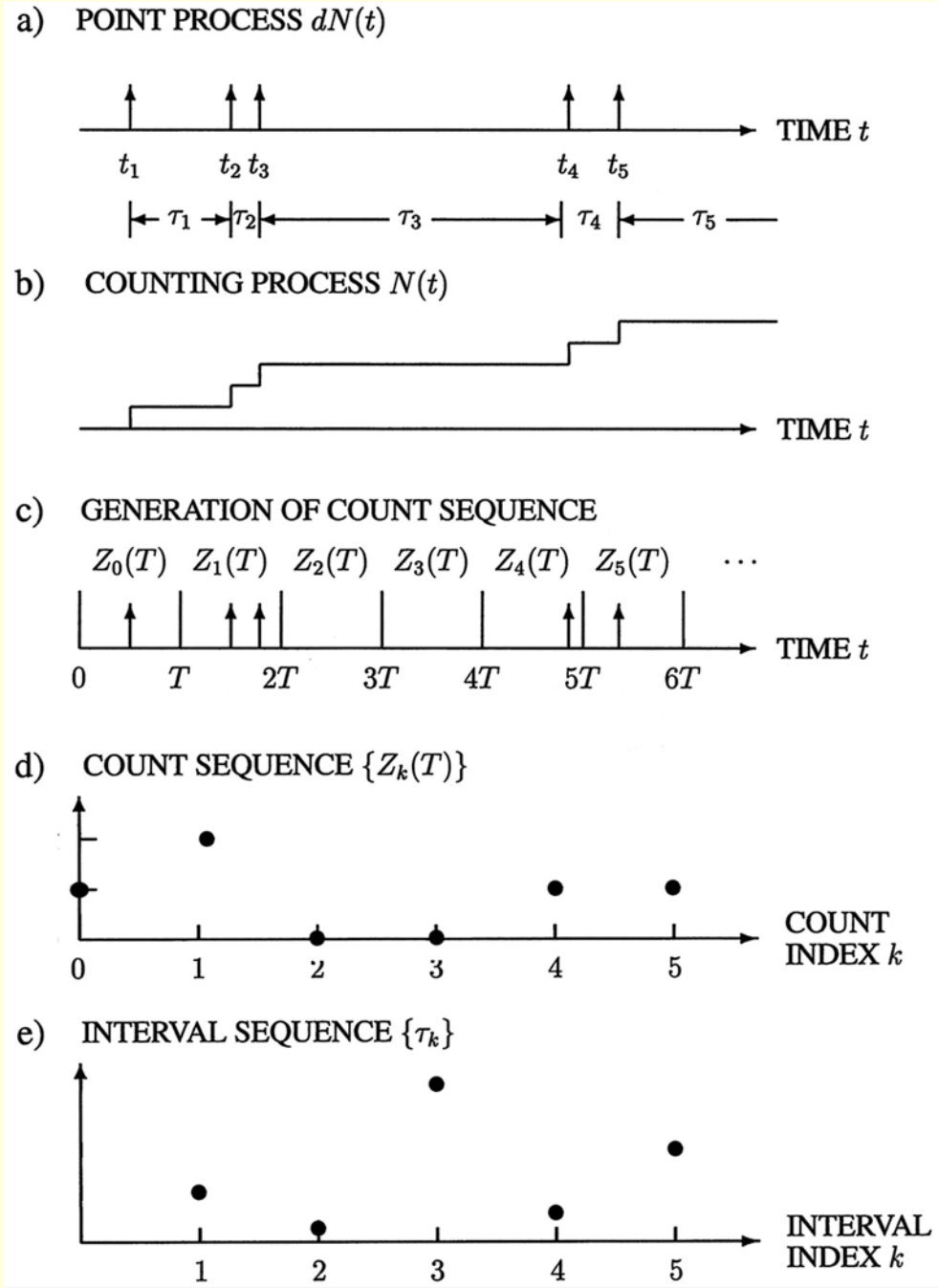
Workshop on New Themes & Techniques in Complex Systems 2005

FRACTALS

$$d \propto s^{-0.30}$$

MEASUREMENT SCALE s (km)	MEASURED LENGTH d (km)
· 0.694	534
— 6.94	314
——— 69.4	133

POINT PROCESSES

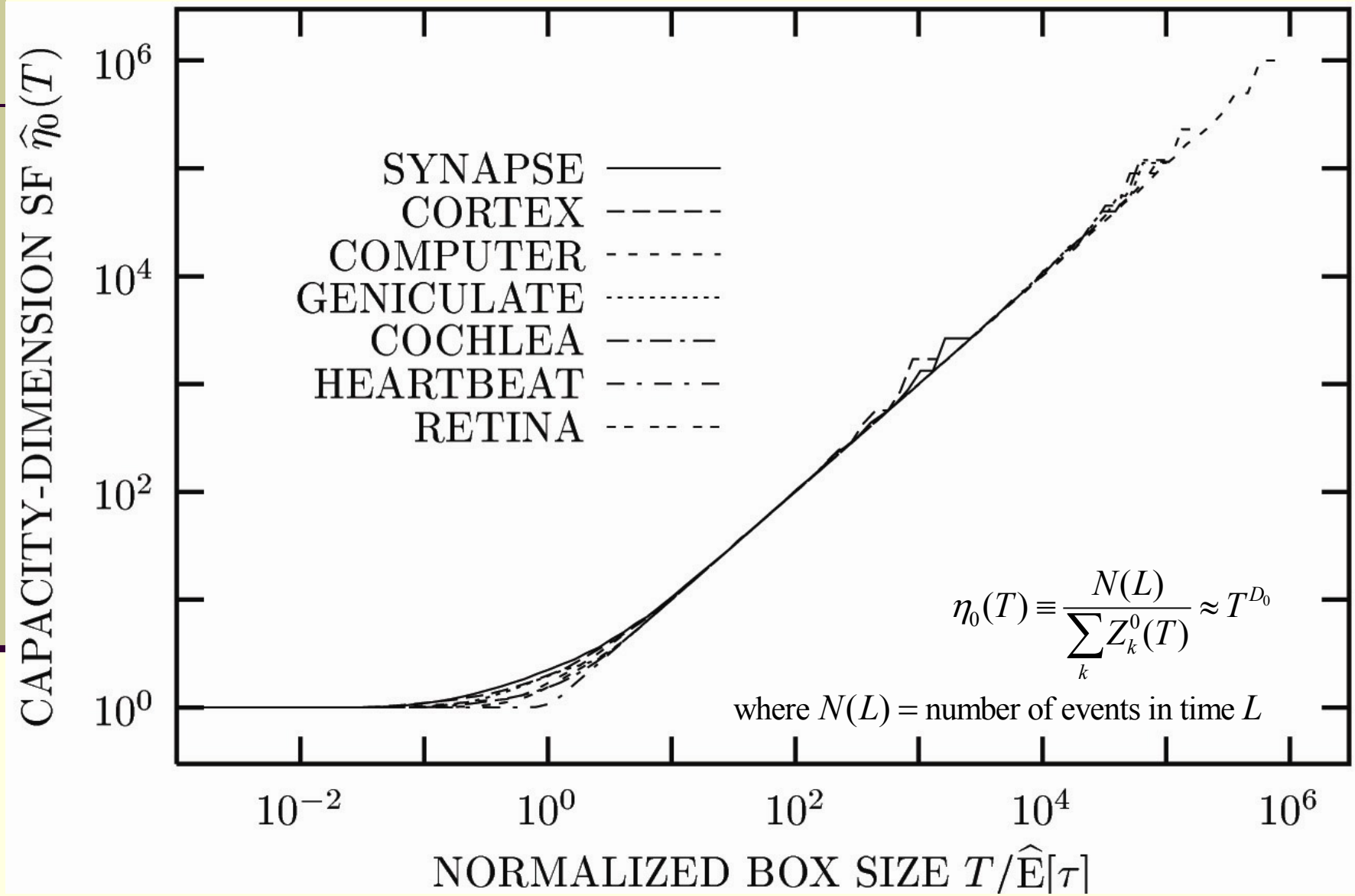

a) POINT PROCESS $dN(t)$
TIME t
t_1
t_2 t_3
t_4 t_5
τ_1 τ_2 τ_3 τ_4 τ_5
b) COUNTING PROCESS $N(t)$
TIME t
c) GENERATION OF COUNT SEQUENCE
$Z_0(T)$ $Z_1(T)$ $Z_2(T)$ $Z_3(T)$ $Z_4(T)$ $Z_5(T)$ ...
TIME t
0 T $2T$ $3T$ $4T$ $5T$ $6T$
d) COUNT SEQUENCE $\{Z_k(T)\}$
COUNT INDEX k
0 1 2 3 4 5
e) INTERVAL SEQUENCE $\{\tau_k\}$
INTERVAL INDEX k
1 2 3 4 5

FRACTAL-BASED POINT PROCESSES

- Fractal point processes
- Fractal-rate point processes

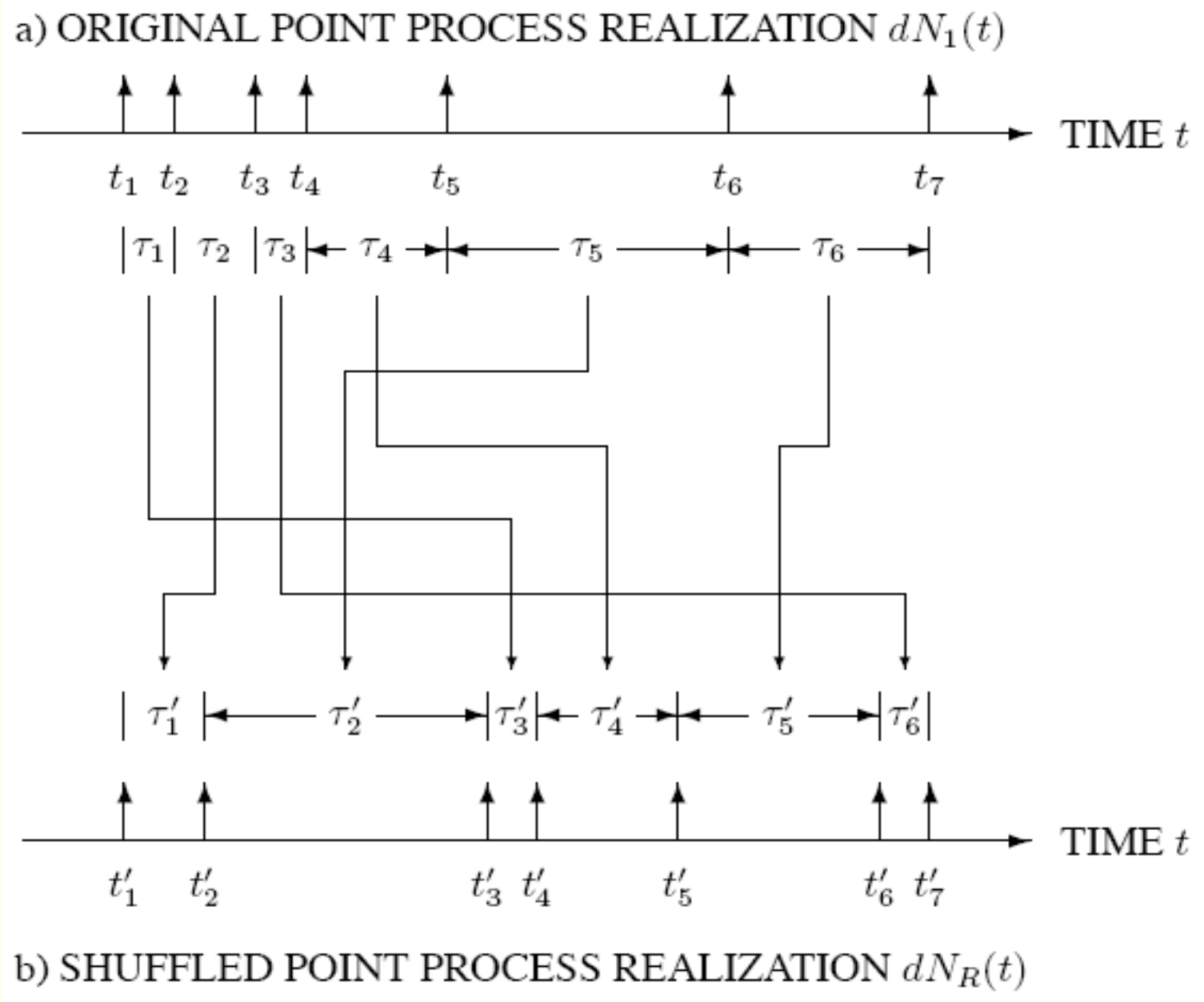
S. B. Lowen and M. C. Teich, *Fractal-Based Point Processes* (Wiley, Hoboken, NJ, 2005).

CAPACITY-DIMENSION SCALING FUNCTION



SURROGATE DATA ANALYSIS

SHUFFLING


a) ORIGINAL POINT PROCESS REALIZATION $dN_1(t)$
TIME t
t_1 t_2 t_3 t_4 t_5 t_6 t_7
τ_1 τ_2 τ_3 τ_4 τ_5 τ_6
τ'_1 τ'_2 τ'_3 τ'_4 τ'_5 τ'_6
TIME t
t'_1 t'_2 t'_3 t'_4 t'_5 t'_6 t'_7
b) SHUFFLED POINT PROCESS REALIZATION $dN_R(t)$

CONGESTIVE HEART FAILURE

INABILITY OF HEART TO INCREASE CARDIAC OUTPUT IN PROPORTION TO METABOLIC DEMANDS

Symptom complex:
Many different presentations and etiologies

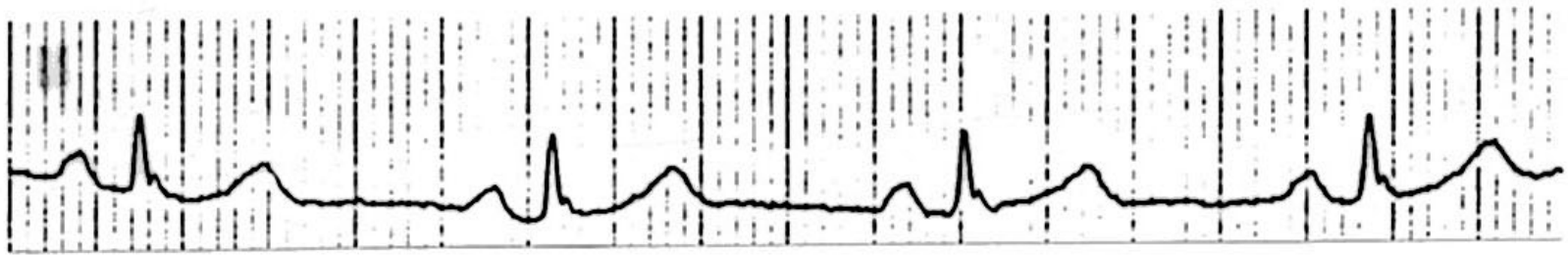
Typical symptoms:

- Shortness of breath
- Swelling in legs
- General fatigue and weakness

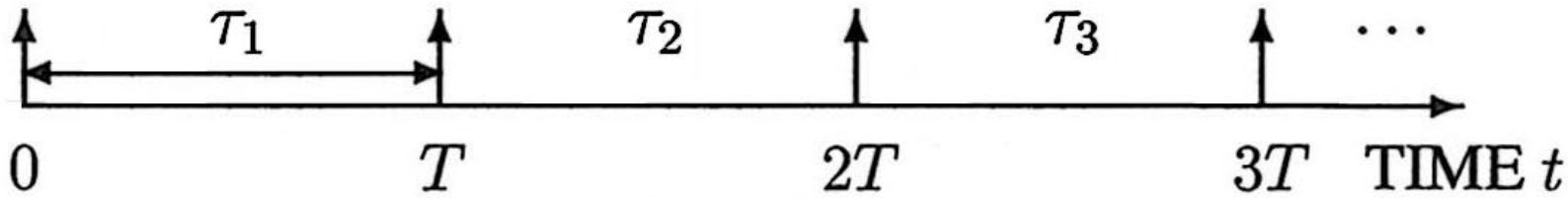
- Clinical diagnostics:
- Ascultate heart
- Carotid pulse
- Electrocardiogram
- Chest radiograph

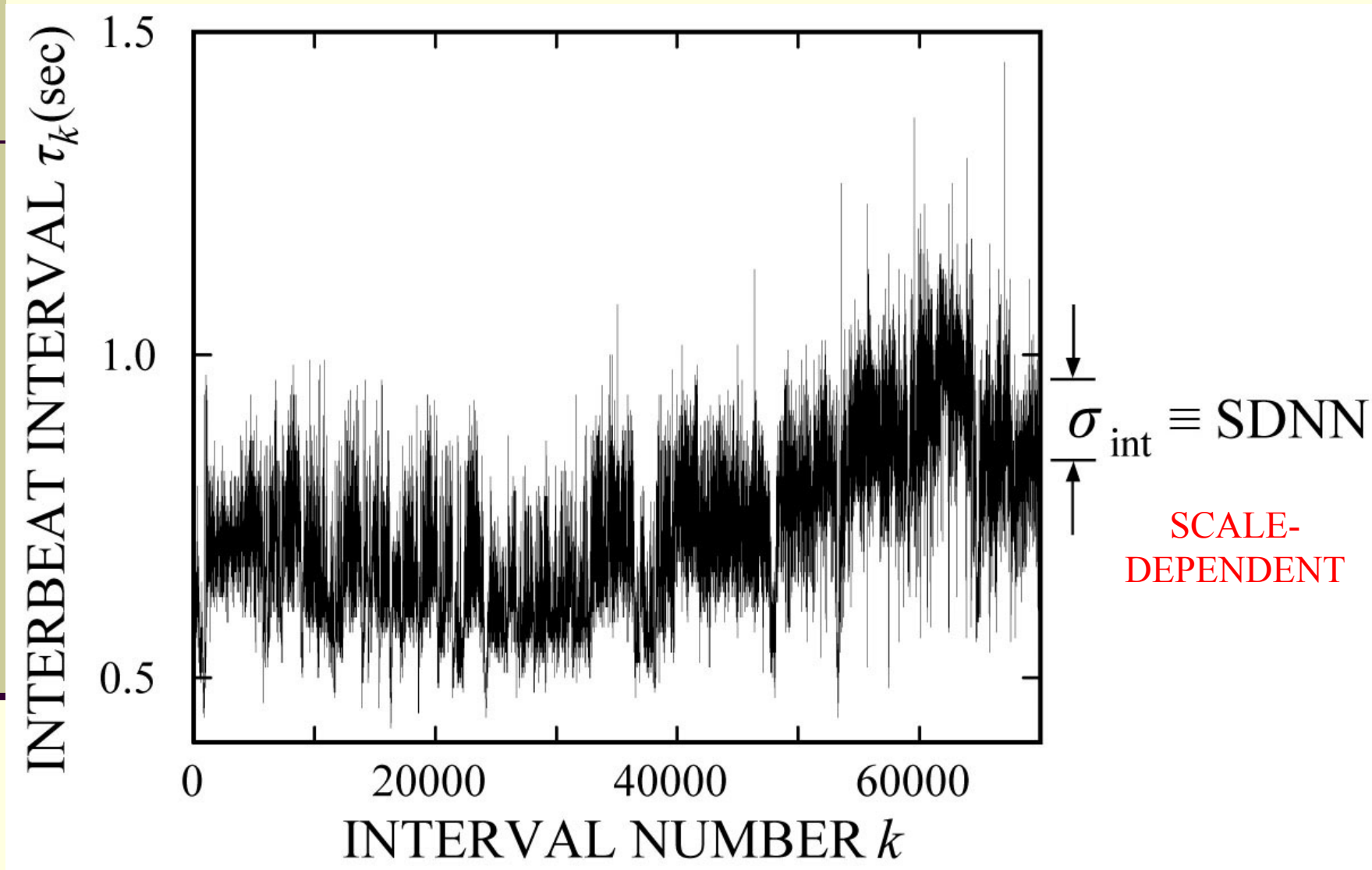
INTERVAL-BASED MEASURES

a)

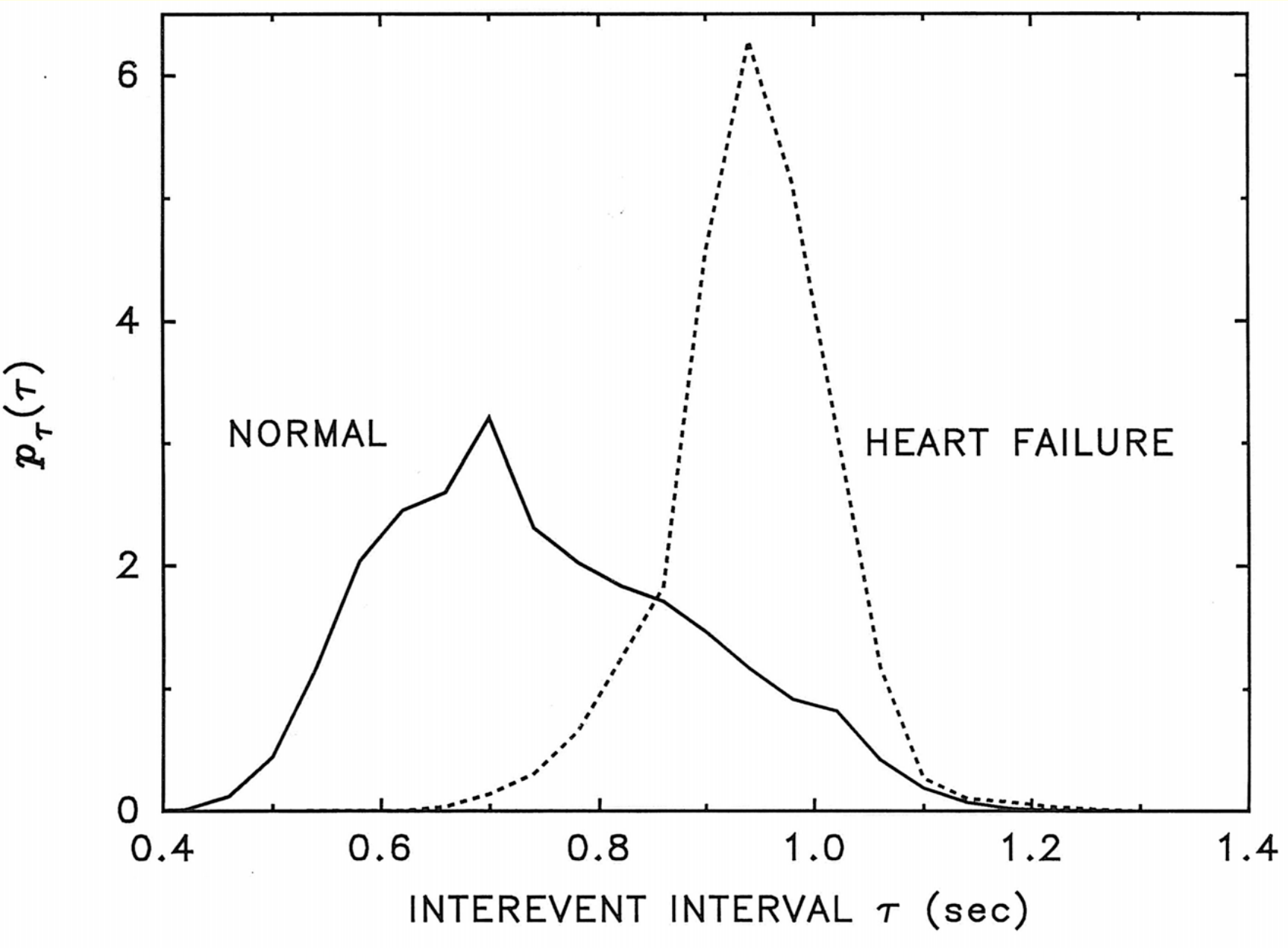


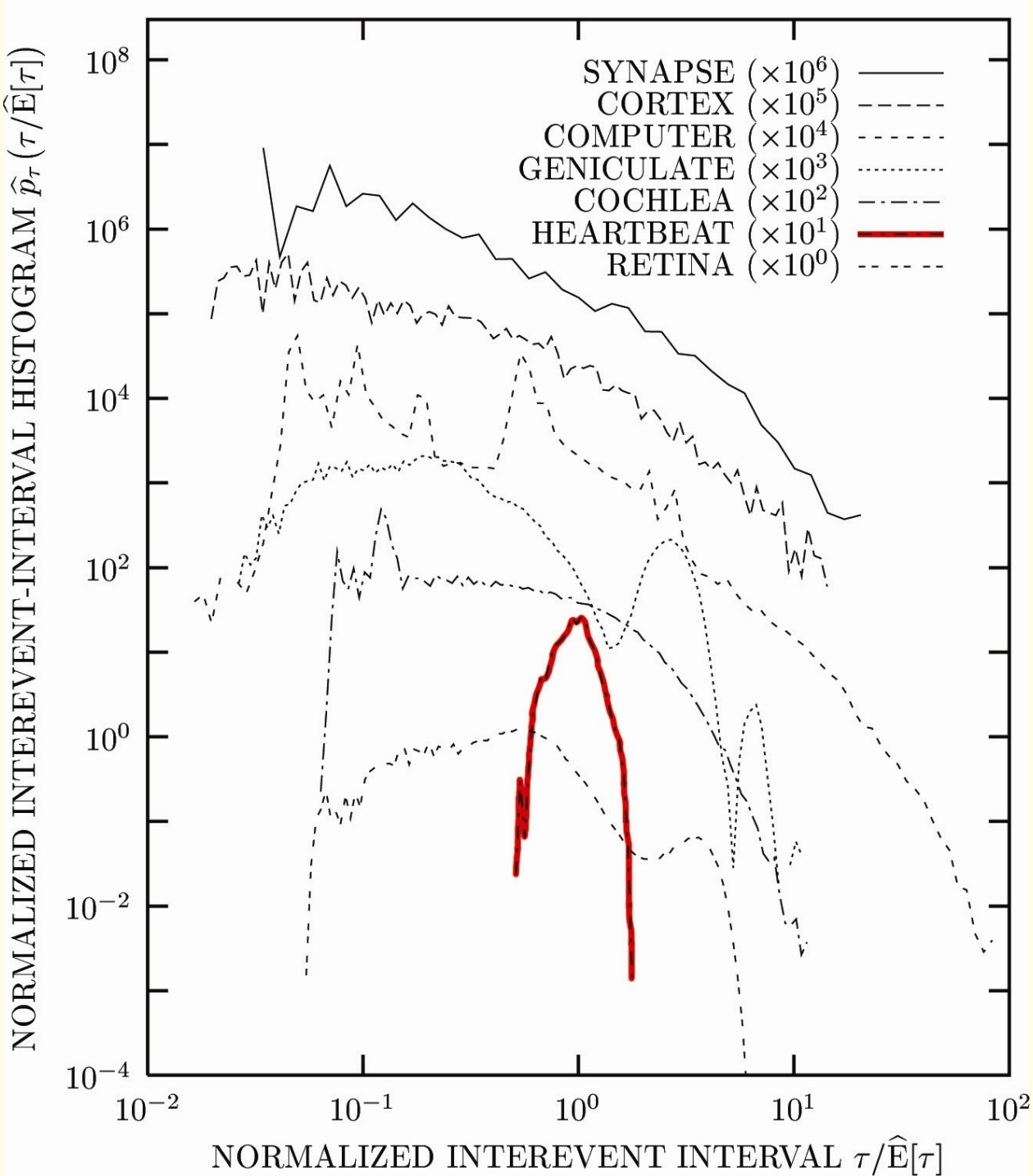
b)



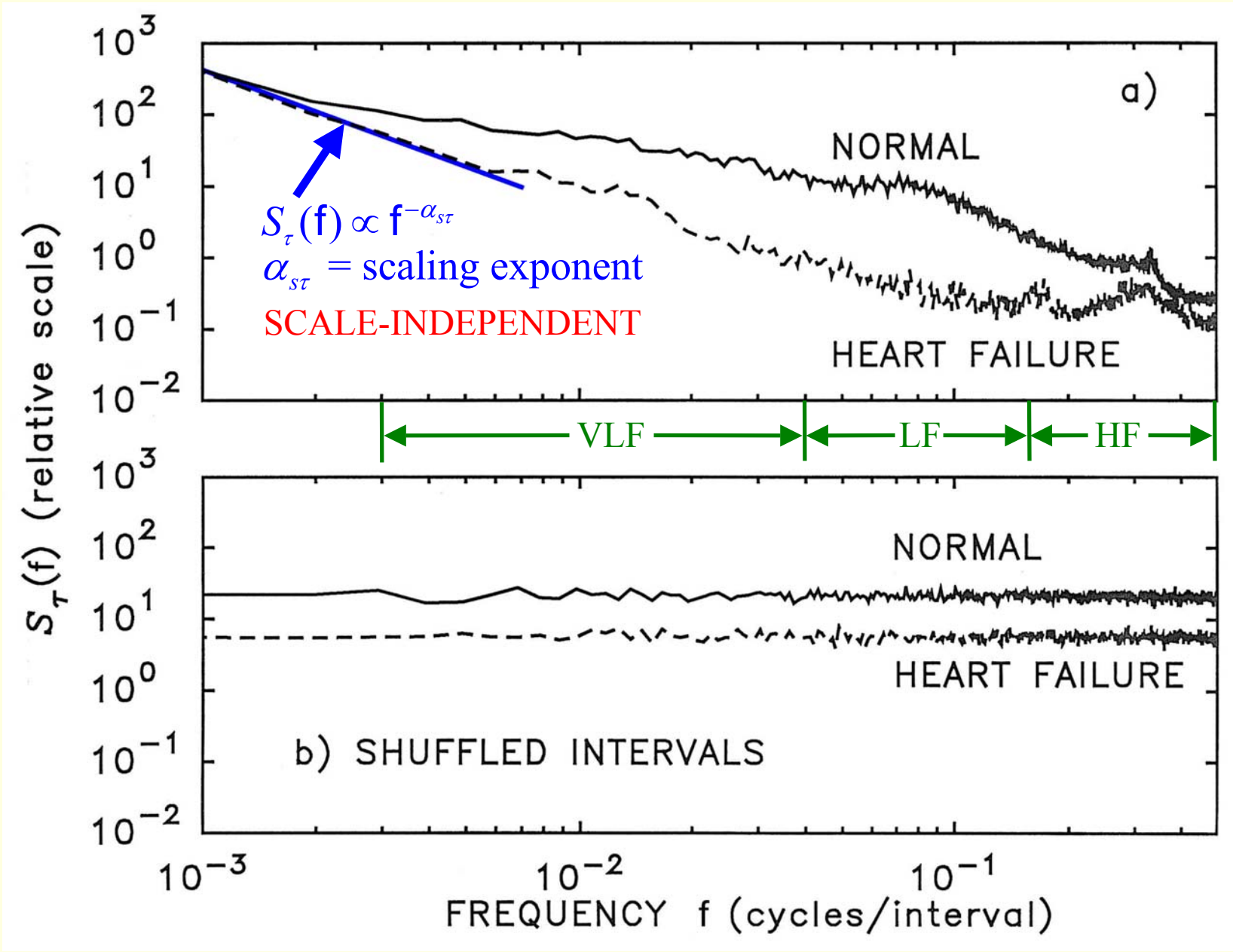

INTERBEAT INTERVAL τ_k(sec)
1.5
1.0
0.5
0
20000
40000
60000
INTERVAL NUMBER k
$\sigma_{\text{int}} \equiv$ SDNN
SCALE-DEPENDENT

INTEREVENT-INTERVAL HISTOGRAM

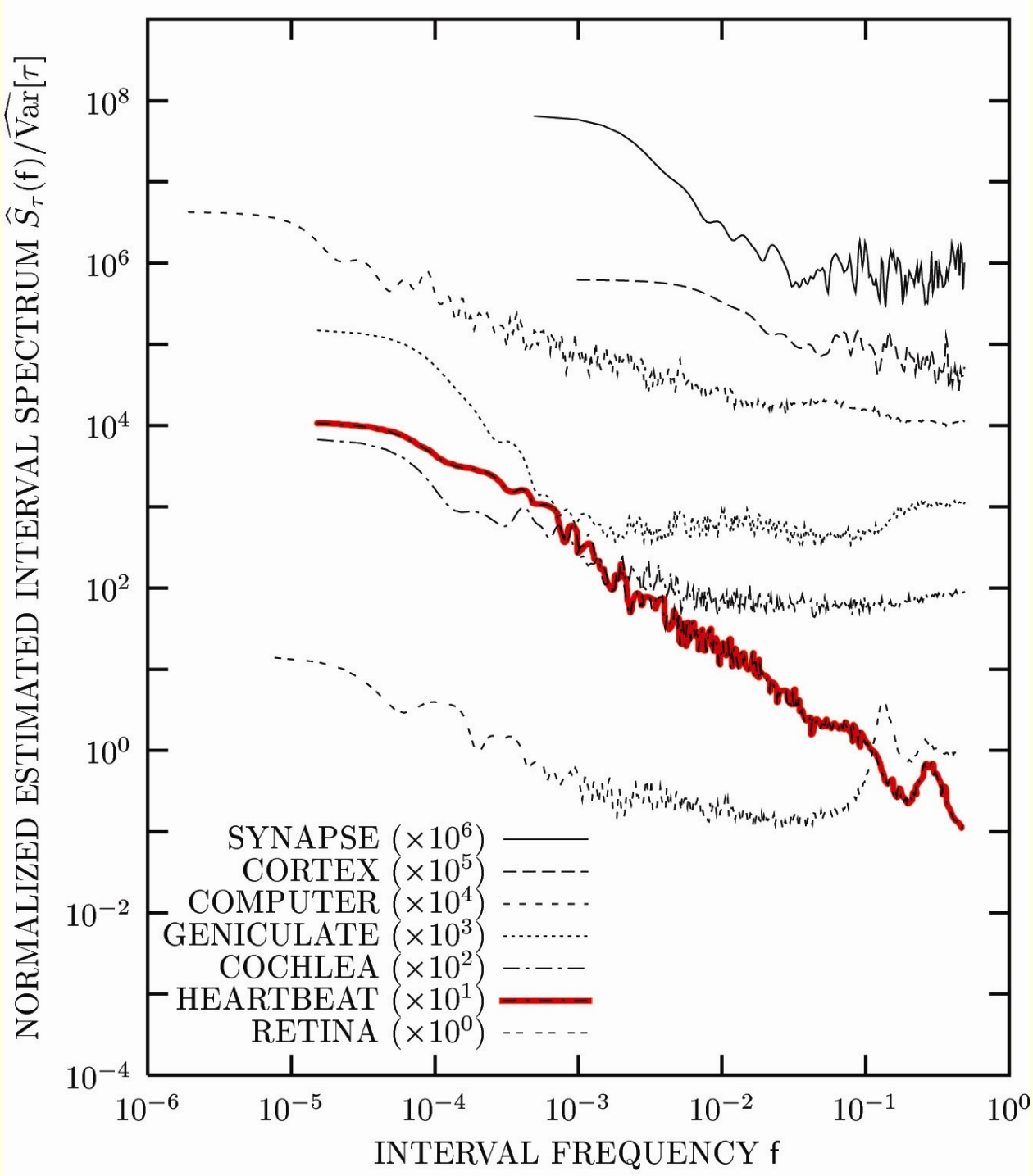



SYNAPSE (×10^6)
CORTEX (×10^5)
COMPUTER (×10^4)
GENICULATE (×10^3)
COCHLEA (×10^2)
HEARTBEAT (×10^1)
RETINA (×10^0)
NORMALIZED INTEREVENT-INTERVAL HISTOGRAM $\hat{p}_\tau\,(\tau/\hat{\mathrm{E}}[\tau])$
NORMALIZED INTEREVENT INTERVAL $\tau/\hat{\mathrm{E}}[\tau]$
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

INTERVAL-BASED SPECTRUM



After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).


NORMALIZED ESTIMATED INTERVAL SPECTRUM $\widehat{S}_\tau(f)/\widehat{\mathrm{Var}}[\tau]$
INTERVAL FREQUENCY f
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
SYNAPSE ($\times 10^{6}$)
CORTEX ($\times 10^{5}$)
COMPUTER ($\times 10^{4}$)
GENICULATE ($\times 10^{3}$)
COCHLEA ($\times 10^{2}$)
HEARTBEAT ($\times 10^{1}$)
RETINA ($\times 10^{0}$)

INTERVAL-BASED TIME-SCALE ANALYSIS

DISCRETE WAVELET TRANSFORM

EXAMINES ALL SCALES

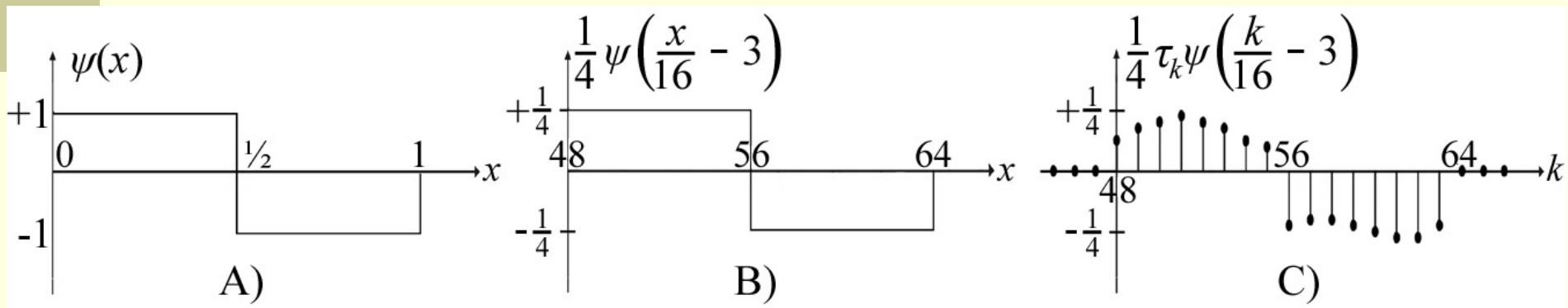
MITIGATES AGAINST NONSTATIONARITIES

m = scale index; 2^m = scale

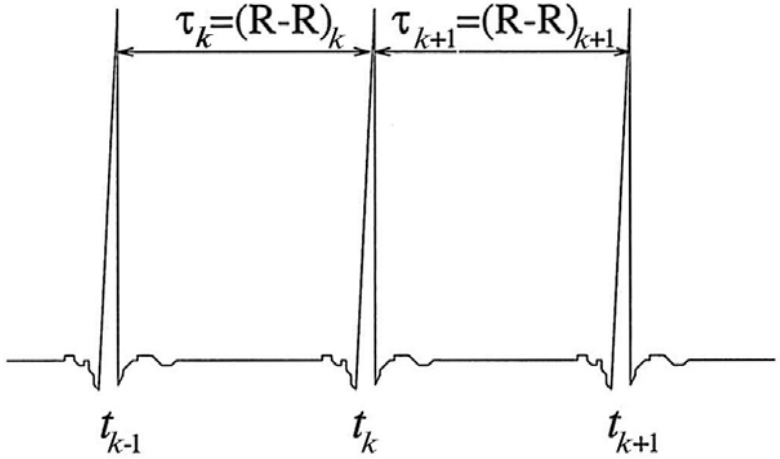
$$W_{\psi,\tau}^{\text{wav}}(m,i) = \sum_k 2^{-m/2}\,\psi(2^{-m}k - i)\,\tau_k$$

$$\sigma_{\text{wav}}^2 \equiv \text{Var}\left[W_{\psi,\tau}^{\text{wav}}(m,i)\right] = 2^{-m}\sum_k\sum_l \psi(2^{-m}k - i)\,\psi(2^{-m}l - i)\,R_\tau(l-k)$$

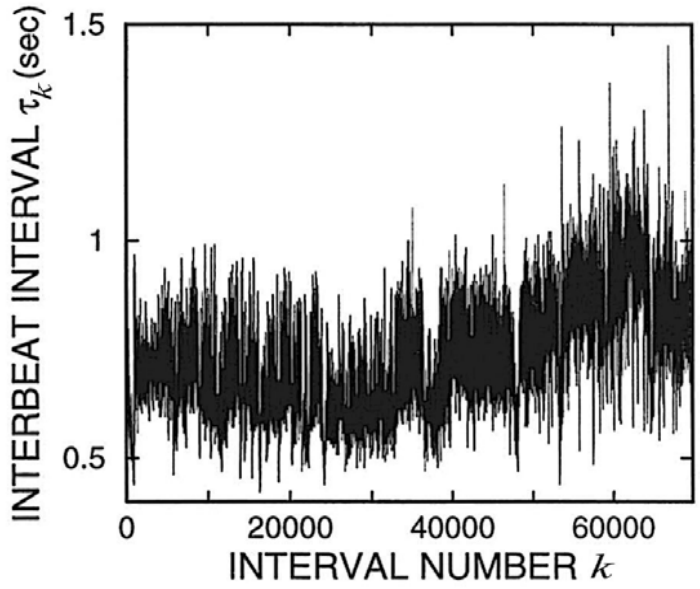
$$A_\tau(k) \equiv \text{Var}\left[W_{\psi,\tau}^{\text{wav}}(m,i)\right] \Big/ \text{Var}\,[\tau]$$



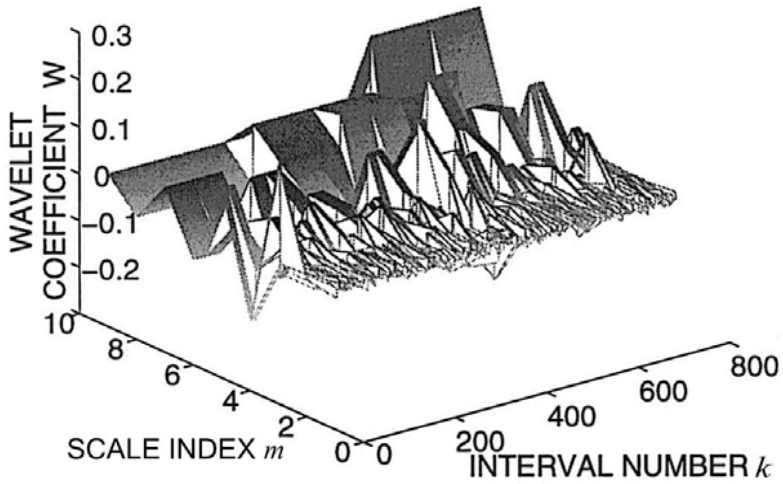
NORMAL


τ_k=(R-R)$_k$
τ_{k+1}=(R-R)$_{k+1}$
t_{k-1}
t_k
t_{k+1}

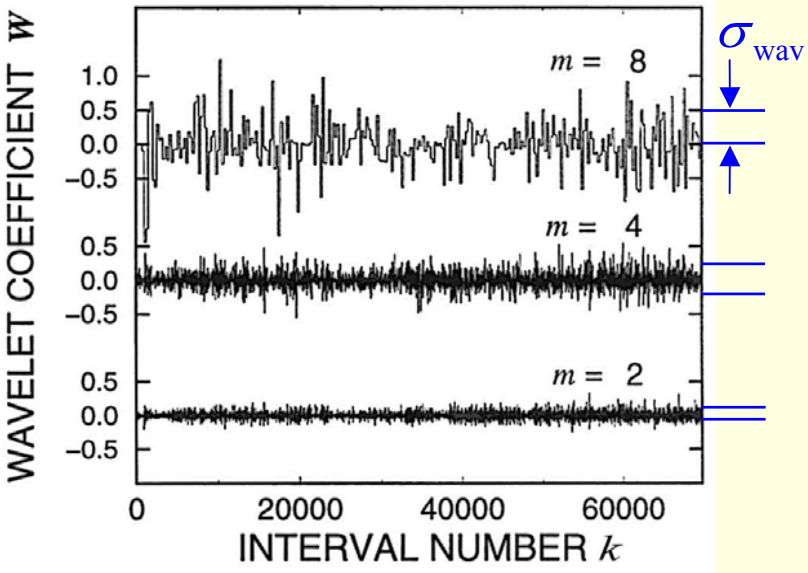
a)


INTERBEAT INTERVAL τ_k (sec)
1.5
1
0.5
0
20000
40000
60000
INTERVAL NUMBER k

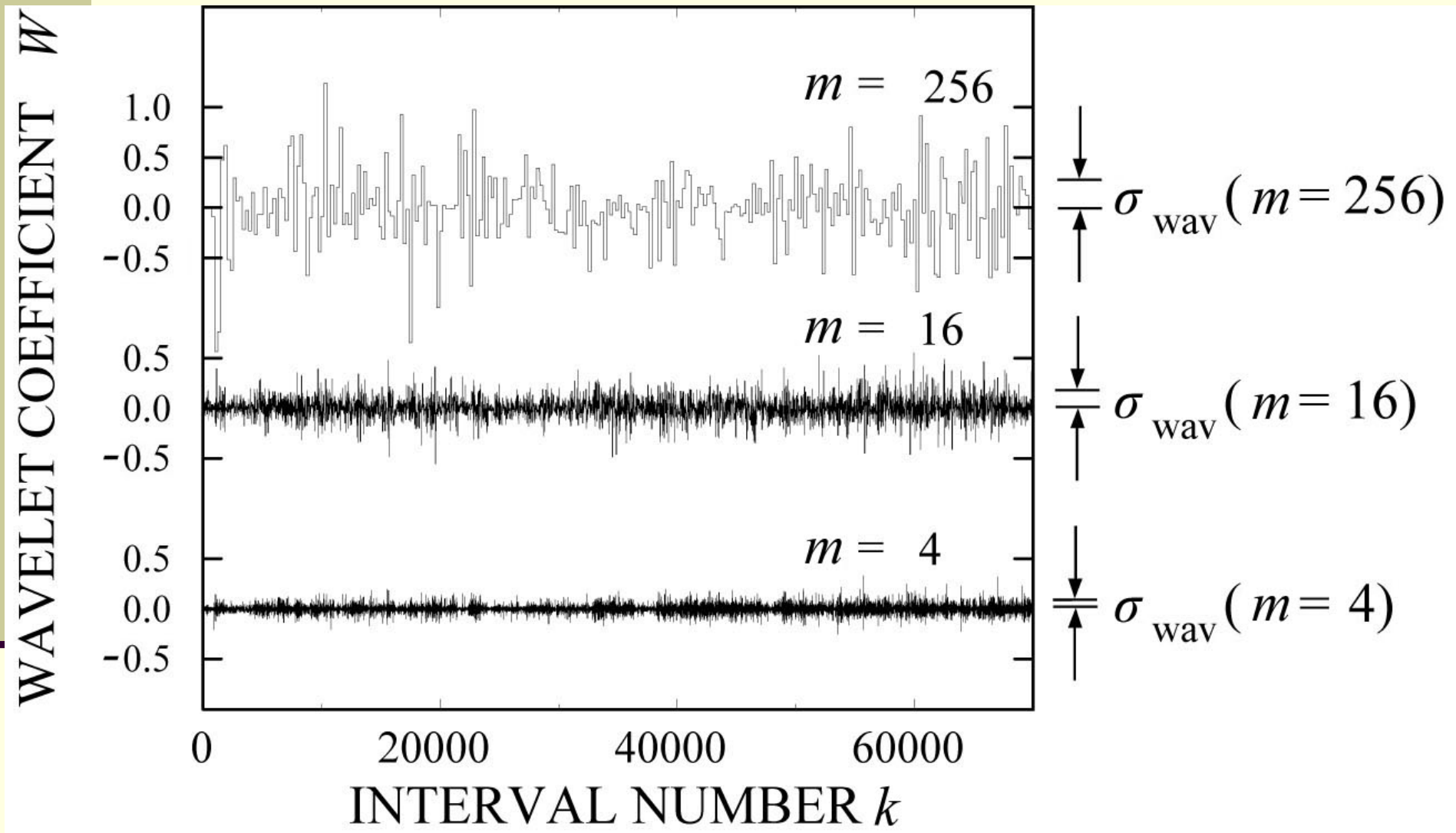
b)


WAVELET COEFFICIENT W
0.3
0.2
0.1
0
−0.1
−0.2
10
8
6
4
2
0
SCALE INDEX m
0
200
400
600
800
INTERVAL NUMBER k

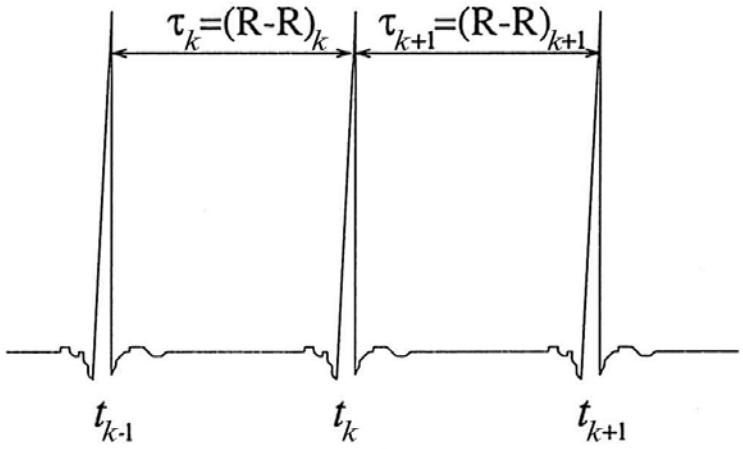
c)


WAVELET COEFFICIENT W
m = 8
m = 4
m = 2
1.0
0.5
0.0
−0.5
0
20000
40000
60000
INTERVAL NUMBER k
σ_{wav}

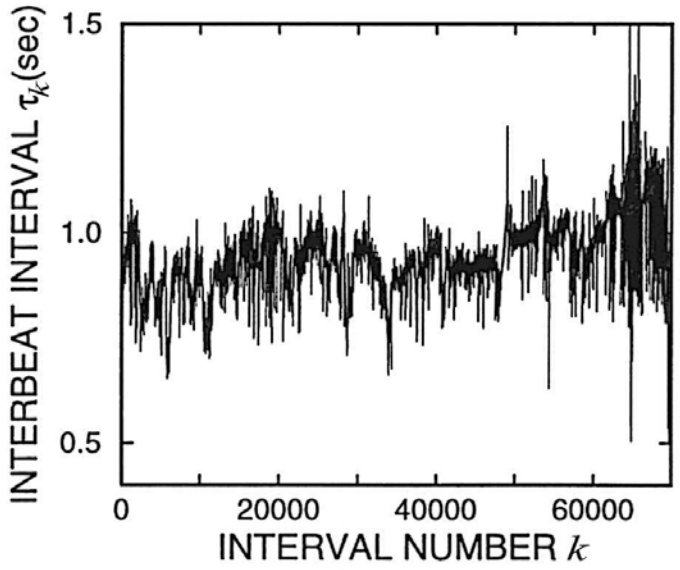
d)


WAVELET COEFFICIENT W
m = 256
m = 16
m = 4
1.0
0.5
0.0
-0.5
0.5
0.0
-0.5
0.5
0.0
-0.5
σ wav (m = 256)
σ wav (m = 16)
σ wav (m = 4)
0
20000
40000
60000
INTERVAL NUMBER k

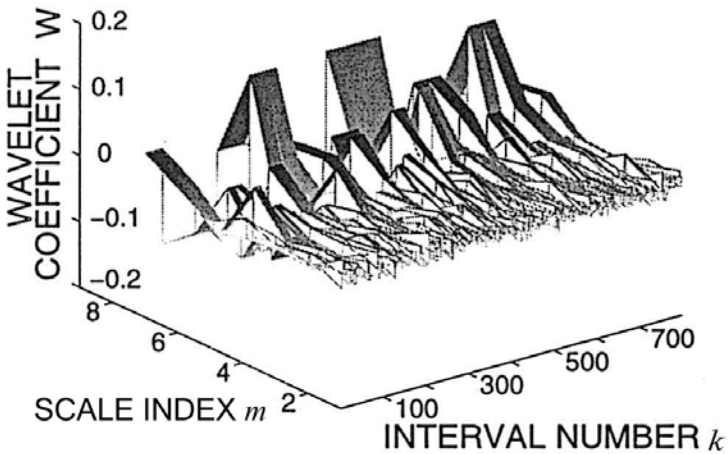
HEART-FAILURE



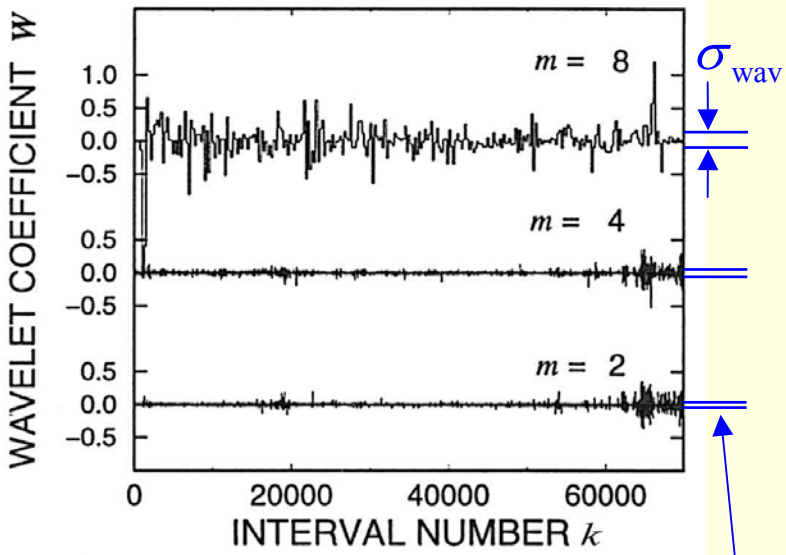
a)



b)



c)



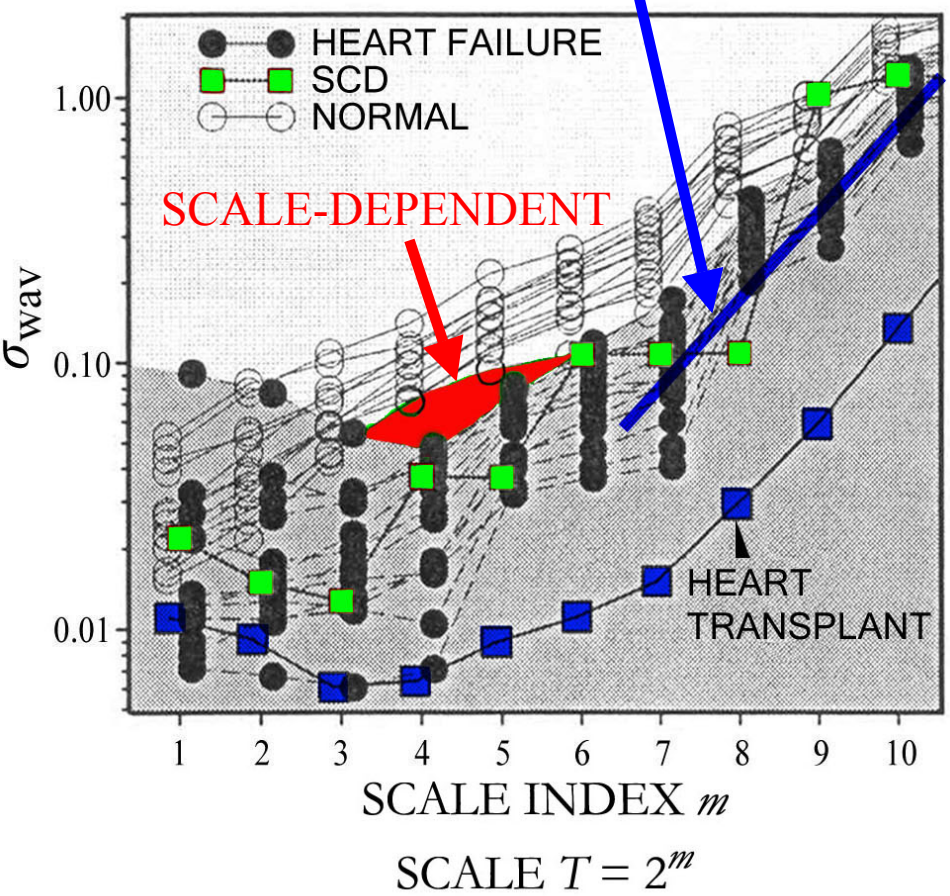
d)

SMALLER VALUES OF σ_{wav} THAN FOR NORMAL SUBJECTS

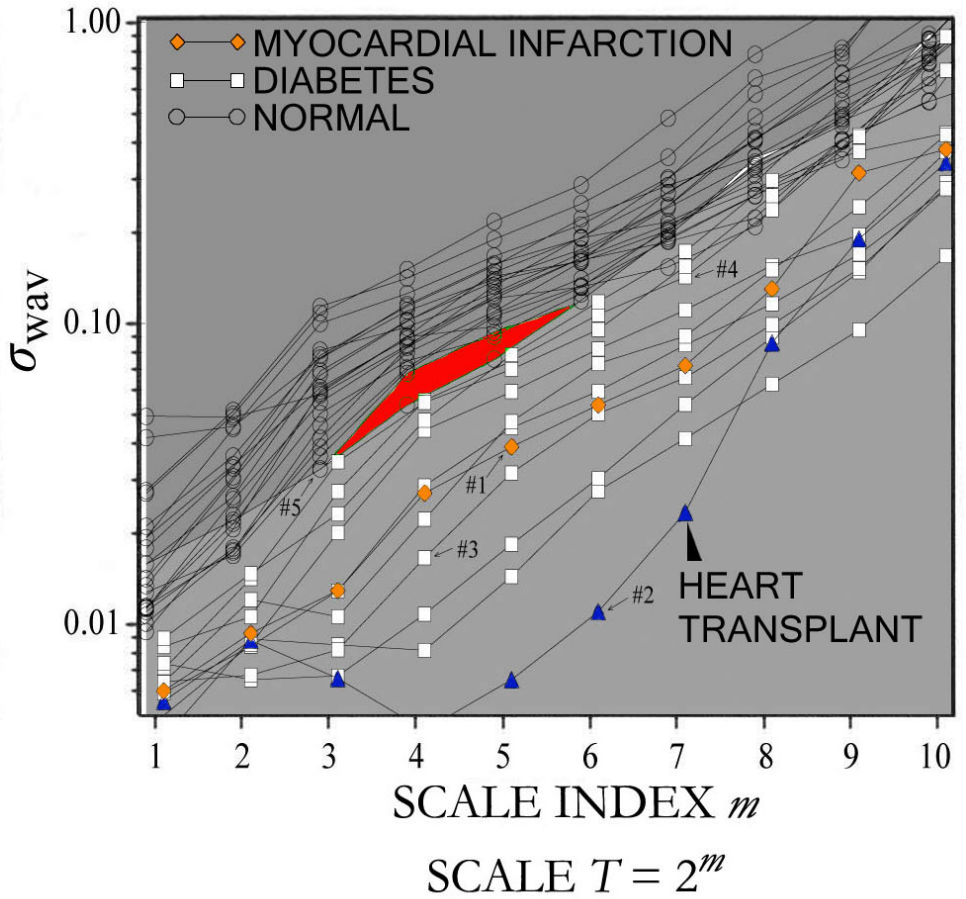
$$\sigma^2_{\text{wav}}(T) \propto T^{\alpha_{A\tau}}$$

$\alpha_{A\tau}$ = scaling exponent

SCALE-INDEPENDENT

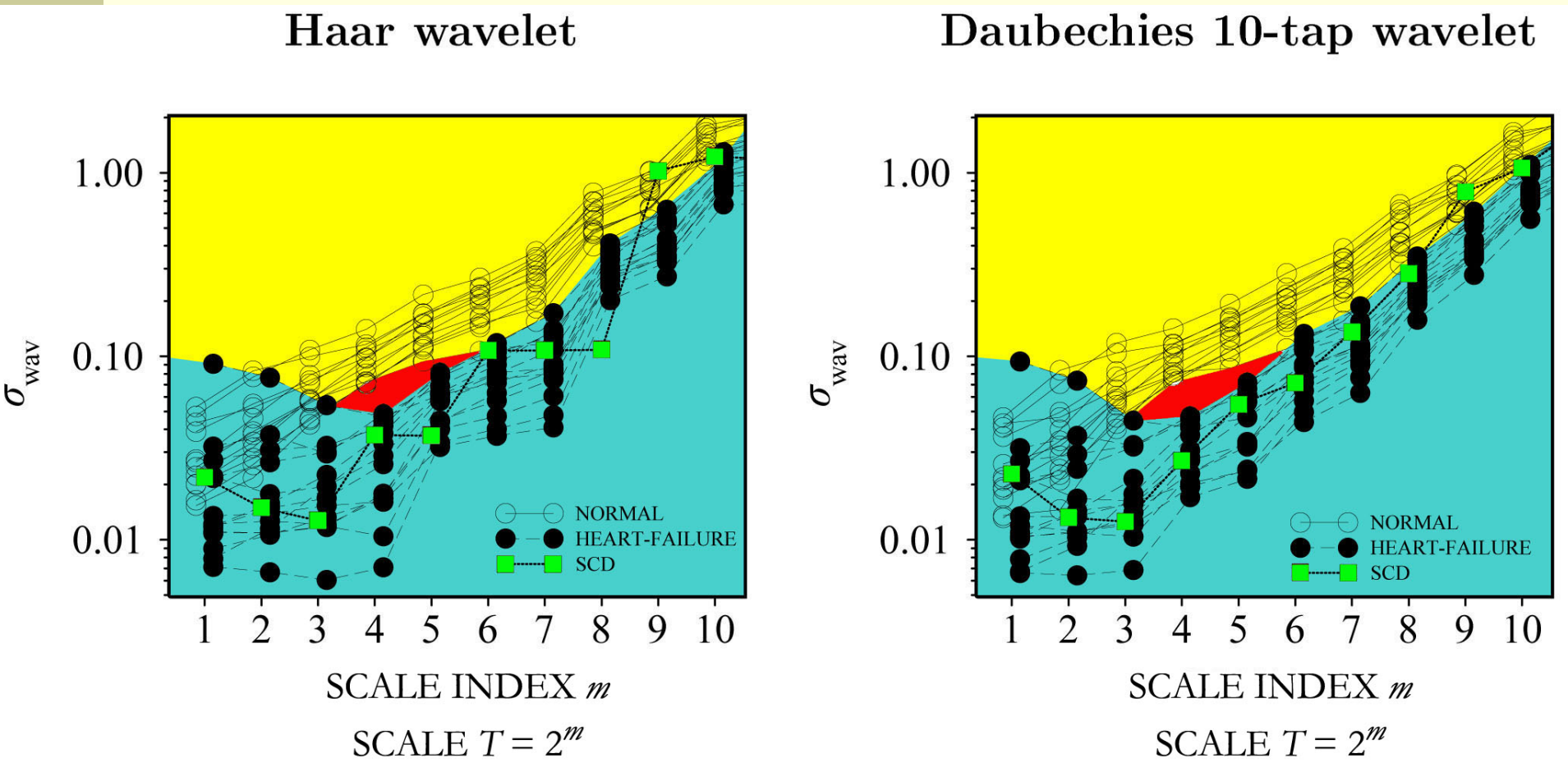


After Teich, *Proc. Int. Conf. IEEE Eng. Med. Biol. Soc.* **20**, 1136-1141 (1998).

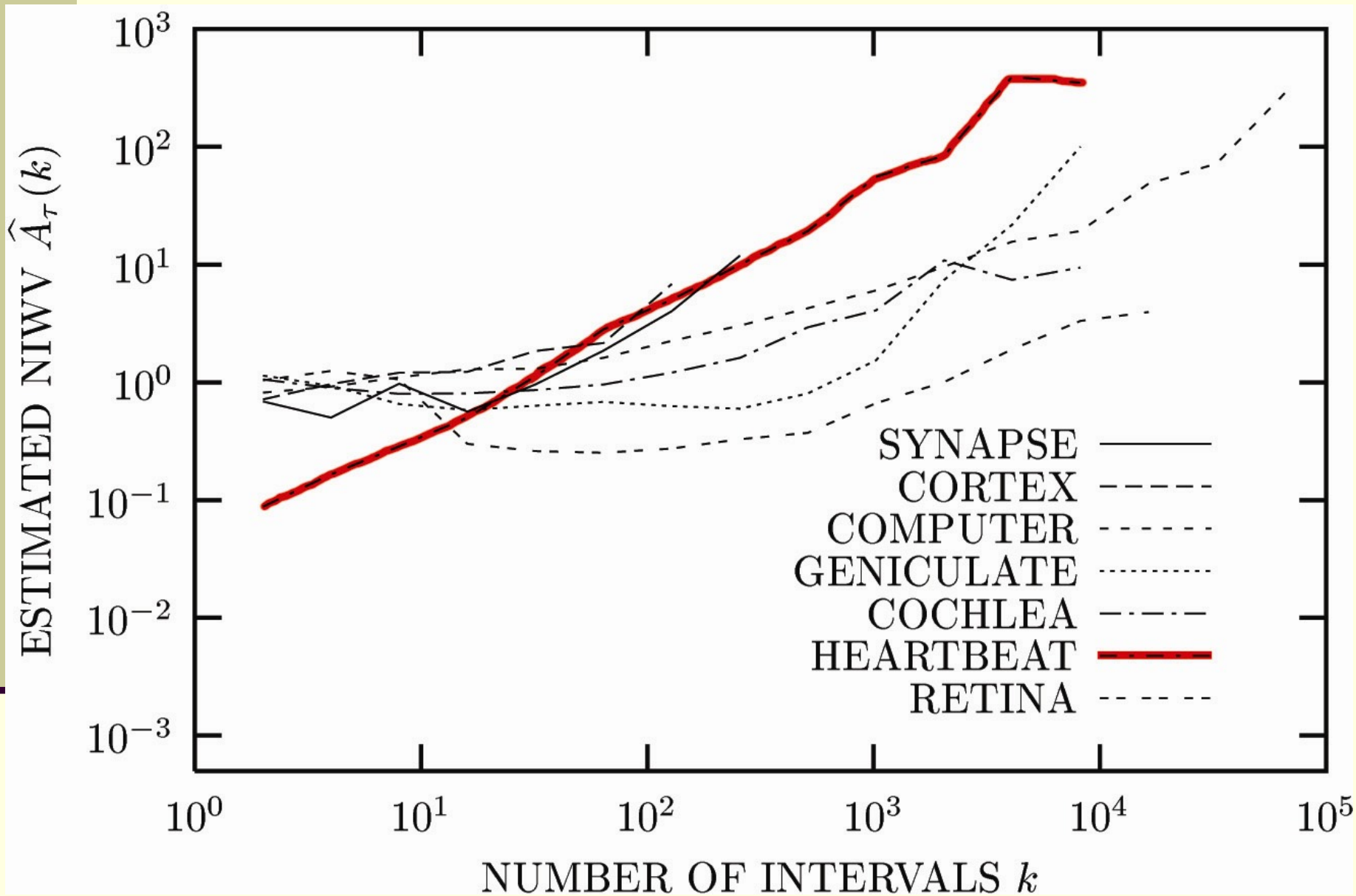


After Ashkenazy *et al.*, *Fractals* **6**, 197-203 (1998).

ROBUSTNESS WITH FORM OF WAVELET

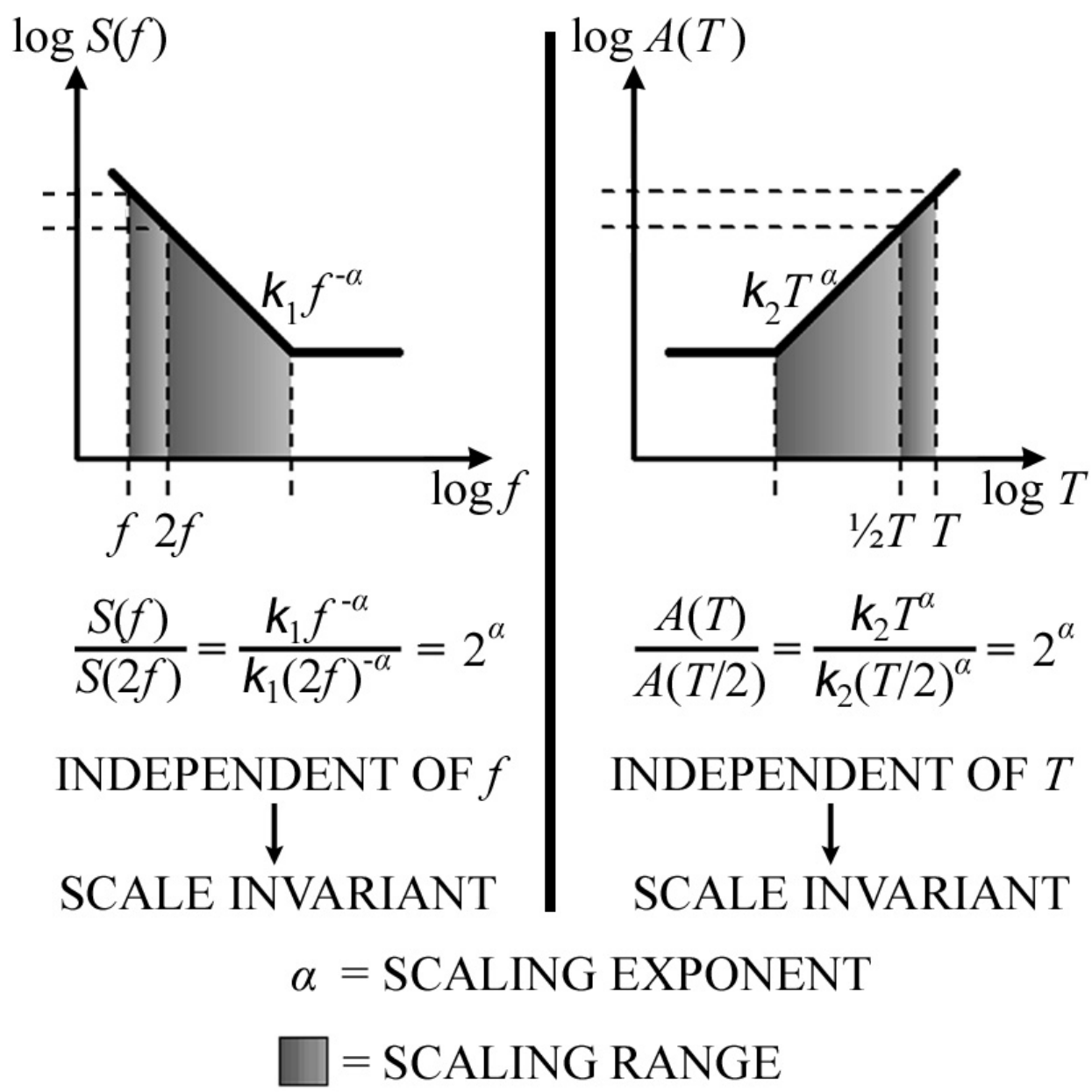


After Thurner, Feurstein & Teich, *Phys. Rev. Letters* **80**, 1544-1547 (1998).

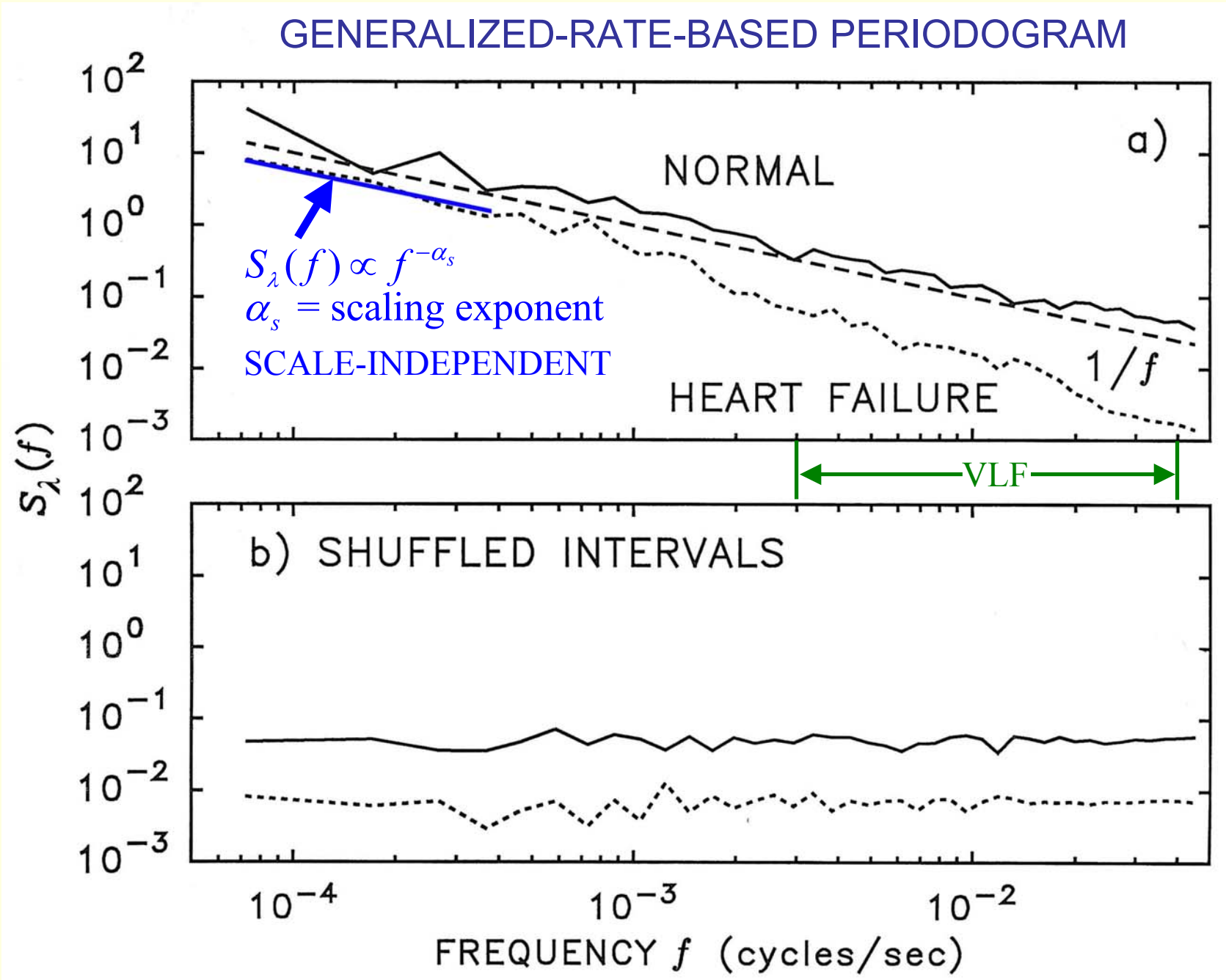

ESTIMATED NIWV $\widehat{A}_\tau(k)$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^0
10^1
10^2
10^3
10^4
10^5
NUMBER OF INTERVALS k
SYNAPSE
CORTEX
COMPUTER
GENICULATE
COCHLEA
HEARTBEAT
RETINA

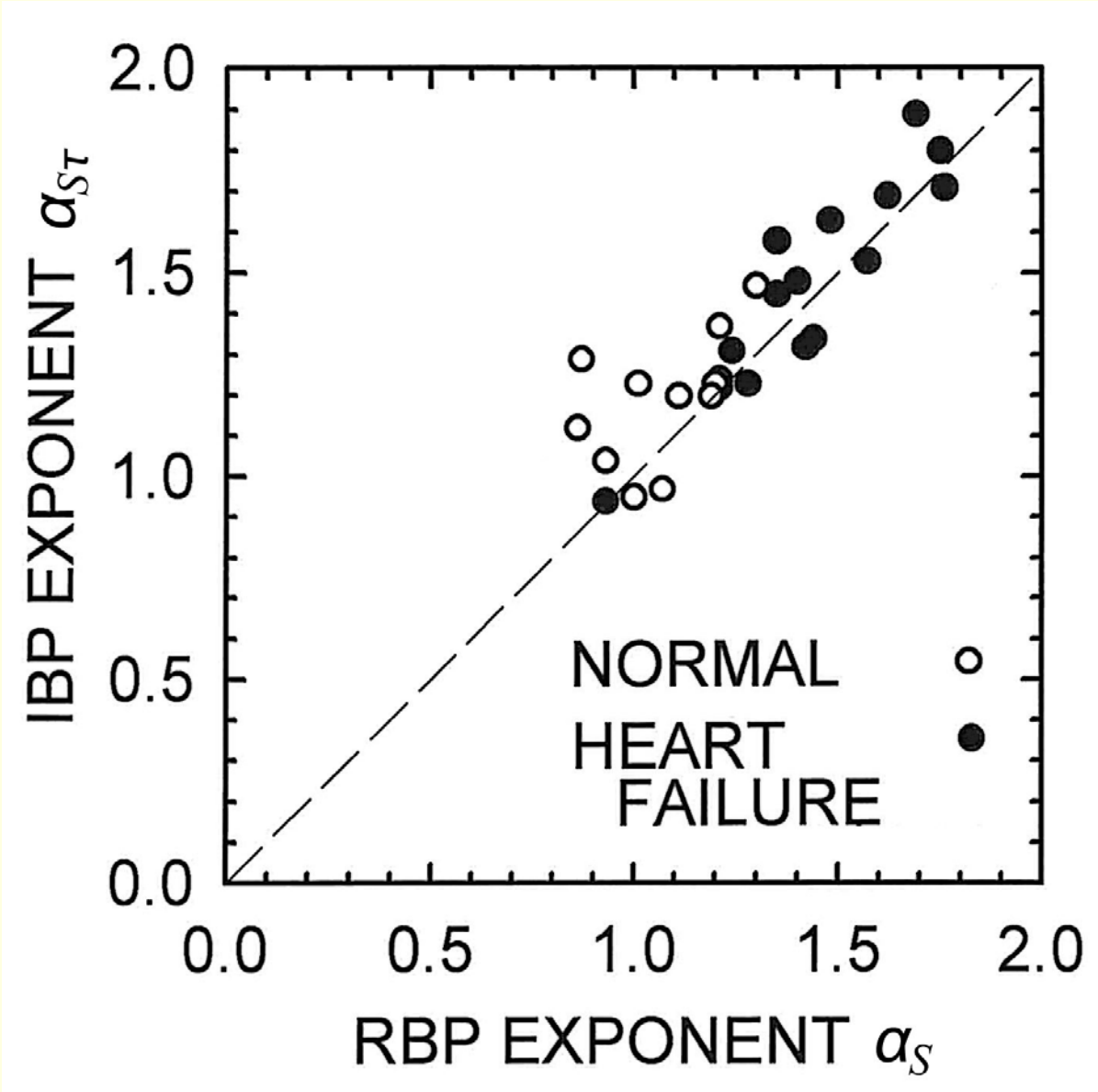
COUNT-BASED MEASURES

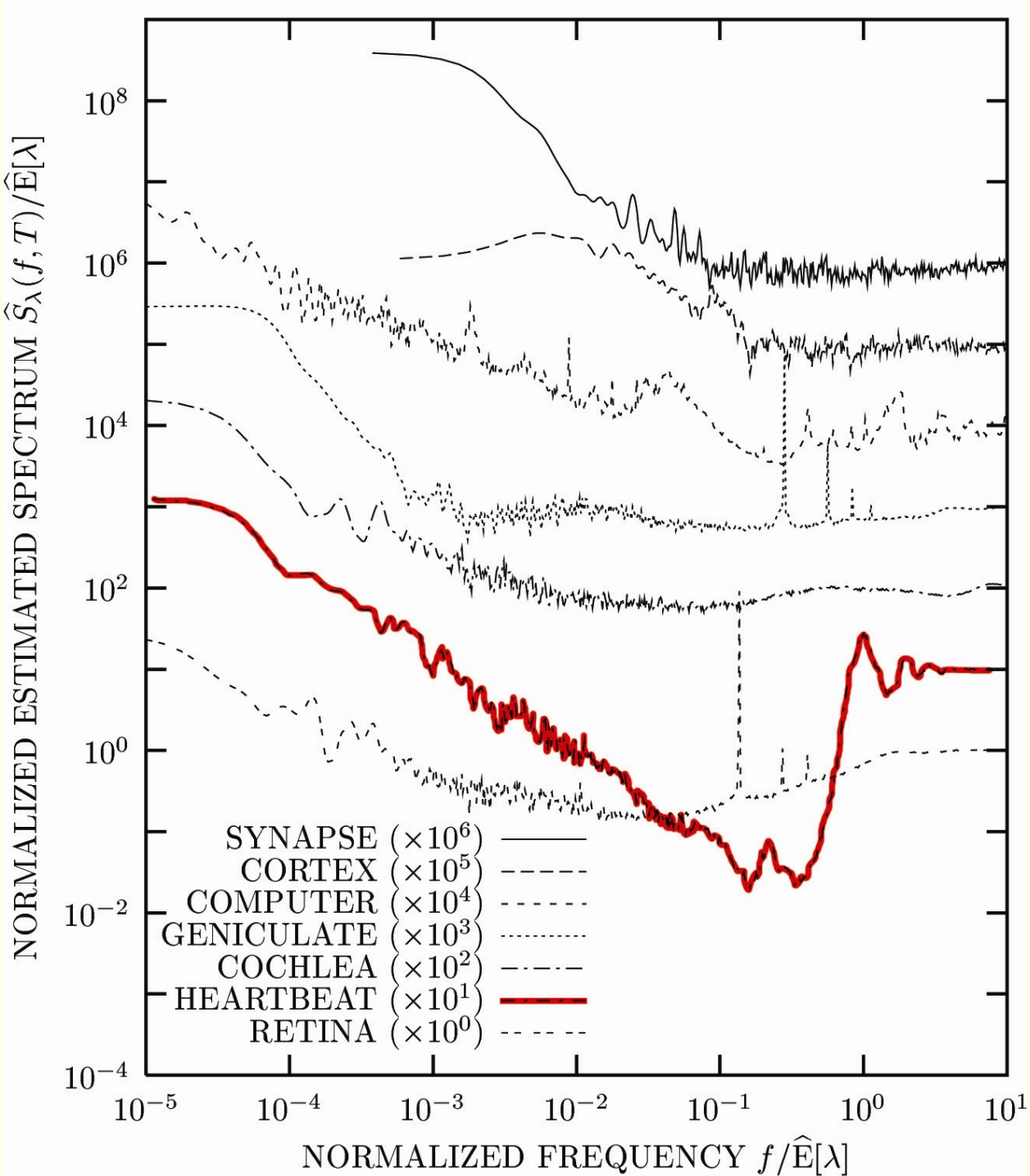
POWER-LAW SCALE INVARIANCE



RATE-BASED SPECTRUM

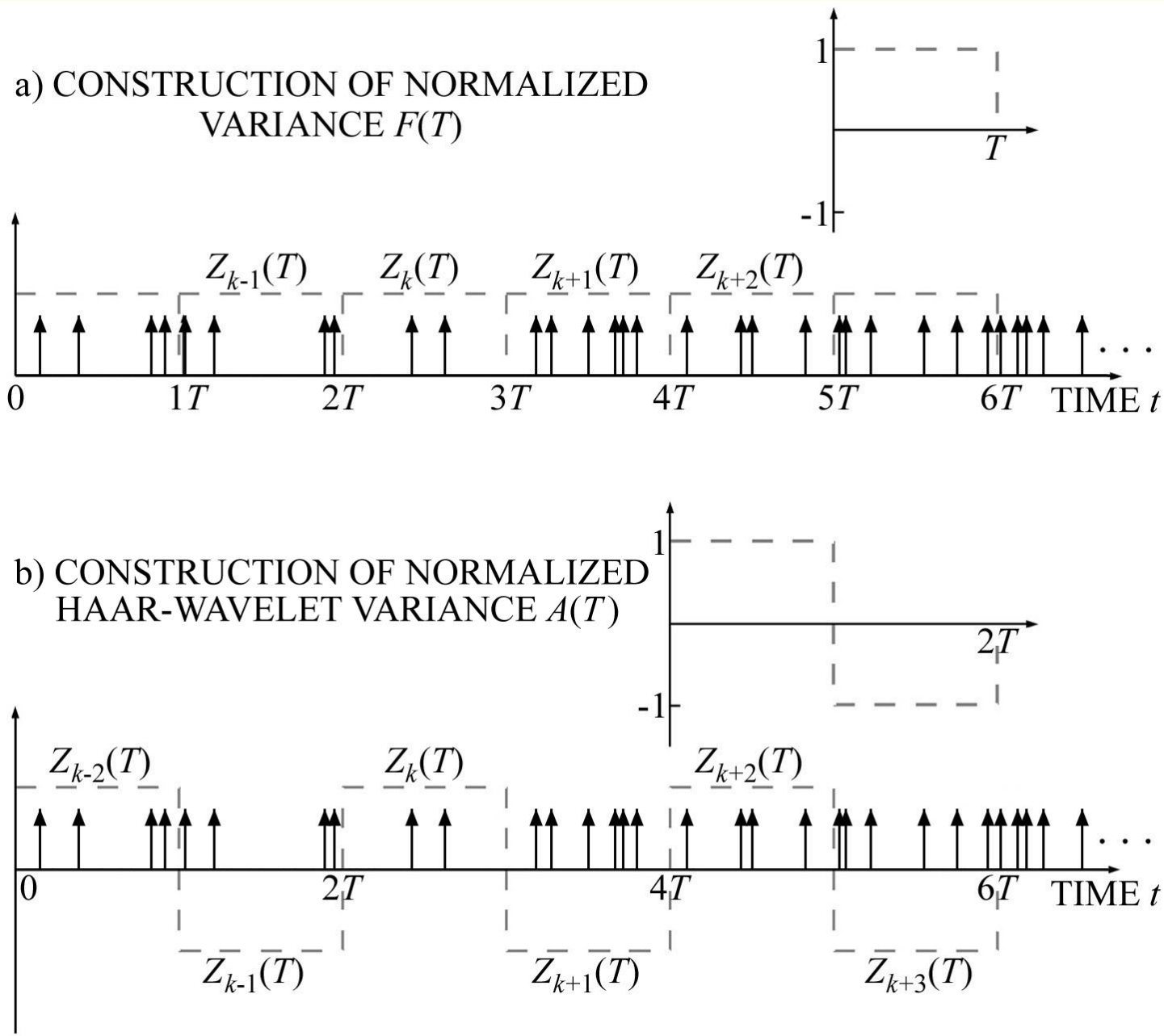



2.0
1.5
1.0
0.5
0.0
IBP EXPONENT $\alpha_{S\tau}$
0.0
0.5
1.0
1.5
2.0
RBP EXPONENT α_S
NORMAL
HEART
FAILURE

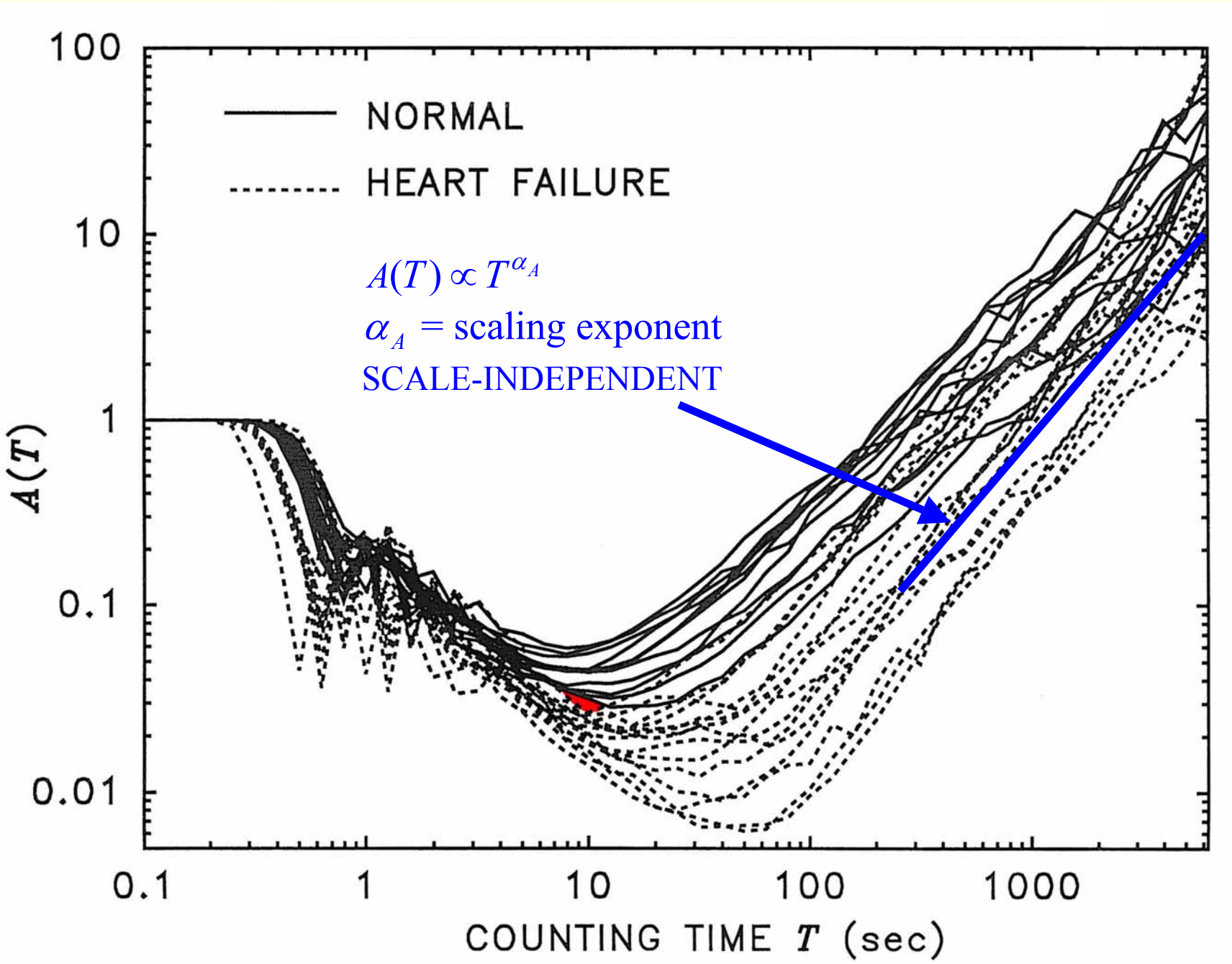

NORMALIZED ESTIMATED SPECTRUM $\widehat{S}_\lambda(f,T)/\widehat{\mathrm{E}}[\lambda]$
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
NORMALIZED FREQUENCY $f/\widehat{\mathrm{E}}[\lambda]$
SYNAPSE $(\times 10^{6})$
CORTEX $(\times 10^{5})$
COMPUTER $(\times 10^{4})$
GENICULATE $(\times 10^{3})$
COCHLEA $(\times 10^{2})$
HEARTBEAT $(\times 10^{1})$
RETINA $(\times 10^{0})$

COUNT-BASED TIME-SCALE ANAYLSIS

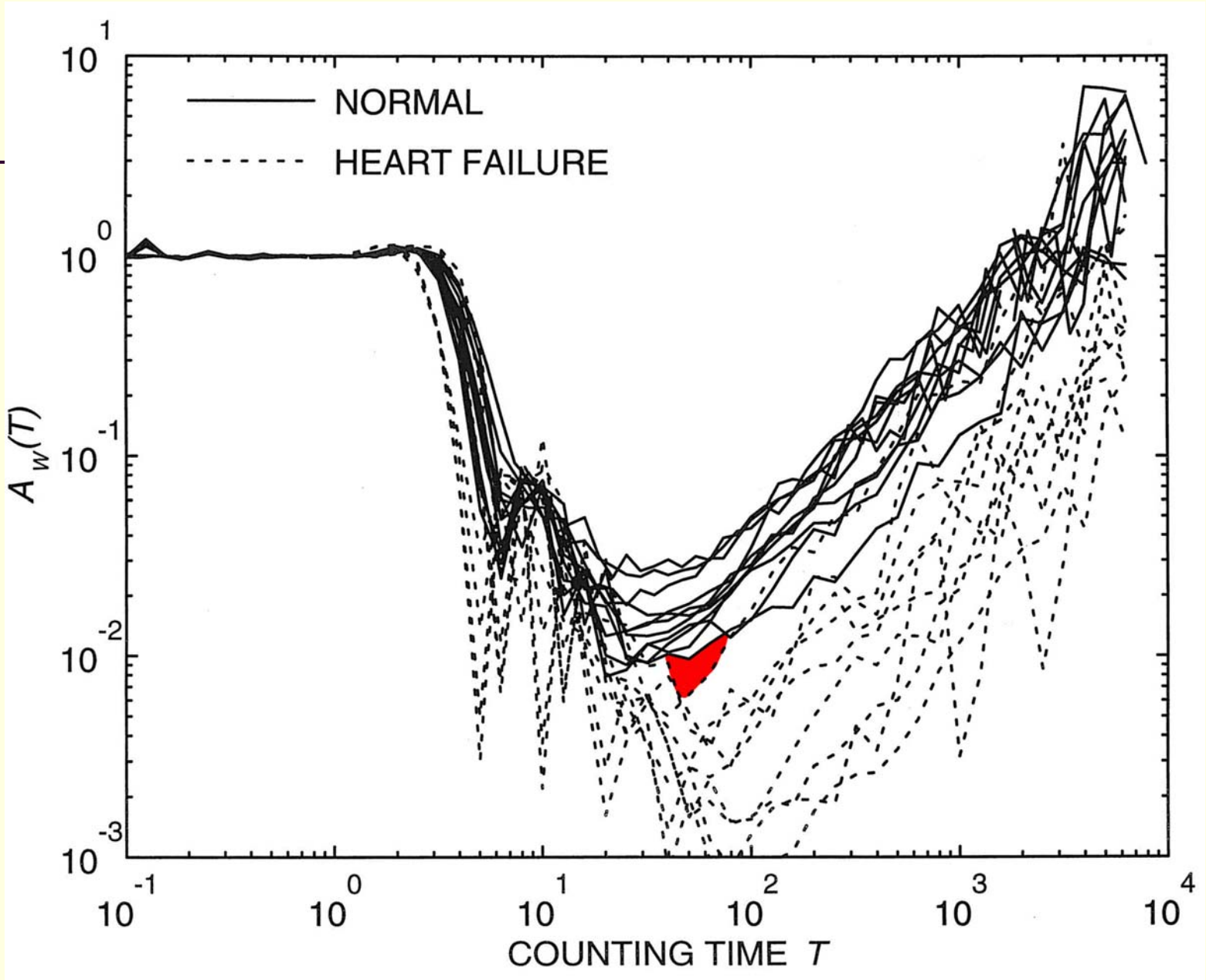
DISCRETE WAVELET TRANSFORM



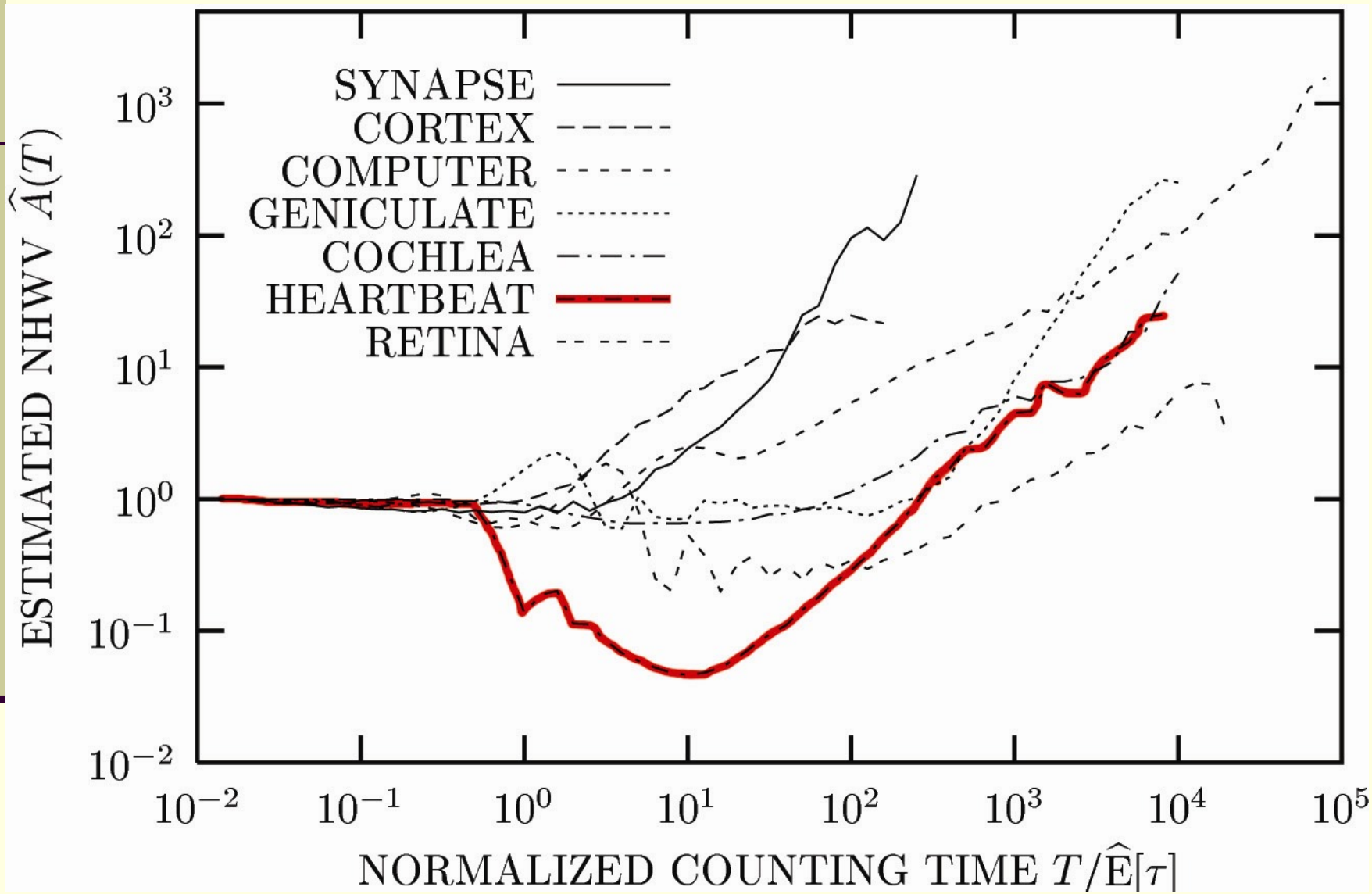
NORMALIZED HAAR-WAVELET VARIANCE



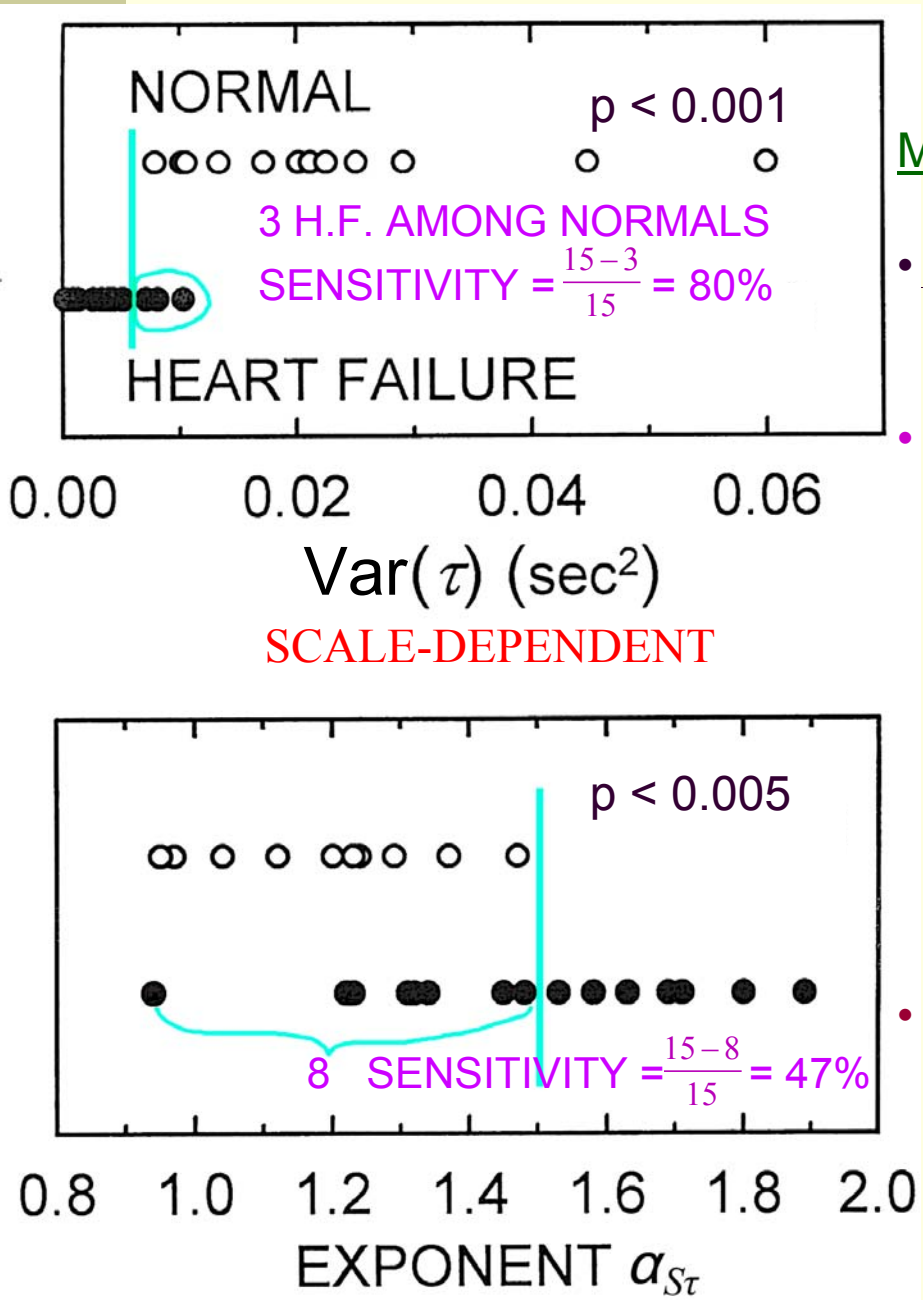
NORMALIZED DAUBECHIES-WAVELET VARIANCE



After Teich, *Proc. 18th Intern. Conf. IEEE Eng. Med. Biol. Soc.* **18**, 1128-1129 (1996).


SYNAPSE
CORTEX
COMPUTER
GENICULATE
COCHLEA
HEARTBEAT
RETINA
ESTIMATED NHWV $\widehat{A}(T)$
NORMALIZED COUNTING TIME $T/\widehat{\mathrm{E}}[\tau]$
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}

IDENTIFYING PATIENTS WITH CARDIAC DYSFUNCTION



MEASURES OF STATISTICAL SIGNIFICANCE

- p VALUE, d′, AND VARIANTS
  (rely on Gaussian assumption)

- SENSITIVITY/SPECIFICITY
  MEASURES OF CLINICAL SIGNIFICANCE
  (distribution free)

  SENSITIVITY ≡ proportion of heart-failure patients that are properly identified

  e.g., Hypothesis that all normal patients are so identified ≡ 100% SPECIFICITY

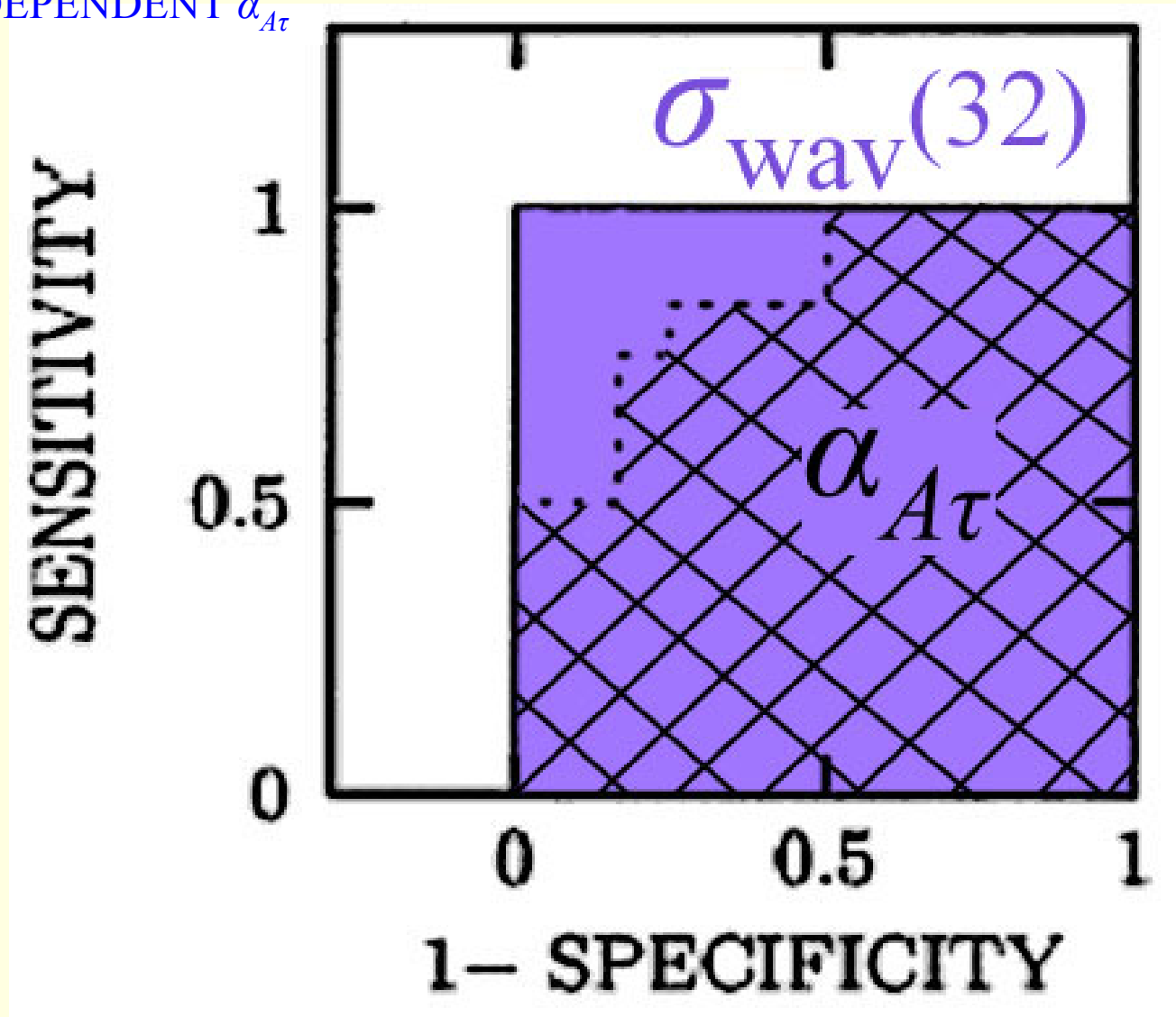
- ROC CURVES & AREA UNDER ROC

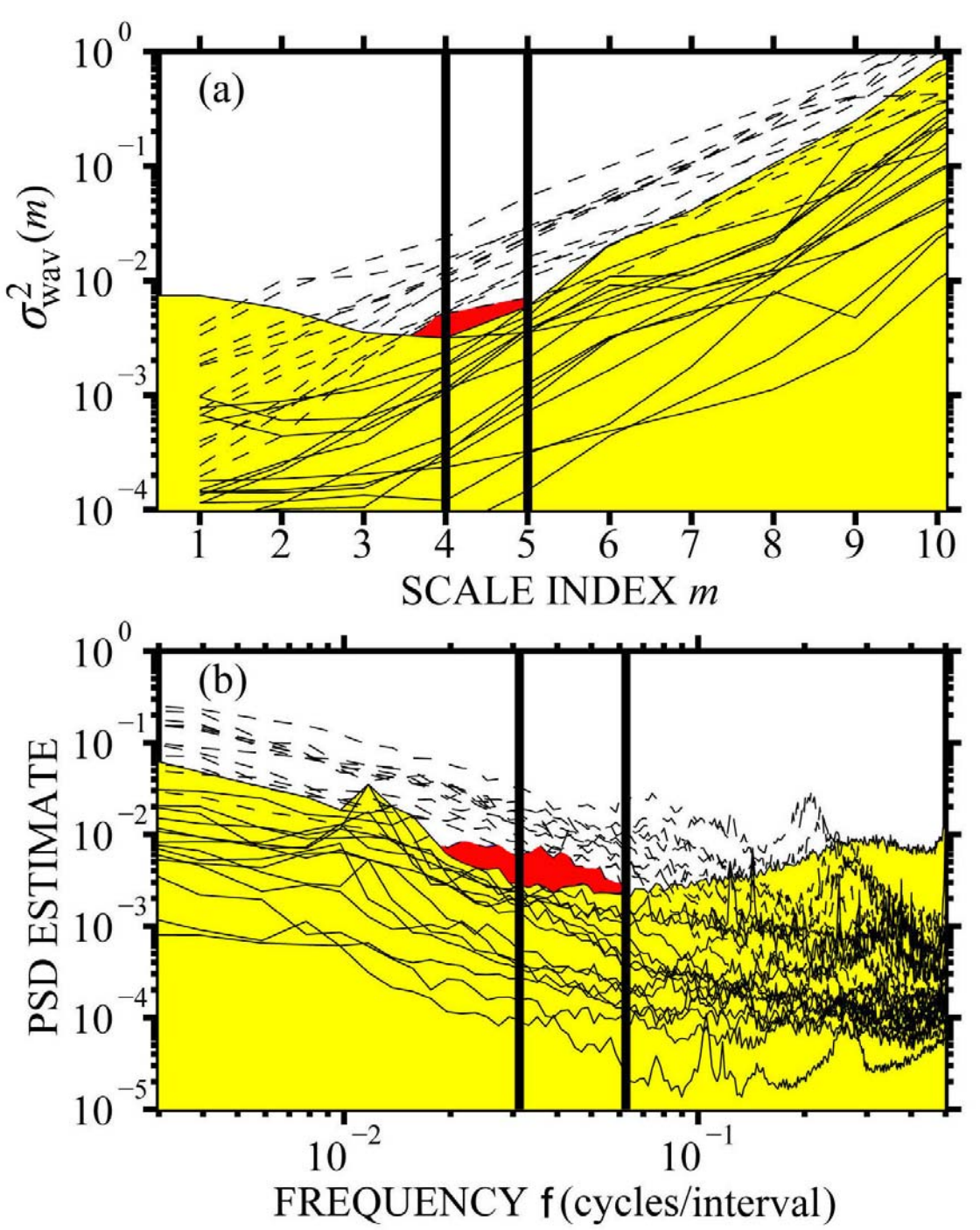
ROC CURVES & AREA UNDER ROC

SCALE-DEPENDENT σ_{wav} (32)

SCALE-INDEPENDENT $\alpha_{A\tau}$

HAAR WAVELET



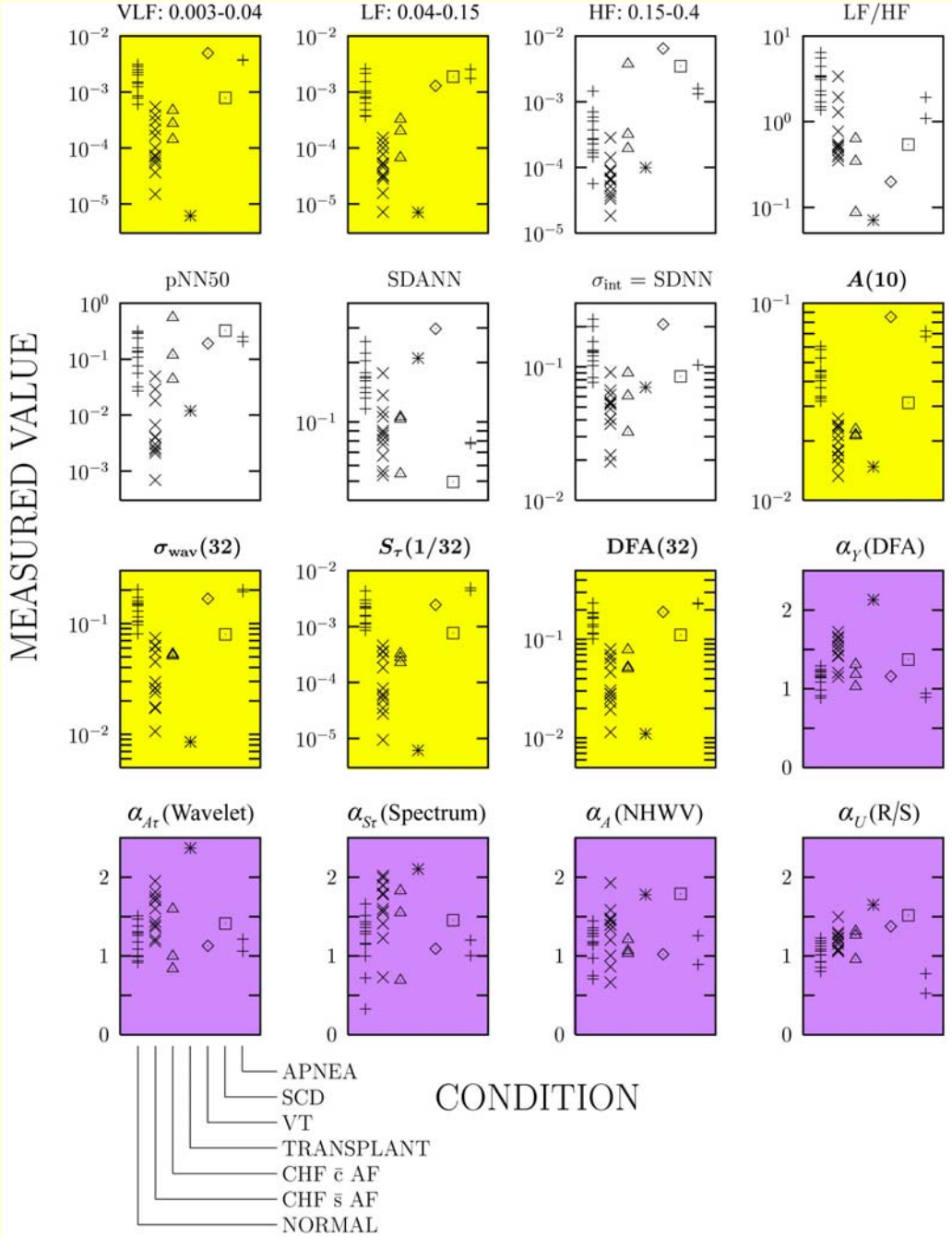


$$16 \le 2^m \ \text{(scale)} \le 32$$

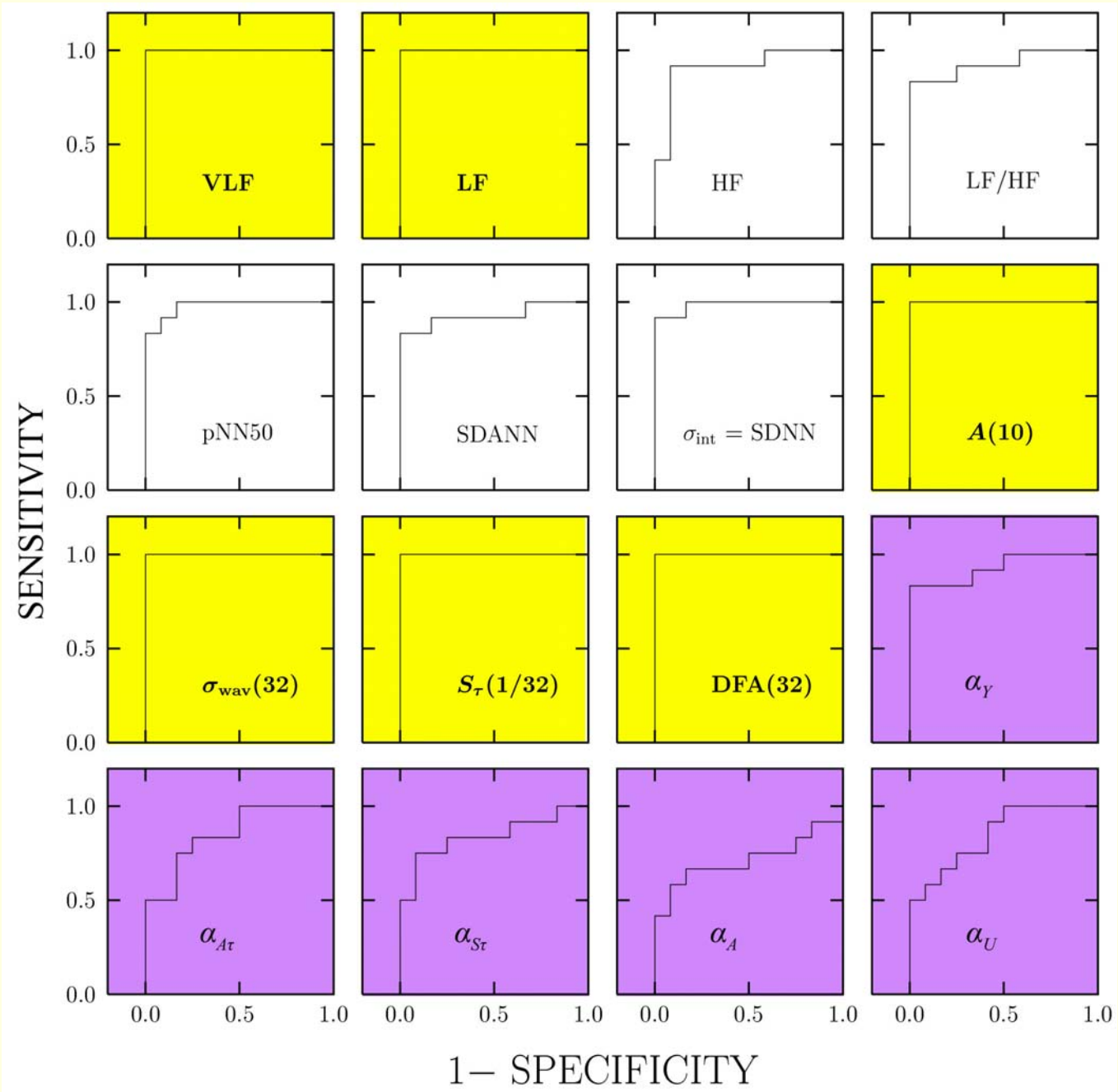
$$\frac{1}{32} < \text{f (cycles/interval)} < \frac{1}{16}$$

After Heneghan, Lowen, & Teich
Proc. 1999 ICASSP (Phoenix, AZ)
paper SPTM-8.2.

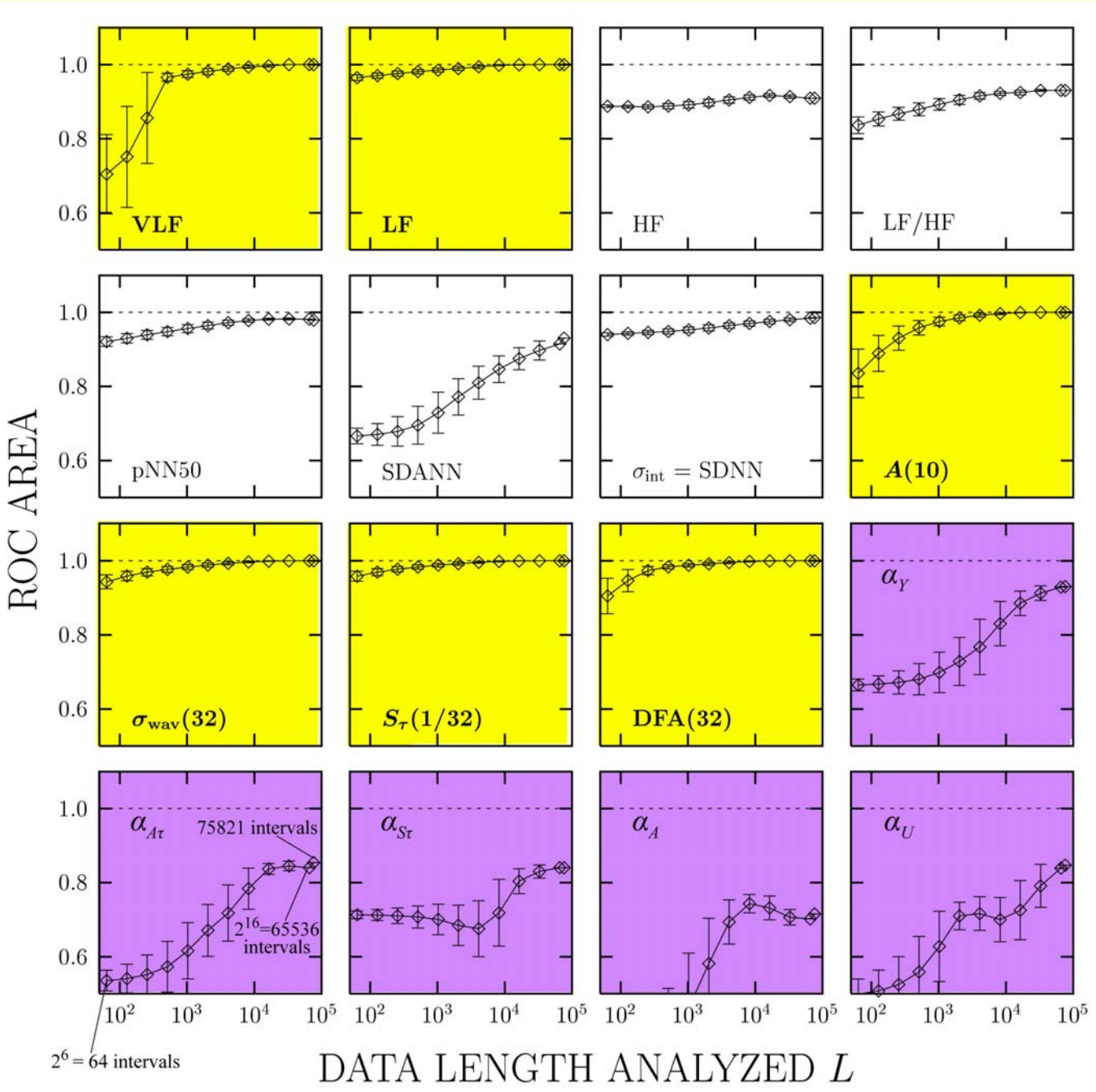
INDIVIDUAL VALUES: DATA

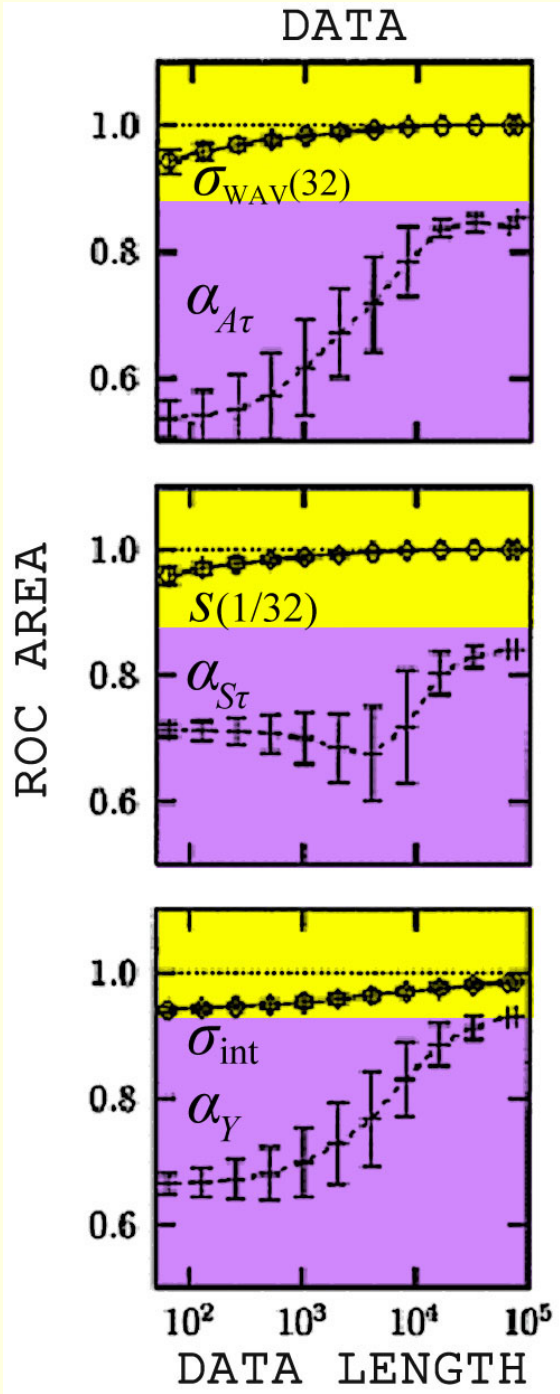


ROC CURVES: NORMAL & CHF DATA ($\bar{s}$ AF; 75821 intervals)

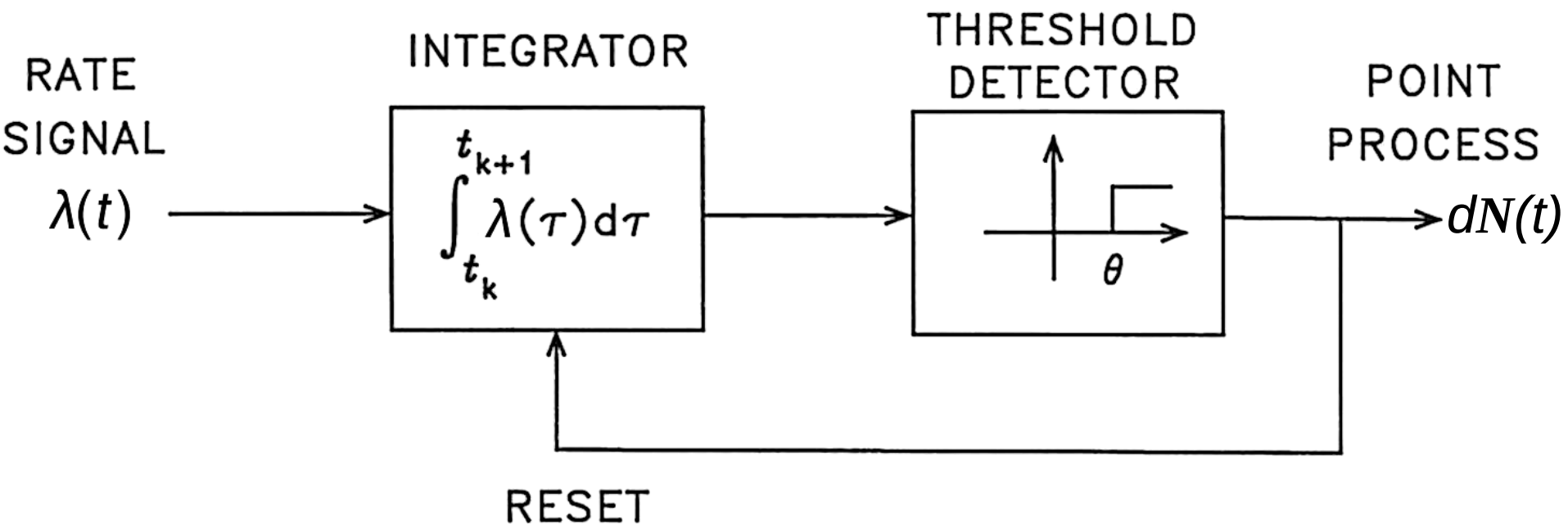


ROC-AREA CURVES: NORMAL & CHF DATA

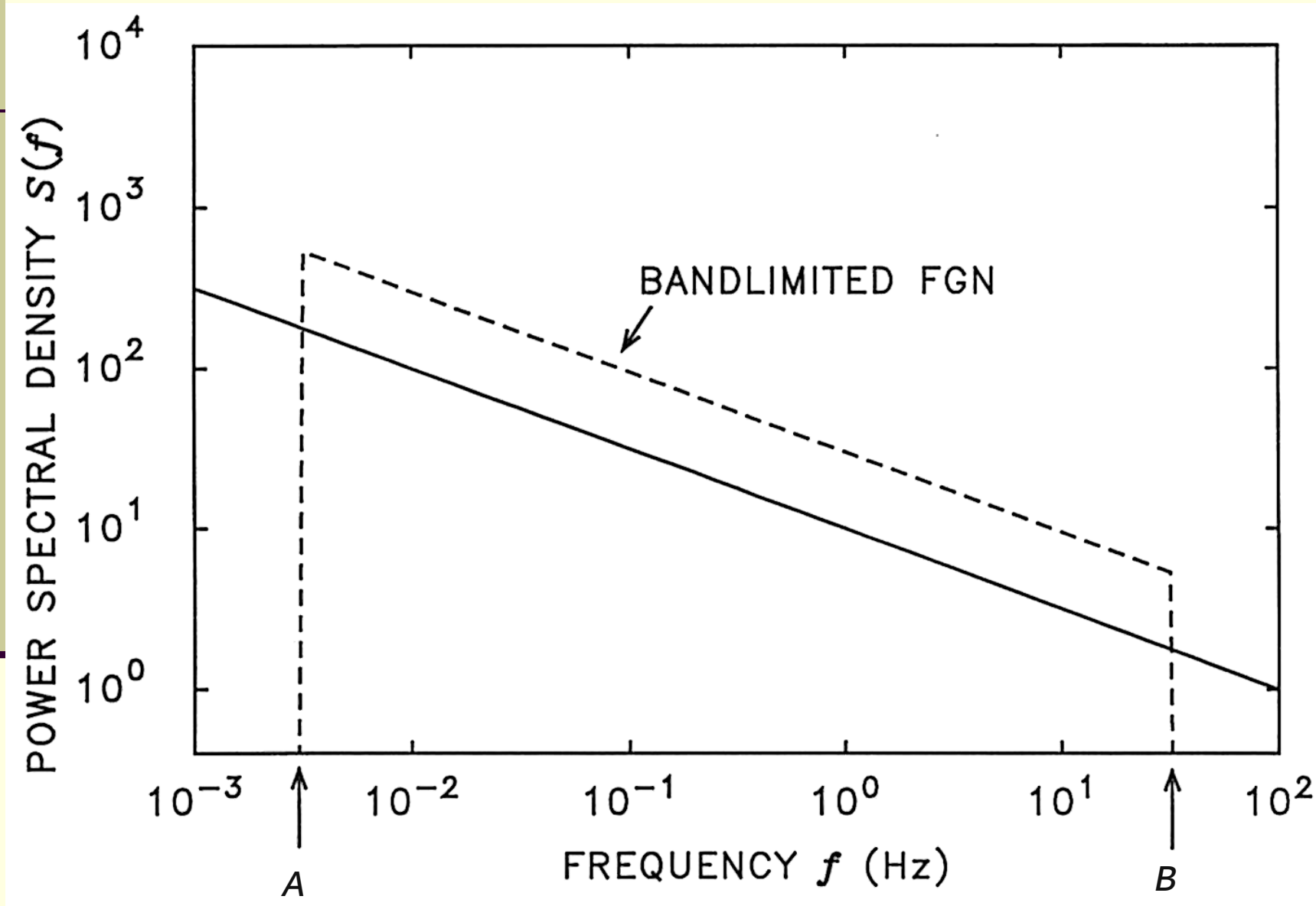


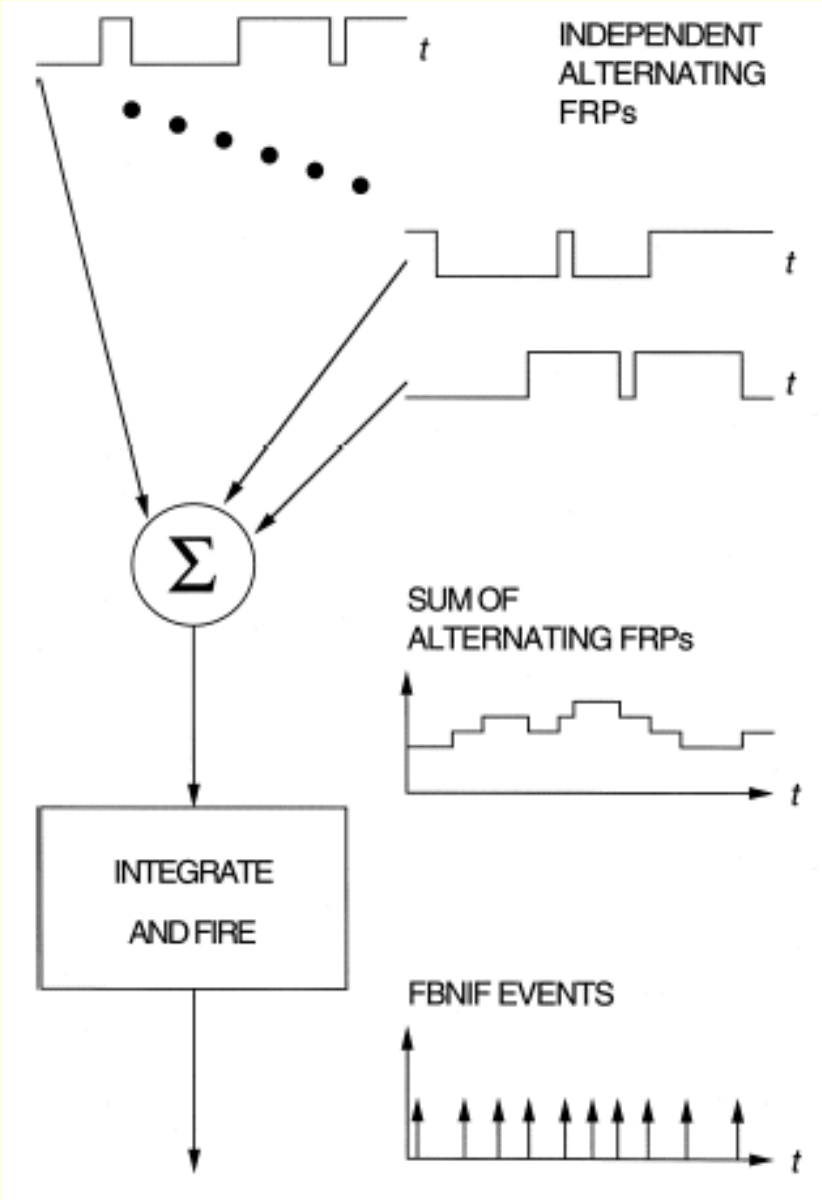

DATA
ROC AREA
1.0
0.8
0.6
$\sigma_{WAV}(32)$
$\alpha_{A\tau}$
$S(1/32)$
$\alpha_{S\tau}$
σ_{int}
α_Y
10^2
10^3
10^4
10^5
DATA LENGTH

THEORY

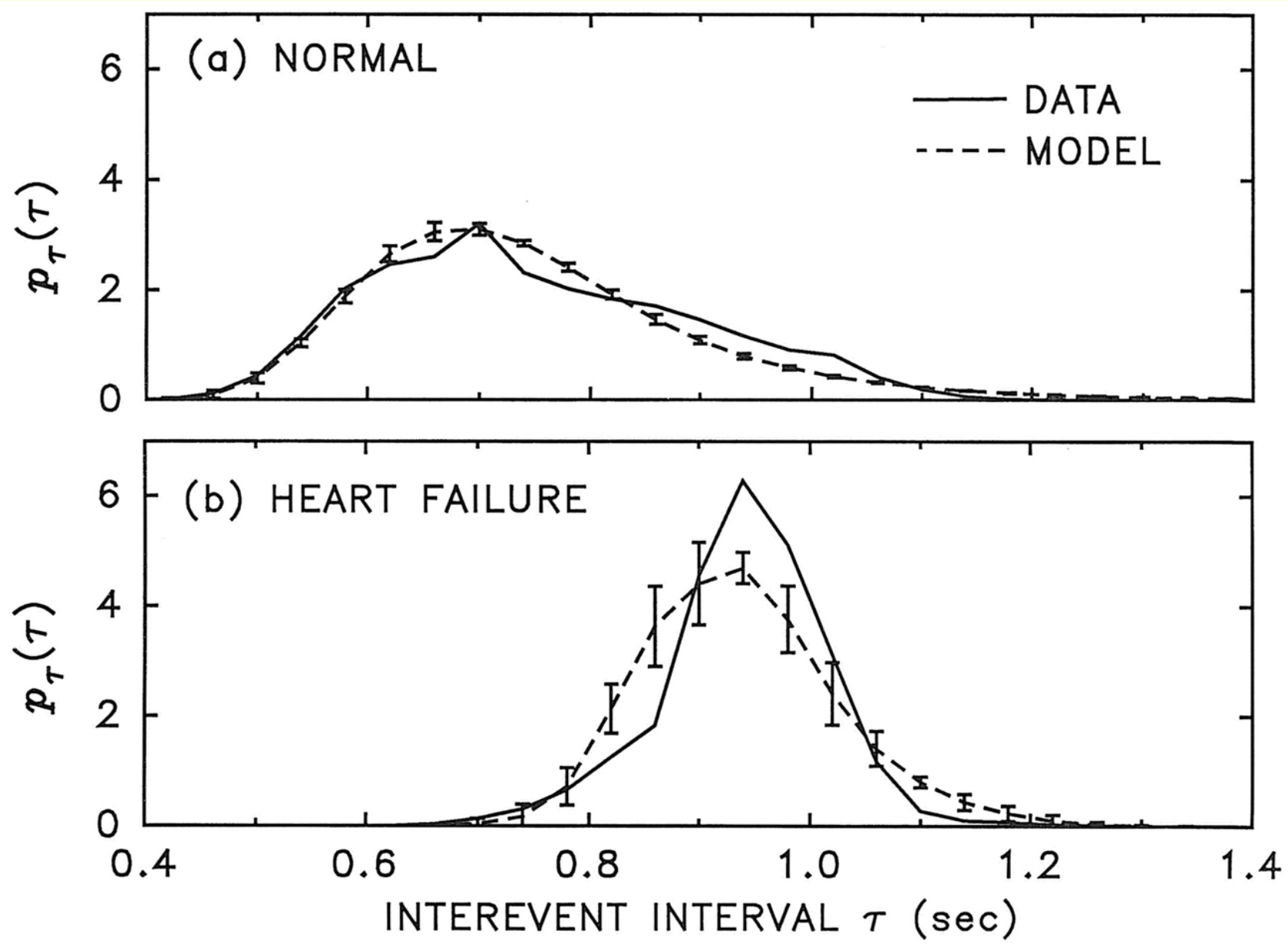

RATE
SIGNAL
$\lambda(t)$
INTEGRATOR
$\int_{t_k}^{t_{k+1}} \lambda(\tau)\,d\tau$
THRESHOLD
DETECTOR
θ
POINT
PROCESS
$dN(t)$
RESET

FREQUENCY-DOMAIN REPRESENTATION OF BANDLIMITED FRACTIONAL GAUSSIAN NOISE (FGN)

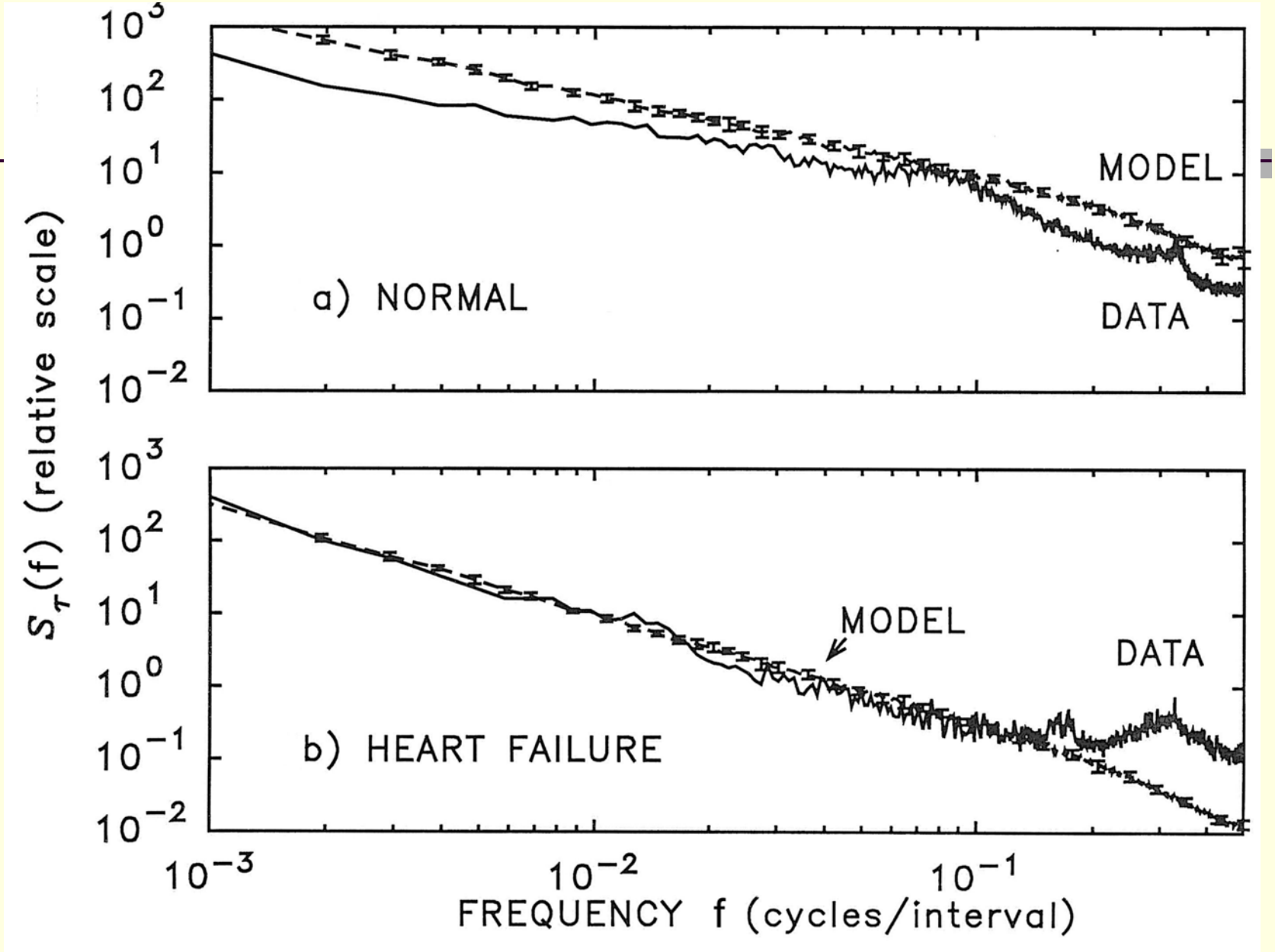



t
INDEPENDENT
ALTERNATING
FRPs
t
t
Σ
SUM OF
ALTERNATING FRPs
t
INTEGRATE
AND FIRE
FBNIF EVENTS
t

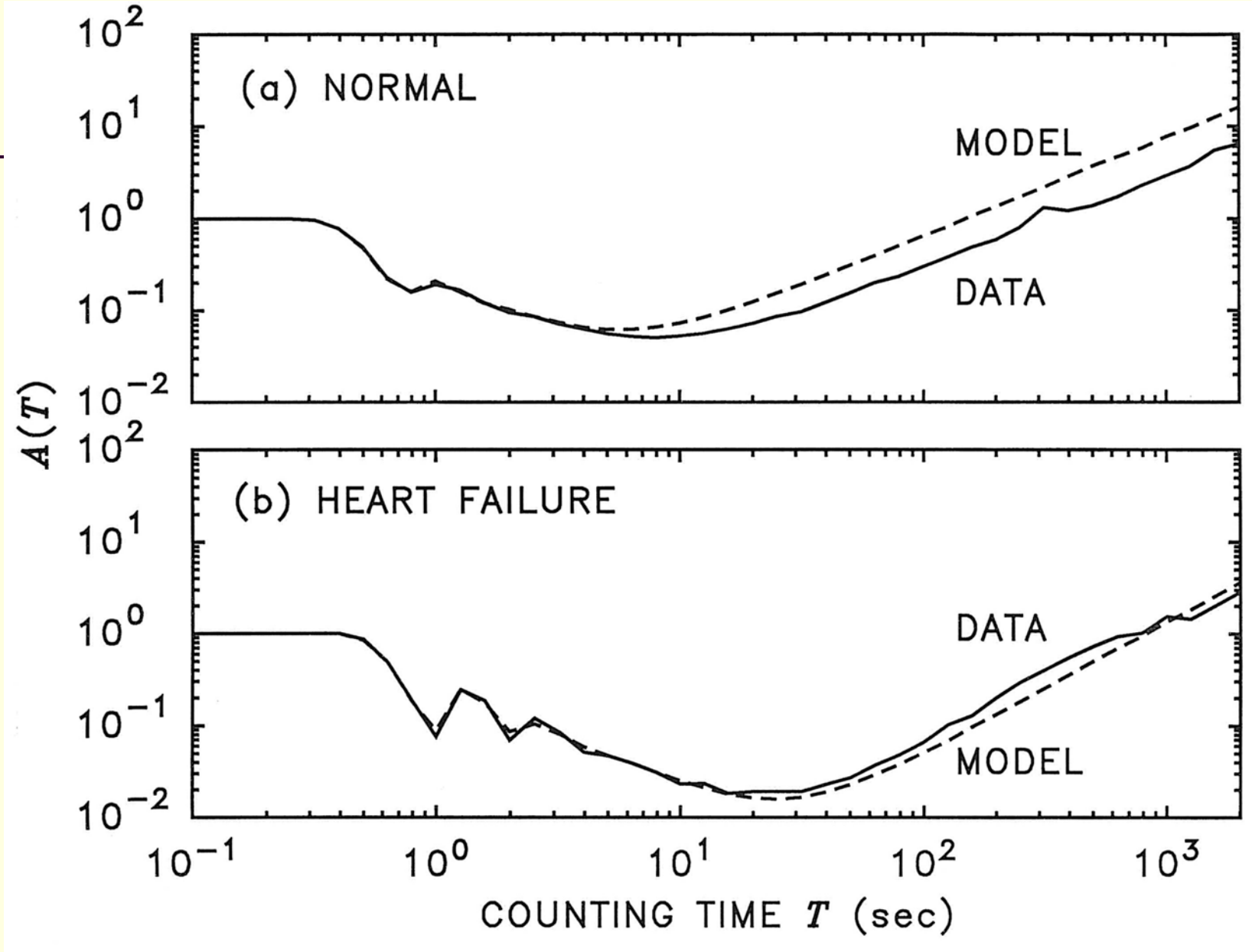
INTEREVENT-INTERVAL HISTOGRAM



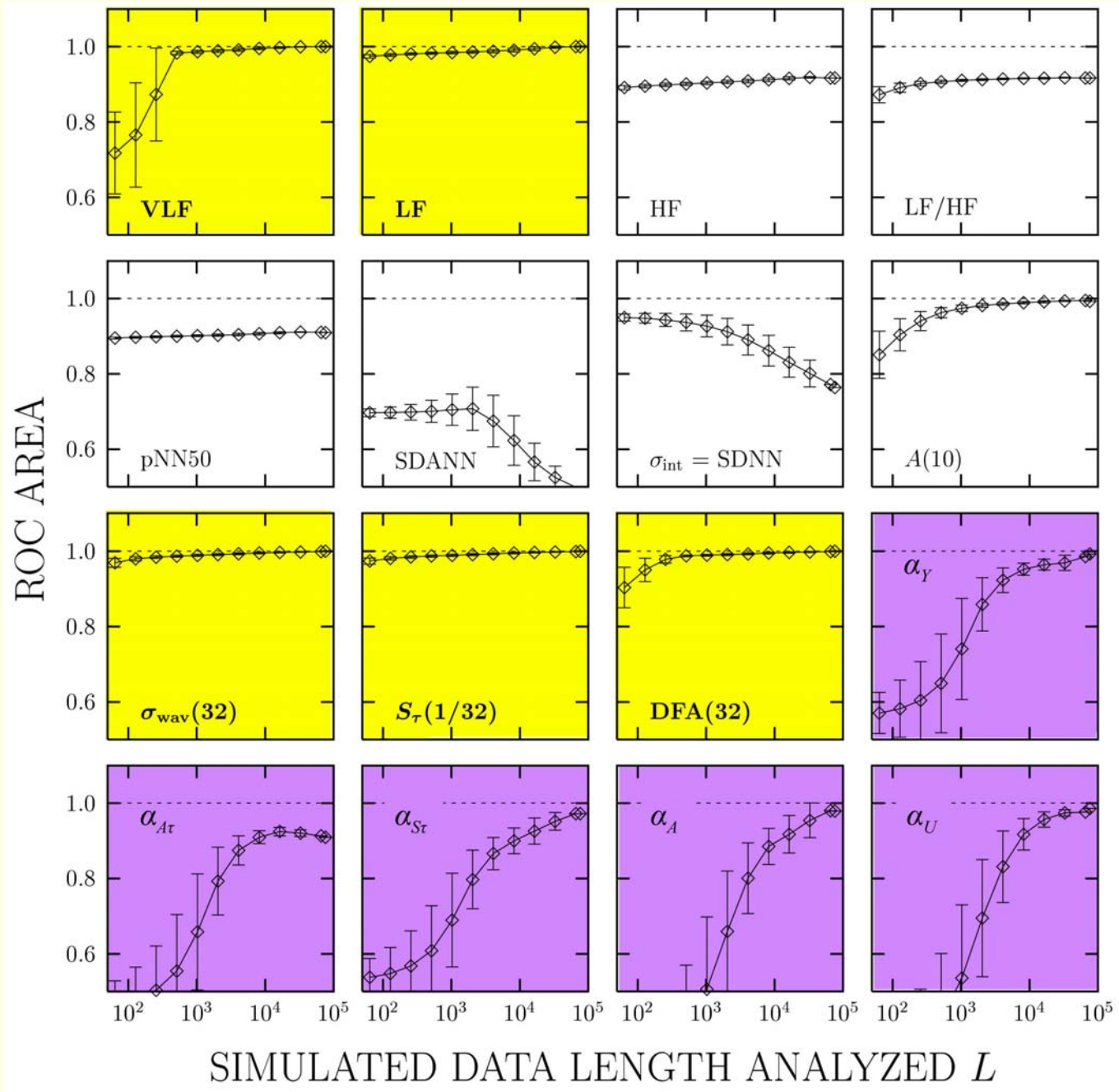
INTERVAL-BASED PERIODOGRAM

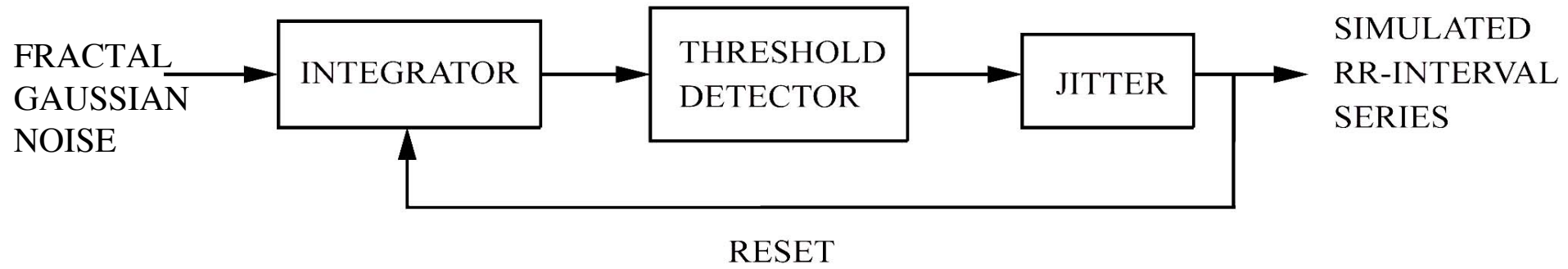


NORMALIZED HAAR-WAVELET VARIANCE

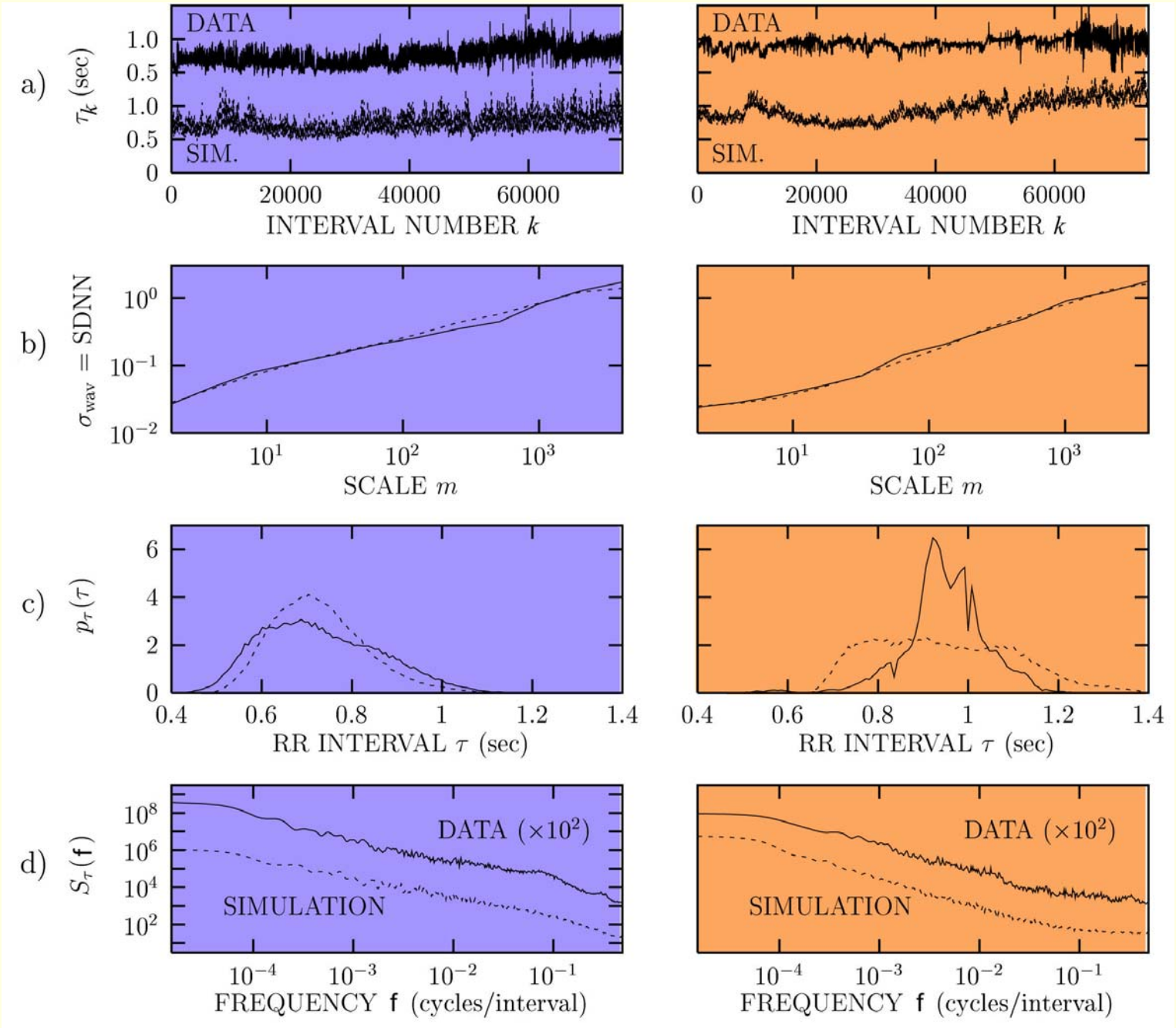


ROC-AREA CURVES: SIMULATION

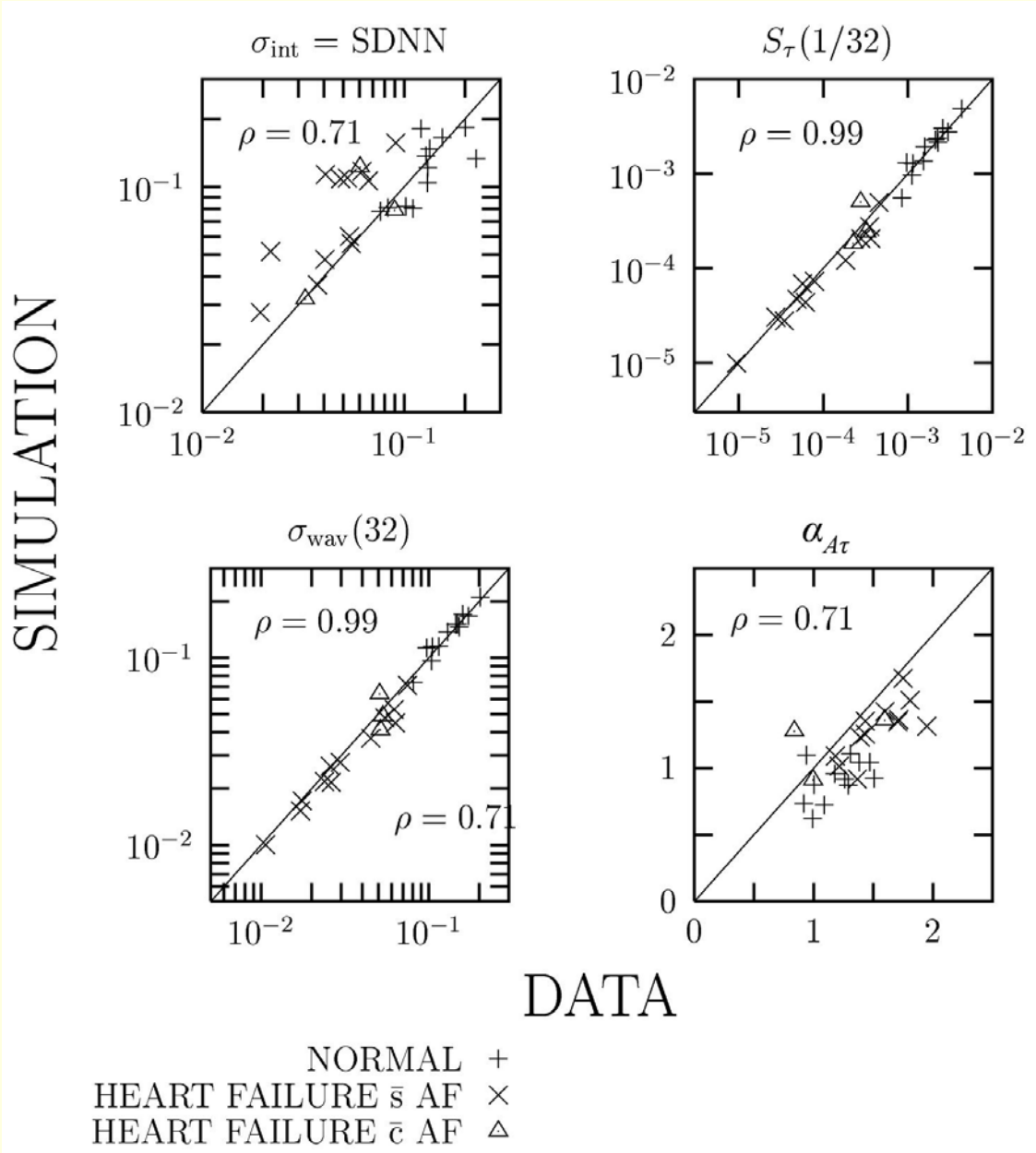



FRACTAL
GAUSSIAN
NOISE
INTEGRATOR
THRESHOLD
DETECTOR
JITTER
SIMULATED
RR-INTERVAL
SERIES
RESET

NORMAL HEART FAILURE

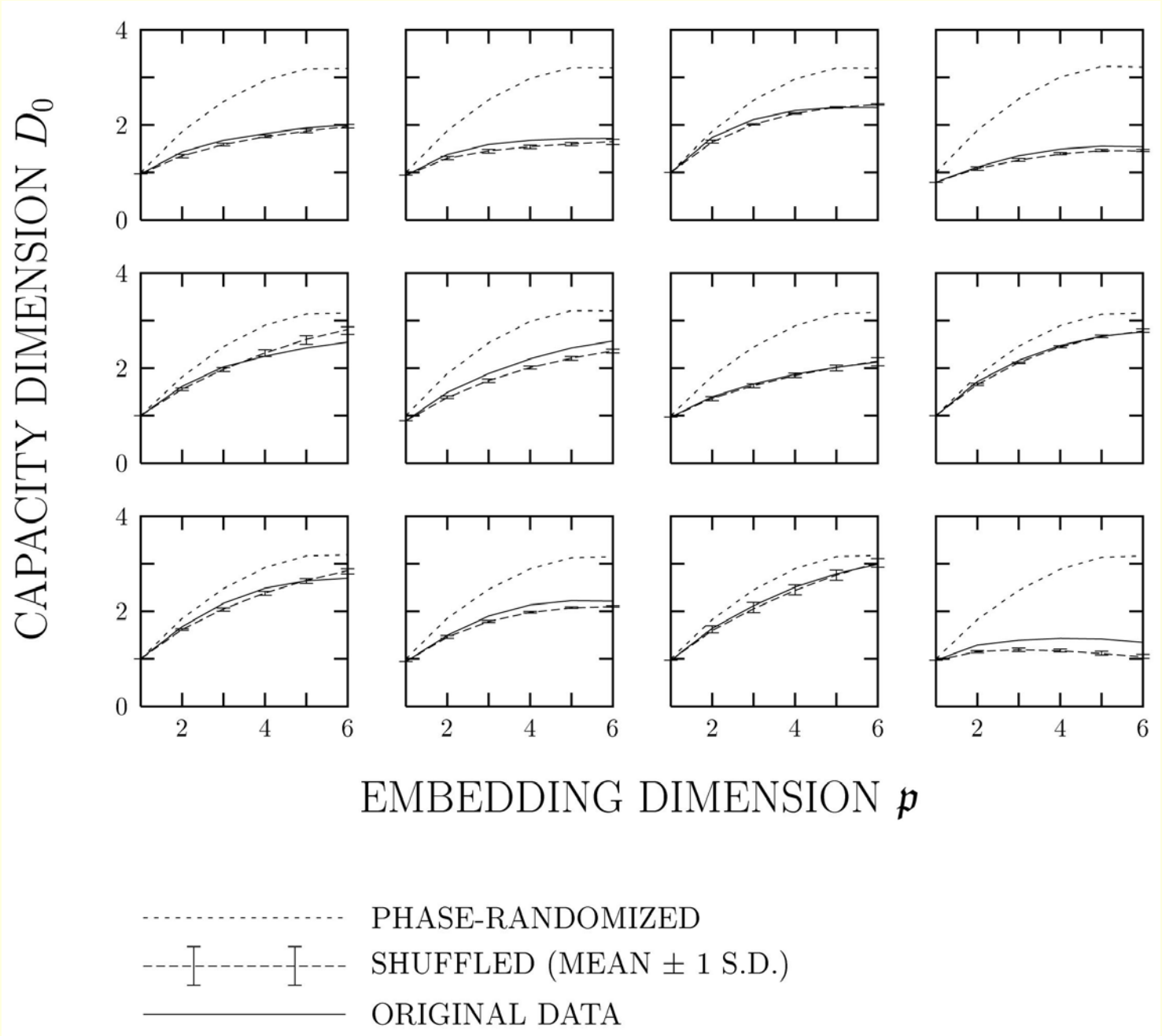


SIMULATION ACCURACY

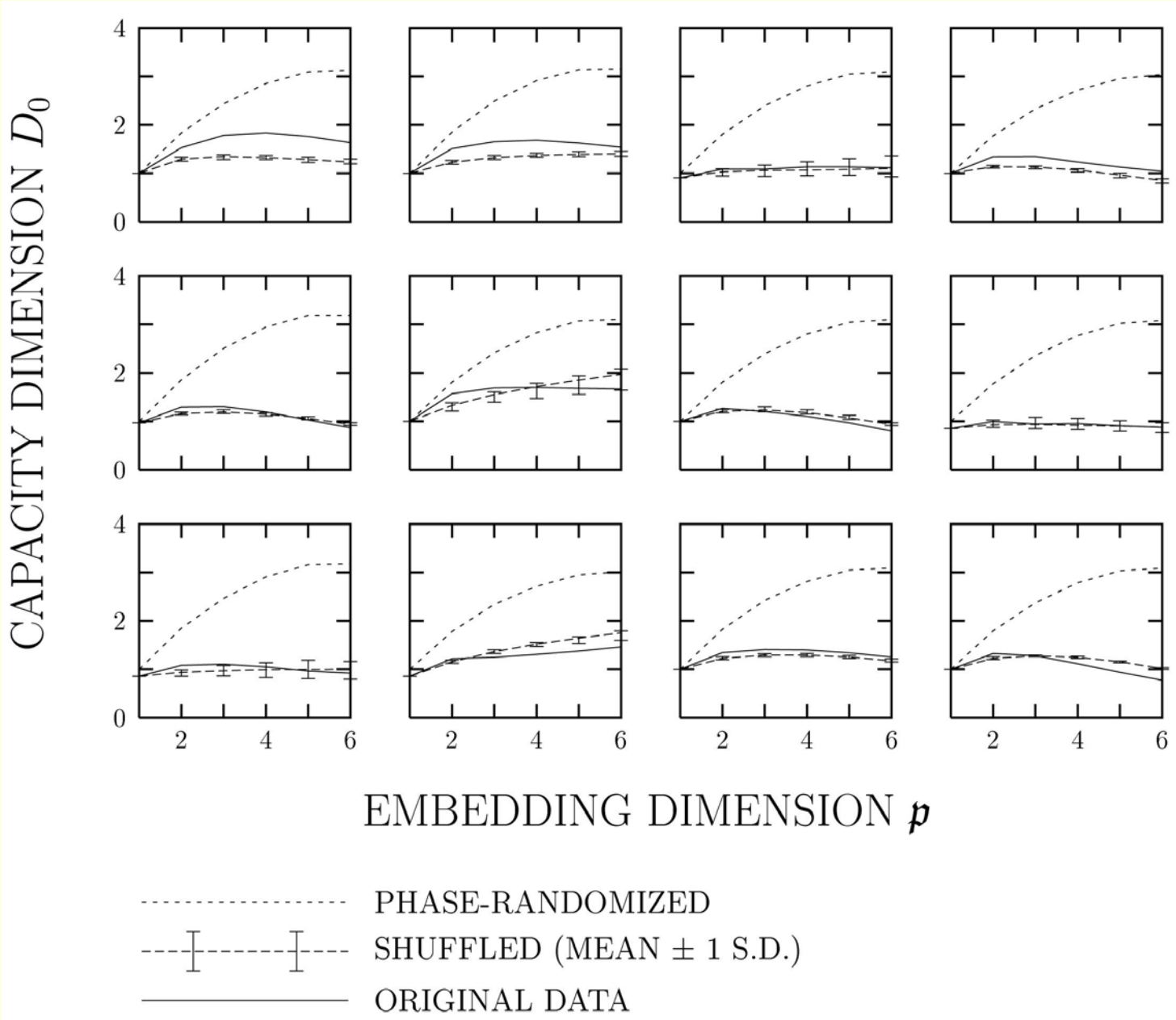


THE HEARTBEAT DOES NOT REFLECT DETERMINISTIC CHAOS

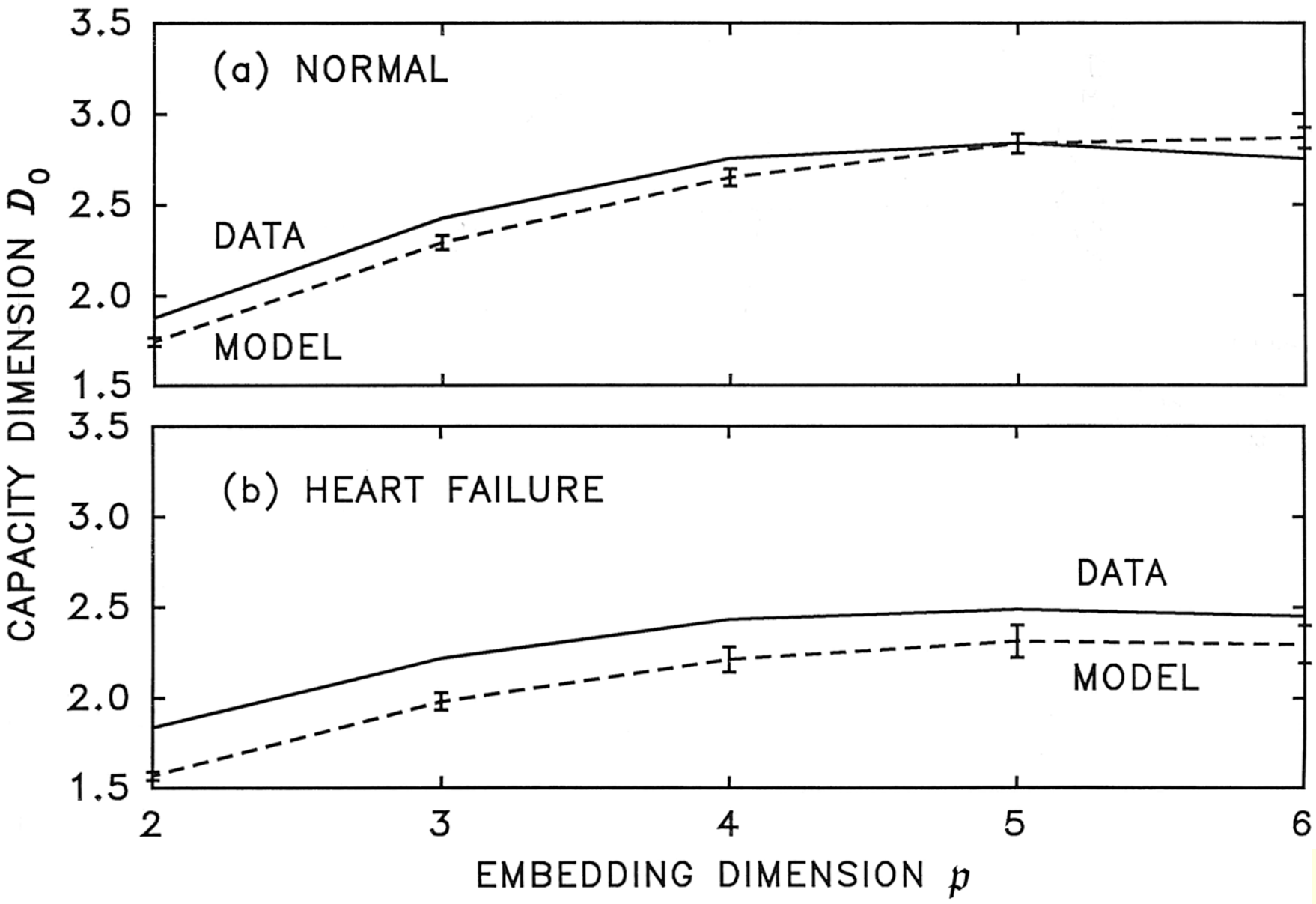
NORMAL


CAPACITY DIMENSION D_0
EMBEDDING DIMENSION p
PHASE-RANDOMIZED
SHUFFLED (MEAN ± 1 S.D.)
ORIGINAL DATA

HEART FAILURE s AF



GENERALIZED-RATE-BASED PHASE-SPACE RECONSTRUCTION: DATA AND MODEL



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M. C. Teich, "Fractal Behavior of the Electrocardiogram: Distinguishing Heart-Failure and Normal Patients Using Wavelet Analysis," *Proc. 18th Intern. Conf. IEEE Eng. Med. Biol. Soc.* **18**, 1128-1129 (1996).

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C. Heneghan, S. B. Lowen, and M. C. Teich, "Analysis of Spectral and Wavelet-Based Measures Used to Asses Cardiac Pathology," in *Proceeding of 1999 IEEE International Conference on Acoustic, Speech, and Signal Processing* (*ICASSP*) (Phoenix, AZ, 1999), paper SPTM-8.2.

M. C. Teich, S. B. Lowen, B. M. Jost, K. Vibe-Rheymer, and C. Heneghan, "Heart Rate Variability: Measures and Models," in *Nonlinear Biomedical Signal Processing*, **II**, edited by M. Akay (IEEE Press, New York, 2001), 159-213.

S. B. Lowen and M. C. Teich, *Fractal-Based Point Processes* (Wiley, Hoboken, NJ, 2005).