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PHOTOCOUNTING STATISTICS OF RADIATION PROPAGATING THROUGH PASSIVE AND ACTIVE RANDOM MEDIA

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The photocounting statistics of radiation propagating through active, quadratic random media and through passive media are considered on the basis of a recently developed quantum dynamic theory. We show that the counting statistics for lossless random active media and those for passive weakly-inhomogeneous random media are similar, although there are some differences. The peak of the photocounting distribution shifts to lower count number n and is found to broaden with increasing turbulence level, as in the case of passive media. In lossy active media, the shift of the peak to lower n is more pronounced, in pumped active media below threshold it is less so, and in pumped active media above threshold, there is a shift of the peak to higher n as a consequence of self-radiation. Severe broadening occurs in this latter case. Photocounting statistics for special states of the incident field are also discussed.

1. INTRODUCTION

Recently the statistics of radiation propagating through an active quadratic random medium have been systematically investigated in a series of papers [1–4] using the coherent state technique. The Heisenberg field equations have been solved to obtain the quantum characteristic functions and quasi-distributions. This approach was based on the Louisell-Yariv-Siegman formulation [5] in terms of real mode functions, and includes the effect of the quadratic nonlinearity in the Hamiltonian. While the results of papers [1,2] are appropriate for the case of undamping active media, papers [3,4] take into account a loss mechanism in the medium. This provides a more realistic description; moreover, pumped media may also be simply treated within the framework of this theory.

The loss mechanism is introduced through a reservoir system of damping oscillators in interaction with the radiation. This formulation is, in part, equivalent to including the coupling of radiative modes in the theory presented in [1,2]; this could not be fully done because of mathematical difficulties, however. The results obtained are appropriate for describing the radiation propagating through an active random medium with a quadratic nonlinearity. If the damping of the radiation is smaller than the self-radiation of the medium and the contribution of the reservoir, then there is amplification of the incident radiation and we speak of pumped active media with gain. If the damping is stronger than the self-radiation and the reservoir contribution, then there is attenuation of the incident radiation and we speak of lossy active media.

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If the frequency of the incident radiation is such that it excites no transitions in the medium, then the medium behaves passively and the corresponding "passive" descriptions proposed in [6] and [7,8] are appropriate (their interrelation is considered in [9]). A preliminary discussion of the relation between the "active" and "passive" descriptions can be found in [2,4].

In this paper, we present a comparison of the photocounting distributions for radiation propagating through both active and passive random media. We show that in the lossless active medium, the "active" description provides results quite close to those obtained using the Diament-Teich or generalized Tatarski "passive" descriptions for a weakly inhomogeneous random medium. The "passive" descriptions therefore approximately describe self-radiation, damping, and the reservoir contribution. The "active" description for the lossless medium shifts the peak of the counting distribution to lower count number n and has the effect of broadening the distribution with increasing level of turbulence, as in passive media [6,9]. In lossy active media, the shift toward lower n is more pronounced, in pumped active media exhibiting gain (below threshold) it is less so, while for pumped active media exhibiting gain above threshold there is a shift of the peak to higher n as a consequence of self-radiation. In this case, furthermore, very strong broadening of the distribution (uncertainty) occurs.

We have also observed a tendency of the active medium to conserve Poisson statistics, as in nonlinear optics in the case of second harmonic generation. Of course, the "active" description is applicable to a broader variety of cases, including the attenuation and amplification of incident radiation. We particularly consider photocounting distributions for incident radiation comprising a Fock state, a coherent state, a chaotic state, and a superposition of coherent and chaotic radiation. Numerical results for coherent and chaotic radiation are presented.

2. THEORETICAL RESULTS

Basic formulas for the description of the photocounting statistics of radiation propagating through a random active medium have been obtained in [4]. In the following, we provide a number of basic results necessary for a discussion of the numerical data and give some additional comments pertinent to the general theory and to the connection between the "active" and "passive" descriptions.

The normal characteristic function [10] takes the form of a product of two characteristic functions for the superposition of coherent and chaotic fields,

$$(1)\qquad C_{\mathcal{N}}^{(W)}(\mathrm{i}s, t) = \left\langle \left[1 - \mathrm{i}s\left(\frac{1}{F} - 1\right)\right]^{-M/2} \left[1 - \mathrm{i}s\left(\frac{1}{E} - 1\right)\right]^{-M/2} . \right.$$

$$\left. . \exp\left[\frac{\mathrm{i}s\bar{A}_1}{1 - \mathrm{i}s\left(\frac{1}{F} - 1\right)} + \frac{\mathrm{i}s\bar{A}_2}{1 - \mathrm{i}s\left(\frac{1}{E} - 1\right)}\right]\right\rangle,$$

provided that we assume a weakly inhomogeneous medium (inhomogeneities are much larger than the wavelength so that fluctuations of the direction of propagation may be neglected [11]), and further provided that fluctuations are slow in comparison with the period of the radiation. All modes are assumed to be equally damped, only a mean frequency is considered, and the phase of the incident complex amplitude is assumed to be random. In (1), we choose

$$\frac{1}{E} - 1 = |v| (|v| + |u|) + \langle n_d \rangle (1 - \exp(-\gamma t)), \tag{2}$$

$$\frac{1}{F} - 1 = |v| (|v| - |u|) + \langle n_d \rangle (1 - \exp(-\gamma t)),$$

$$\bar{A}_{1,2} = (|u| \mp |v|)^2 W_0/2,$$

in which u and v are given by

$$u(t) = \exp(-\gamma t/2) [\cos(\mathscr{K}^{1/2}\omega t) - \mathrm{i}\,\mathrm{ch}(\varphi/2) \sin(\mathscr{K}^{1/2}\omega t)],$$

$$v(t) = \mathrm{i} \exp(-\gamma t/2)\, \mathrm{sh}(\varphi/2) \sin(\mathscr{K}^{1/2}\omega t).$$

Since u and v are time-dependent functions with oscillating components $\sim 10^{15}$ sec^{-1}, and since the period is random [4], we must average the mean integrated intensity $\langle W \rangle_{\mathcal{N}}$ over time. Thus $\langle \cos^2 \omega \mathscr{K}^{1/2} t \rangle = \langle \sin^2 \omega \mathscr{K}^{1/2} t \rangle = 1/2$ and in this situation we speak of the "average times" case. Then

$$|u|^2 = \exp(-\gamma t) \left(1 + \tfrac{1}{2} sh^2 \frac{\varphi}{2}\right), \quad |v|^2 = \tfrac{1}{2} \exp(-\gamma t)\, \mathrm{sh}^2 \frac{\varphi}{2} \tag{3a}$$

are the observable quantities.

The maximum effect of the medium and maximum deviation from the free-field statistics occur for "special times" [3,4], such that $\cos^2 \omega \mathscr{K}^{1/2} t = 0$, $\sin^2 \omega \mathscr{K}^{1/2} t = 1$ (for times such that $\cos^2 \omega \mathscr{K}^{1/2} t = 1$, $\sin^2 \omega \mathscr{K}^{1/2} t = 0$, the free-field photodetection equation with the Poisson kernel applies). Then,

$$|u|^2 = \exp(-\gamma t)\, \mathrm{ch}^2 \frac{\varphi}{2}, \quad |v|^2 = \exp(-\gamma t)\, \mathrm{sh}^2 \frac{\varphi}{2}. \tag{3b}$$

In (1), (2) and (3) the quatity is is a parameter, the time $t = z/c$, where z is the distance travelled in the medium, c is the velocity of light, M is the number of degrees of freedom, γ is the damping constant which is much lower than the radiative frequency ω, W_0 is the integrated intensity of the incident radiation, $\langle n_d \rangle = (\exp(\hbar\psi/KT) - 1)^{-1}$ is the mean number of reservoir chaotic oscillators, $\hbar$ is Planck's constant divided by 2π, K is Boltzmann's constant, T is the absolute temperature of the reservoir, and ψ is the reservoir frequency. The quantity $\varphi = \ln \mathscr{K}$, where $\mathscr{K}$ is a typical quantity fluctuating in the medium as a consequence of fluctuations of permittivity $\varepsilon(\mathbf{x}, t)$ [1–4], such that $\langle \mathscr{K} \rangle = \langle \exp \varphi \rangle = 1$. Assuming that the permittivity fluctuations $\varepsilon'(\mathbf{x}, t)$ are Gaussian ($\varepsilon(\mathbf{x}, t) = 1 + \varepsilon'(\mathbf{x}, t)$, $|\varepsilon'| \ll 1$, we assume for simplicity that $\langle \varepsilon \rangle = 1$), we can show [4] that the probability distribution $\bar{P}(\mathscr{K})$ is lognormal,

$$\bar{P}(\mathscr{K}) = \frac{1}{(2\pi)^{1/2}\,\sigma \mathscr{K}} \exp\left[-\frac{(\ln \mathscr{K} + \sigma^2/2)^2}{2\sigma^2}\right], \tag{4}$$

i.e. $\bar{P}(\varphi)$ is a Gaussian distribution centered at $\varphi = -\sigma^2/2$. Furthermore,

$$\langle \mathscr{K}^j \rangle = \langle \exp j\varphi \rangle = \exp\left[\frac{\sigma^2}{2} j(j-1)\right], \quad j = 0, \pm 1, \ldots, \tag{5}$$

where σ is the standard deviation of $\ln \mathscr{K}$. The average in (1) is taken over W_0 (the probability distribution of the incident radiation is $P(W_0)$) and over $\mathscr{K}$ or φ with $\bar{P}(\mathscr{K})$ or $\bar{P}(\varphi)$. We see from (1) and (2) that the quantities $(1/E - 1)$ and $(1/F - 1)$ describe the vacuum contribution of chaotic radiation energy and the contribution of chaotic energy from the reservoir into radiation, while the signal numbers $\bar{A}_{1/2}$ are connected with the incident radiation.

It is clear that (1) has the form of a product of two generating functions for Laguerre polynomials; thus the photocounting distribution $p(n, z/c) = \mathrm{d}^n C_{\mathscr{N}}^{(W)}(\mathrm{i}s, z/c) : n'!\, \mathrm{d}(\mathrm{i}s)^n|_{\mathrm{i}s=-1}$ and its factorial moments $\langle W^k \rangle_{\mathscr{N}} = \mathrm{d}^k C_{\mathscr{N}}^{(W)}(\mathrm{i}s, z/c)/\mathrm{d}(\mathrm{i}s)^k|_{\mathrm{i}s=0}$ can be expressed as a finite convolution of the well-known expressions for the superposition of coherent and chaotic fields [4]. These are rather complicated expressions; fortunately, calculations using the exact expressions are not necessary. It turns out that if we neglect the vacuum contribution of radiation in comparison with the contribution of the reservoir and the signal component, assuming that $\langle W_0 \rangle$ is sufficiently high, we can obtain simplified equations which are sufficiently accurate.

From (1) we have

$$\langle n \rangle = \langle W \rangle_{\mathscr{N}} = \left.\frac{\mathrm{d}C_{\mathscr{N}}^{(W)}(\mathrm{i}s, z/c)}{\mathrm{d}(\mathrm{i}s)}\right|_{\mathrm{i}s=0} = \langle |u|^2 + |v|^2 \rangle \langle W_0 \rangle + M\langle |v|^2 \rangle + M\langle n_d \rangle (1 - \exp(-\gamma z/c)) . \tag{6}$$

Since from (3a, b), $|u|^2 + |v|^2 = \exp(-\gamma z/c)(1 + \mathrm{ch}\,\varphi)/2$ and $\exp(-\gamma z/c)\,\mathrm{ch}\,\varphi$ for the "average times" and "special times" respectively, we may use (5) to obtain

$$\langle n \rangle = \langle W \rangle_{\mathscr{N}} = \exp(-\gamma z/c)\left(\frac{\exp \sigma^2 + 3}{4}\langle \mathrm{W}_0 \rangle + \frac{\exp \sigma^2 - 1}{8} M\right) + M\langle n_d \rangle (1 - \exp(-\gamma z/c)), \tag{7a}$$

$$= \exp(-\gamma z/c)\left(\frac{\exp\sigma^2 + 1}{2}\langle W_0\rangle + \frac{\exp\sigma^2 - 1}{4}M\right) + \tag{7b}$$

$$+ M\langle n_d\rangle\,(1 - \exp(-\gamma z/c))\,,$$

for "average times" and "special times" respectively. For the turbulent atmosphere [6,12–14,9], we choose $\sigma = 1/2$ as an intermediate level of turbulence, and $\sigma = 3/2$ as the saturation value. The coefficients of $\langle W_0\rangle$ and of M in (7a,b) are then

$$\frac{\exp\sigma^2 + 3}{4} = \begin{cases}1{\cdot}071, \\ 3{\cdot}122,\end{cases} \quad \frac{\exp\sigma^2 - 1}{8} = \begin{cases}0{\cdot}0355, & \text{for} \quad \sigma = 1/2\,, \\ 1{\cdot}061, & \text{for} \quad \sigma = 3/2\,,\end{cases} \tag{8a}$$

for "average times", and

$$\frac{\exp\sigma^2 + 1}{2} = \begin{cases}1{\cdot}142, \\ 5{\cdot}244,\end{cases} \quad \frac{\exp\sigma^2 - 1}{4} = \begin{cases}0{\cdot}0710, & \text{for} \quad \sigma = 1/2\,. \\ 2{\cdot}122, & \text{for} \quad \sigma = 3/2\,,\end{cases} \tag{8b}$$

for "special times".

We see from (7a,b) and (8a,b) that there are three kinds of contribution to $\langle n\rangle = \langle W\rangle_{\mathcal{N}}$. The first term $\langle|u|^2 + |v|^2\rangle\,\langle W_0\rangle$ represents the response to the incident field, while the quantity $\exp(\gamma z/c)\,\langle|u|^2 + |v|^2\rangle$ provides the self-radiation enhancement of the incident radiation which is subsequently diminished by the damping factor $\exp(-\gamma z/c)$ in lossy media. We observe that this enhancement is small for $\sigma = 1/2$, while it is quite large for $\sigma = 3/2$. In lossy media $\langle|u|^2 + |v|^2\rangle\,\langle 1$, for small z, σ is small so that the enhancement is rather small; for high z, the self-radiation enhancement is saturated but the damping is strong. In general, there always exists a z such that $\langle|u|^2 + |v|^2\rangle < 1$ ($\gamma > 0$, if e.g. $\gamma = 3 \times 10^{10} \ll \omega = 10^{15}\ \text{sec}^{-1}$, then $z \gtrsim 1$ cm). The second term $M\langle|v|^2\rangle$ represents the vacuum contribution, while the third term $M\langle n_d\rangle\,(1 - \exp(-\gamma z/c))$ provides the contribution of the chaotic energy from the reservoir into the field.

In the case of pumped active media, it is known (cf. e.g. [16, 17]) that the pumping compensates the damping constant $\gamma/2$. Thus $\gamma/2 - g$ replaces $\gamma/2$, where g is the pumping gain parameter. At threshold, $\gamma/2$ is just equal to g, while above threshold $\gamma/2 - g < 0$. If we allow γ to include the pumping parameter, then $\gamma = 0$ at threshold for a non-damping pumped medium, while $\gamma < 0$ for a pumped medium above threshold. In this case, $\exp(-\gamma z/c) > 1$, with $\langle n_d\rangle = [\exp(\hbar\psi/KT) - 1]^{-1} < 0$, since above threshold the medium has a negative temperature [18] (this entails the existence of distributions). In general, including both the vacuum contribution of the radiation and the reservoir contribution, we obtain $\langle n\rangle < \langle W_0\rangle$ for lossy active media and an attendant attenuation of the incident signal. For pumped active media with gain, $\langle n\rangle > \langle W_0\rangle$, and there is amplification of the incident signal. Passive weakly inhomogeneous media and lossless active media yield $\langle n\rangle = \langle\mathcal{K}\rangle\,\langle W_0\rangle = \langle W_0\rangle$ [6–8], and provide a basis for comparing the "active" and "passive" descriptions.

The main assumption of the damping theory is that only those reservoir frequencies $|\psi_l| \approx \omega$ are strongly coupled to radiation frequencies ω_j [5,3], and M is given by the number of degrees of freedom of the incident radiation. Measuruments are most sensitive in the region $\Gamma T = 1$ (Γ is a half-width of the radiation spectrum and T is the detection time), which corresponds to $M \doteq 1{\cdot}5$ [15] (for $\Gamma T \to 0$, $M \to 1$). For single-mode radiation, $M = 1$, choosing $\langle W_0 \rangle = 20$ allows us to neglect the vacuum contribution $M\langle |v|^2 \rangle$ in (6) (i.e. we neglect 1·061 in comparison with 62·5 for "average times" and $\sigma = 3/2$, etc.). The vacuum term corresponds to terms $|v| (|v| \pm |u|)$ in $({}^1/E, F - 1)$ of (1) (cf. (2)). Neglecting these terms, we obtain

$$C_{\mathcal{N}}^{(W)}(\mathrm{i}s, z/c) = [1 - \mathrm{i}s \langle n_d \rangle (1 - \exp(-\gamma z/c))]^{-1} \, . \left\langle \exp \left[\frac{\mathrm{i}s(|u|^2 + |v|^2) W_0}{1 - \mathrm{i}s\langle n_d \rangle (1 - \exp(-\gamma z/c))} \right] \right\rangle . \tag{9}$$

Numerical results for the photocounting distributions which will be given in Sec. 4 indeed show that the vacuum contribution is negligible. In this case the signal component is $(|u|^2 + |v|^2) W_0$ and is fluctuating in the medium. The noise component is given by the reservoir noise $\langle n_d \rangle (1 - \exp(-\gamma z/c))$, and is deterministic. Thus, setting $M = 1$, the photocounting distribution becomes [10]

$$p(n, z/c) = \frac{1}{n!} \left(1 + \frac{1}{\langle n_d \rangle (1 - \exp(-\gamma z/c))} \right)^{-n} . \, (1 + \langle n_d \rangle (1 - \exp(-\gamma z/c))^{-1} \left\langle \exp \left[- \frac{(|u|^2 + |v|^2) W_0}{1 + \langle n_d \rangle (1 - \exp(-\gamma z/c))} \right] . \, L_n^0 \left(- \frac{(|u|^2 + |v|^2) W_0}{\langle n_d \rangle (1 - \exp(-\gamma z/c)) [1 + \langle n_d \rangle (1 - \exp(-\gamma z/c))]} \right) \right\rangle , \tag{10}$$

with factorial moments

$$\langle W^k \rangle_{\mathcal{N}} = [\langle n_d \rangle (1 - \exp(-\gamma z/c))]^k \left\langle L_k^0 \left(- \frac{(|u|^2 + |v|^2) W_0}{\langle n_d \rangle (1 - \exp(-\gamma z/c))} \right) \right\rangle , \tag{11}$$

where L_k^λ is the Laguerre polynomial.

We have assumed for simplicity that the photoefficiency $\eta = 1$. For $\eta \neq 1$, we make the replacement $\mathrm{i}s \to \eta\, \mathrm{i}s$ in (1), thus $(1/E, F - 1) \to \eta(1/E, F - 1)$ and $\bar{A}_{1,2} \to \eta \bar{A}_{1,2}$. For the most general exact formula for the photocounting distribution, we again use the generating function for the Laguerre polynomials [2,4] (for $\eta = 1$ we obtain results given in [2,4]). If the vacuum contribution can be neglected, it is clear from (9) that it suffices to set $W_0 \to \eta W_0$ and $\langle n_d \rangle \to \eta \langle n_d \rangle$.

Furthermore, we note that the characteristic function (9) corresponds to the Glauber-Sudarshan weighting function

$$\Phi_{\mathcal{N}}(\alpha, t) = \frac{1}{\langle n_d \rangle (1 - \exp(-\gamma z/c))} \left\langle \exp \left(- \frac{|\alpha - \xi(t)|^2}{\langle n_d \rangle (1 - \exp(-\gamma z/c))} \right) \right\rangle , \tag{12}$$

where $\xi(t) = u\xi + v\xi^*$. The brackets indicate an average over the incident complex amplitude ξ, and over φ. This corresponds to the general quasi-distribution $\Phi_{\mathscr{A}}(\alpha, t)$ associated with antinormal ordering ($\Phi_{\mathscr{N}}(\alpha, t)$ does not in general exist) obtained in [4] and simplified by neglecting the vacuum contribution. These quasidistributions obey Fokker-Planck equations with neglected rotating terms and quantities of the order of the vacuum contribution, in fact. The propagation of radiation through a random medium will be discussed from the point of view of the equivalent master and Fokker-Planck equations in [19].

If $\langle n_d \rangle = 0$, in this case the damping is caused by the reservoir vacuum alone, (9) and (10) yield

$$p(n, z/c) = \left\langle \frac{[(|u|^2 + |v|^2)\, W_0]^n}{n!} \exp\left[-(|u|^2 + |v|^2)\, W_0\right] \right\rangle . \tag{13}$$

This is the average of the Poisson kernel which formally yields the Diament-Teich description for $|u|^2 + |v|^2 \to \exp \varphi$. This is the effective value for $|u|^2 + |v|^2 = \exp(-\gamma z/c)(1 + \mathrm{ch}\, \varphi)/2$, and $\exp(-\gamma z/c)\, \mathrm{ch}\, \varphi$ for "average times" and „special times" respectively.

Before discussing the connection between the "passive" and "active" descriptions, we express (10) and (13) in a form useful for numerical calculation. We have used (10) for the incident Poisson (coherent) radiation in the form

$$p(n, z/c) = \left(\frac{2}{\pi}\right)^{1/2} \frac{n!}{\sigma} \frac{a^n}{(1 + a)^{n+1}} \int_0^\infty \sum_{k=0}^{n} \frac{1}{(k!)^2 (n - k)!} \cdot \tag{14}$$

$$\cdot \left[\frac{b\, F(x)}{a(1 + a)}\right]^k \exp\left[-\left(\frac{b\, F(x)}{1 + a} + \frac{2x^2}{\sigma^2} + \frac{\sigma^2}{8}\right)\right] (1 + \exp 2x) \exp(-x)\, \mathrm{d}x\,,$$

with $a = \langle n_d \rangle (1 - \exp(-\gamma z/c))$, $b = \langle n_c \rangle \exp(-\gamma z/c)$ ($\langle n_c \rangle = \langle W_0 \rangle$) and $F(x) = \mathrm{ch}^2\, x$ and $\mathrm{ch}\, 2x$ for "average times" and "special times" respectively.

Assuming the incident radiation to be in the form of a superposition of coherent and chaotic fields [10] (Sec. 17.3), we obtain from (13) that

$$p(n, z/c) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\Gamma(n + M)}{\sigma} \int_0^\infty f(x) \exp(-x)\, \mathrm{d}x\,, \tag{15a}$$

where

$$f(x) = (a\, F(x) + 1)^{-M-n} (a\, F(x))^n \exp\left(-\frac{b\, F(x)}{a\, F(x) + 1}\right) . \tag{15b}$$

$$\cdot \sum_{j=0}^{n} \frac{1}{j!(n - j)!\, \Gamma(j + M)} \left[\frac{b}{a(a\, F(x) + 1)}\right]^j \exp\left[-\left(\frac{2x^2}{\sigma^2} + \frac{\sigma^2}{8}\right)\right] (1 + \exp 2x)\,.$$

Here $a = \langle n_{ch} \rangle \exp(-\gamma z/c)/M$ and $b = \langle n_c \rangle \exp(-\gamma z/c)$. The quantities $\langle n_{ch} \rangle$ and $\langle n_c \rangle$ represent the mean photon numbers in the chaotic and coherent components respectively, and $F(x)$ is given above. For Poisson (coherent) incident radiation, $\langle n_{ch} \rangle = 0$ and we obtain a simpler expression. This also follows from (14) with $\langle n_d \rangle = 0$.

In order to estimate more carefully the effect of the physical vacuum, we have excluded damping ($\gamma = \langle n_d \rangle = 0$) in the exact formula for $p(n, z/c)$ given in [4]:

$$(16) \quad p(n, z/c) = \left\langle |u|^{-M} \left(\frac{|v|}{|u|} \right)^n \exp(-W_0) \sum_{k=0}^{n} \frac{(-1)^k}{\Gamma(k + M/2)\, \Gamma(n - k + M/2)} \,. \right.$$

$$\left. . \, L_k^{M/2-1} \left(\frac{W_0}{2|u|\,|v|} \right) L_{n-k}^{M/2-1} \left(-\frac{W_0}{2|u|\,|v|} \right) \right\rangle .$$

By use of the characteristic function and the generating function for the Laguerre polynomials, this can be written in the form

$$(17a) \qquad p(n, z/c) = \sum_{\substack{m+i=n \\ i \leq n/2}} C_{mi}\, p_0(m - i)\,,$$

with

$$(17b) \qquad C_{mi} = \prod_{k=1}^{i} \frac{M/2 + m - k}{k} \left\langle \frac{|v|^{2i}}{|u|^{M+2m}} \right\rangle ,$$

which is useful for computer calculation. Here p_0 is the photocounting distribution of the incident radiation. Comparing the results obtained with the use of (13) and (16) ($\langle n_d \rangle = 0$) with $\gamma = 0$, the difference caused by the physical vacuum is directly obtained. To obtain further information, we set $\gamma = \langle n_d \rangle = 0$ in (6) and obtain a value of $\langle W_0 \rangle < 20$ such that $\langle n \rangle = 20$. Within the framework of the theory including damping, this is equivalent to the replacement $\langle n \rangle \to \exp(-\gamma z/c)\, \langle n \rangle$ which is approximately equivalent to $\langle W_0 \rangle \to \exp(-\gamma z/c)\, \langle W_0 \rangle$, neglecting the vacuum contribution. The theory without damping can therefore be used approximately for calculations when damping is present. However, in the exact theory, the ratio of the vacuum contribution to the signal term remains the same in the presence or absence of damping, viz. 1·061 : 62·5 for "average times" and $\sigma = 3/2$ (with both signal and vacuum noise damped). The approximate procedure leads to a different noise-to-signal ratio of approximately to 1 : 19 (damped signal with undamped vacuum noise). But we shall see (Sec. 4) that curves for $p(n, z/c)$ obtained in this way, and those neglecting the vacuum contribution are similar; the exact curves will be closer to those neglecting the vacuum contribution, as for $\gamma = 0$.

3. RELATION TO "PASSIVE" DESCRIPTIONS

Although this point has been discussed in [2, 4, 9] where it has been generally found that the "passive" descriptions neglect self-radiation, we provide a number of further insights on the relationship of the "active" and "passive" descriptions here. Some of these are contained in a preliminary form in [4].

The "active" description proposed in [1–4] includes the effect of quadratic terms in the Hamiltonian. It is therefore a quantum active nonlinear description. If we do not account for the reservoir, which is not considered for a passive medium, then the annihilation operator $\mathrm{a}_j(t)$ at t may be expressed in terms of the annihilation and creation operators at $t = 0$ [1–4], i.e.

$$\mathrm{a}_j(t) = u(t)\,\mathrm{a}_j + v(t)\,\mathrm{a}_j^+ . \tag{18}$$

In the "passive" descriptions, one assumes that the radiation does not interact actively with the medium so there is no exchange of energy between the radiation and the atoms of the medium, between different modes of radiation, etc. In such a "passive" description, complex mode functions $\boldsymbol{u}_l(\boldsymbol{x})$ should be used, yielding coefficients $K'_{jk} = (\omega_j\omega_k)^{1/2}/16\pi c^2 \,.\int \boldsymbol{u}_j(\boldsymbol{x})\,.\,\boldsymbol{u}_k(\boldsymbol{x})\,\varepsilon'(\boldsymbol{x}, t)\,\mathrm{d}^3\boldsymbol{x} \approx 0$ for the nonlinear terms $\mathrm{a}_j\mathrm{a}_k$ in the Hamiltonian [3] (as well as for the coefficients of $\mathrm{a}_j^+\mathrm{a}_k^+$) since the period of $\boldsymbol{u}_j(\boldsymbol{x})$ is of the order of the wavelength λ while the inhomogeneities are assumed to be much larger than λ. The self-radiative terms are therefore absent, and the "passive" descriptions are appropriate. It has been shown in [4] that using (18) in the vector-potential decomposition we can redefine $\mathrm{a}_j(t)$ as

$$\mathrm{a}_j(t) = u\mathrm{a}_j + (v\mathrm{a}_j^+)^+ = (u_j + v_j^*)\,\mathrm{a}_j . \tag{19}$$

This expresses a compensation of the self-radiation when the reservoir is not present. The solutions for u and v are such, that for "special times" [3,4], $|u + v^*|^2 = \exp\varphi$ (for "average times" there is an additional smoothing effect caused by the average over time). Thus

$$\mathrm{n}(t) = \sum_j \mathrm{a}_j^+(t)\,\mathrm{a}_j(t) = \exp(\varphi)\sum_j \mathrm{a}_j^+\mathrm{a}_j = \exp(\varphi)\,\mathrm{n}_0 \tag{20a}$$

and the corresponding c-number equation (averaged in the coherent state) is

$$W(t) = \exp(\varphi)\,W_0 . \tag{20b}$$

A consequence of energy conservation (in a weakly inhomogeneous medium) $\langle \mathrm{n}(t)\rangle = \langle \mathrm{n}_0\rangle$ is that $\langle[\mathrm{a}_j(t), \mathrm{a}_k^+(t)]\rangle = \langle\exp(\varphi)\rangle\,\delta_{jk} = \delta_{jk}$.

Equation (20a) is typical of the quantum passive linear generalized Tatarski description, since from (20a) $\langle \mathrm{n}^k\rangle = \langle\exp(k\varphi)\rangle\,\langle \mathrm{n}_0^k\rangle$ and consequently $\langle\exp(is\mathrm{n})\rangle = \langle\exp(is\mathrm{n}_0\exp\varphi)\rangle = \langle\mathcal{N}\exp[(\exp(is\exp\varphi) - 1)\,\mathrm{n}_0]\rangle$ ($\mathcal{N}$ denotes the normal ordering operator). This leads to the modified photodetection equation with a shot noise term [8,9]. Treating the medium between the source and the detector as a linear passive filter [6], we can write $\langle\exp(is\mathrm{n})\rangle = \langle\mathcal{N}\exp[(\exp is - 1)\,\mathrm{n}]\rangle$, based on $[\mathrm{a}_j(t), \mathrm{a}_k^+(t)] = \delta_{jk}$. Then, using (20b), we arrive at the Diament-Teich description [6], as discussed in [9]. This is a semiclassical passive linear description (it is quantum for the field; the effect of the medium is introduced through the c-number relation (20b)). Of course, some quantum properties such as the shot noise are lost in this treatment. A comparison of both "passive" descriptions has been made in [9]. The corresponding $p(n, z/c)$ in the Diament-Teich and generalized Tatarski descriptions differ only by a normalization factor. As a consequence of the shot noise term $p_0(0)\,\delta(n)$ in the modified photodetection equation, its regular part, which is non-zero for $n > 0$, is normalized to $1 - p_0(0)$ in the generalized Tatarski description, while in the Diament-Teich description $\sum_{n=0}^{\infty} p(n, z/c) = 1$. Such small deviations are absent if $p_0(0) = 0$, e.g. for Fock state $|N\rangle$ both descriptions are identical.

4. NUMERICAL RESULTS AND DISCUSSION

We have calculated $p(n, z/c)$ for a $p_0(n)$ which is given by $\delta(n - 20)$ (Fock state $|20\rangle$), for Poisson (coherent) incident radiation with $p_0(n) = \langle n_c\rangle^n \exp(-\langle n_c\rangle)/n!$, for the Bose-Einstein case $p_0(n) = \langle n_{ch}\rangle^n/(1 + \langle n_{ch}\rangle)^{1+n}$ with $\langle W_0\rangle$ ($= \langle n_c\rangle$ or $\langle n_{ch}\rangle$) $= 20$, and also for the superposition of coherent and chaotic radiation with the signal-to-noise ratio $\langle n_c\rangle : \langle n_{ch}\rangle = 18 : 2$ ($\langle n_c\rangle + \langle n_{ch}\rangle = 20$), and $\Gamma T = 1$.

For the Fock state $|20\rangle$ we have obtained results for an undamped medium with $\gamma = \langle n_d\rangle = 0$ and we have found that for the state $|N\rangle$ it is generally true that $p(n, z/c) = 0$ for $n < N$ and $n = N + 2k + 1$, $k = 0, 1, \ldots$ as a consequence of the quadratic terms in the Hamiltonian [4]. For $n = N + 2k$, $p(n, z/c)$ is a decreasing function starting from $n = N$; it decreases more slowly for greater values of σ. We have also calculated a number of cases with $\gamma > 0$ according to (13) [4]. However, the Fock state is a typical quantum state having no classical analogue in which vacuum fluctuations play an important role. Thus, neglecting the vacuum contribution, we qualitatively change $p(n, z/c)$ (e.g. in this case $p(n, z/c)$ is non-zero for all $n > N$, which leads to its more rapid decrease) although for $\sigma = 3/2$ and $\exp(-\gamma z/c) = 2/(1 + \exp \sigma^2)$ (for "special times" and $\langle n\rangle = \langle W_0\rangle = N = 20$) good agreement with the corresponding "passive" curve (curve c in Fig. 1 of [9]) has been found. Generally, however, the exact equations should be used for this case. Fortunately, the Fock state plays a minimum role in the optical region, the coherent state being of far more importance.

In Fig. 1 we present $p(n, z/c)$ for $\sigma = 1/2$ and $\sigma = 3/2$, for incident radiation which is in coherent state with a Poisson $p_0(n)$, $\gamma = \langle n_d\rangle = 0$, and $\langle n_c\rangle = 20$. The full curves a, b' and c' represent the Poisson distribution with $\langle n_c\rangle = 20$ ($\sigma = 0$) and the exact $p(n, z/c)$ for $\sigma = 1/2$ and $\sigma = 3/2$ respectively. These were obtained using (16) [(17)] for "average times". The dotted curves b'' and c'' are obtained using (13) [(15)] ($\langle n_{ch}\rangle = 0$) and neglecting the vacuum contribution for „average times". We see that the vacuum effect is indeed small, and, in general, leads to a minimal broadening of the curves. The broken curves $\bar{b}'$ and $\bar{c}'$ represent the exact $p(n, z/c)$ for $\sigma = 1/2$ and $3/2$ obtained from (16) [(17)] for "special times". The case of the undamped medium provides a pure self-radiation effect for "special times" while there is an additional smoothing associated with the average over time for "average times". We see that, in this case, there is an amplification of the incident radiation caused by the nonlinearity, e.g. $\langle n \rangle = 63{\cdot}5$ for $\sigma = 3/2$ and $\langle W_0 \rangle = 20$ (cf. (8a)). Furthermore, we observe that the peak of $p(n, z/c)$ lies approximately at $n = \langle W_0 \rangle$ as it does for $p_0(n)$; this is exactly valid for the Fock state[1]), but some broadening with increasing σ occurs. More generally, the peak will occur approxim-

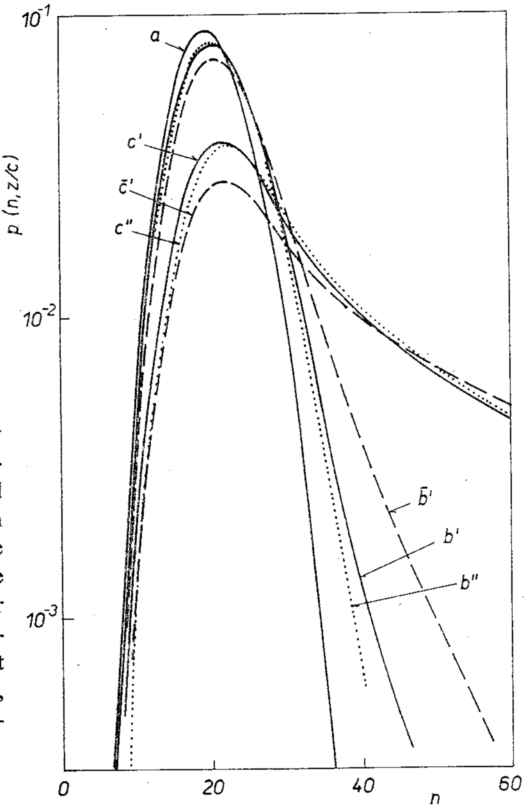

Fig. 1.
The photocounting distribution $p(n, z/c)$ for coherent radiation passing through a non-damping active random medium. The full curve a represents the Poisson distribution with $\langle n_c \rangle = 20$; full curves b' and c' are exact distributions $p(n, z/c)$ for "average times", while broken curves $\bar{b}'$ and $\bar{c}'$ are for "special times" with $\sigma = 1/2$ and $3/2$, respectively. Dotted curves b'' and c'' present $p(n, z/c)$ for $\sigma = 1/2$ and $3/2$ respectively, and for "average times" if the vacuum contribution is neglected.

ately at $n = \exp(-\gamma z/c) \langle W_0 \rangle$. This indicates that the contribution from self-radiative photons caused an increase of the most probable count number in comparison with the passive medium [9]. Moreover we observe a tendency of the active medium to conserve Poisson statistics (this is especially true for $\sigma = 1/2$), as is the case of second harmonic generation in nonlinear optics [20].

In Fig. 2 we compare the photocounting distribution of radiation passing through a passive medium and an active lossless medium assuming that the incident radiation

[1]) Some small shift to higher n is probably caused by an approximation used in obtaining $p(n, z/c)$ in connection with the average over the random phases of the incident radiation [4]; in this we neglect a number of small terms in $p(n, z/c)$ which causes a small broadening of $p(n, z/c)$ provided that the first moments are the same. Thus the exact $p(n, z/c)$ must have its peak slightly shifted to lower n.

is coherent with $\langle n_c \rangle = 20$. We have used the first two factorial moments of (10) (the first two derivatives of (9) for $is = 0$)

$$\langle W \rangle_{\mathcal{N}} = \langle |u_0|^2 + |v_0|^2 \rangle \exp(-\gamma z/c) \langle W_0 \rangle + \langle n_d \rangle (1 - \exp(-\gamma z/c)), \tag{21a}$$

$$\langle W^2 \rangle_{\mathcal{N}} = 2\langle n_d \rangle^2 (1 - \exp(-\gamma z/c))^2 + 4\langle n_d \rangle (1 - \exp(-\gamma z/c)) \,. \tag{21b}$$
$$. \langle |u_0|^2 + |v_0|^2 \rangle \exp(-\gamma z/c) \langle W_0 \rangle + \langle (|u_0|^2 + |v_0|^2)^2 \rangle \exp(-2\gamma z/c) \langle W_0^2 \rangle$$

for comparison with the "passive" moments $\langle W \rangle_{\mathcal{N}} = \langle W_0 \rangle$ and $\langle W^2 \rangle_{\mathcal{N}} = \exp \sigma^2 . \langle W_0^2 \rangle$ (cf. (5)); $\langle W_0 \rangle^2 = \langle W_0^2 \rangle = \langle n_c \rangle^2$ holds for coherent radiation. The quantities u_0 and v_0 represent u and v without the exponential factor $\exp(-\gamma z/c)$. This provides, therefore, that

$$\exp(-\gamma z/c) = \left(\frac{2 - \exp \sigma^2}{2\langle |u_0|^2 + |v_0|^2 \rangle^2 - \langle (|u_0|^2 + |v_0|^2)^2 \rangle} \right)^{1/2}$$

$$= 4 \left(\frac{2 - \exp \sigma^2}{8 + 7 \exp \sigma^2 + 2 \exp 2\sigma^2 - \exp 3\sigma^2} \right)^{1/2} \tag{22a}$$

$$= 2 \left(\frac{2 - \exp \sigma^2}{3 \exp \sigma^2 + 2 \exp 2\sigma^2 - \exp 3\sigma^2} \right)^{1/2} . \tag{22b}$$

The first expression holds for "average times" while the second is appropriate for "special times". The quantity $\langle n_d \rangle$ is simply obtained from (21a) (recalling that $\langle |u_0|^2 + |v_0|^2 \rangle = \langle (1 + \text{ch}\varphi)/2 \rangle = (\exp \sigma^2 + 3)/4$ and that it is equal to $\langle \text{ch}\varphi \rangle = (\exp \sigma^2 + 1)/2$ for "average times" and for "special times" respectively. We have obtained the factor $\exp(-\gamma z/c) \doteq 0{\cdot}796$ with $\langle n_d \rangle \doteq 14{\cdot}4$ for "average times" and the factor $\exp(-\gamma z/c) \doteq 0{\cdot}758$ with $\langle n_d \rangle = 11{\cdot}1$ for "special times" with $\sigma = 1/2$. For $\sigma = 3/2$, the equations have the unphysical solution $\langle n_d \rangle < 0$ (or $1 > \exp(-\gamma z/c) > \langle |u_0|^2 + |v_0|^2 \rangle^{-1}$), and the best correspondence occurs for $\langle n_d \rangle = 0$ and $\exp(-\gamma z/c) = \langle |u_0|^2 + |v_0|^2 \rangle^{-1}$. In general a comparison of the two moments provide good value of $\exp(-\gamma z/c)$ and $\langle n_d \rangle$ up to $\sigma \doteq 0{\cdot}8$; for higher σ, $\langle n_d \rangle = 0$ and only the first moment is to be considered for the comparison which gives rise to the above factor $\exp(-\gamma z/c)$. For $\sigma = 3/2$, this gives second moments of $\langle W^2 \rangle_{\mathcal{N}} \doteq 0{\cdot}6 \exp \sigma^2 \langle W_0 \rangle^2$ and $0{\cdot}8 \exp \sigma^2 \langle W_0 \rangle^2$, in terms of the "passive" moments for "average" times and "special times" respectively. Similar conclusions are valid for the superposition of coherent and chaotic radiation ($\langle n_c \rangle : \langle n_{ch} \rangle = 18 : 2$) when, for example for "special times" and $\sigma = 1/2$, we obtain $\exp(-\gamma z/c) \doteq 0{\cdot}64$, $\langle n_d \rangle \doteq 15$. For chaotic radiation ($\langle W_0 \rangle = \langle n_{ch} \rangle$, $\langle W_0^2 \rangle = 2\langle n_{ch} \rangle^2$) the best fit is for $\langle n_d \rangle = 0$, and the second moment is about 0·8 of the "passive" moment. Slightly different data are obtained within the framework of the generalized Tatarski description [7, 8].

The full curve a in Fig. 2 represents the Poisson distribution with $\langle n_c \rangle = 20$, while the full curves b and c are for the "passive" case with $\sigma = 1/2$ and $\sigma = 3/2$ respectively [9]. The full curves b' and c' represent $p(n, z/c)$ for "average times"; while broken curves $\bar{b}'$ and $\bar{c}'$ are for "special times"; b' and $\bar{b}'$ are obtained from (10) [(14)] with $\sigma = 1/2$ and with the above-given factor $\exp(-\gamma z/c)$ and $\langle n_d \rangle$.

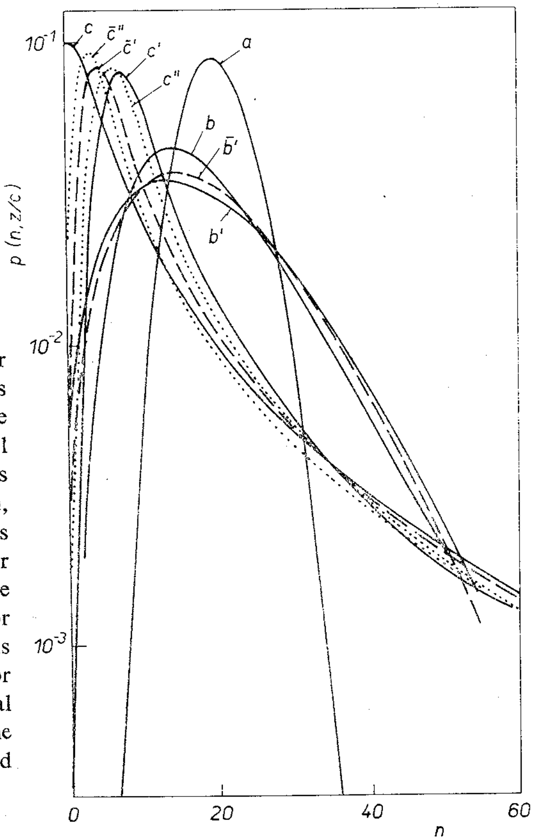

Fig. 2.
The photocounting distribution $p(n, z/c)$ for coherent radiation passing through a lossless active random medium. The full curve a is the Poisson distribution with $\langle n_c \rangle = 20$, full curves b and c represent the "passive" curves obtained from the Diament-Teich description, while full curves b' and c' are "active" curves for $\sigma = 1/2$ and $3/2$, respectively, and for "average times"; broken curves $\bar{b}'$ and $\bar{c}'$ are for $\sigma = 1/2$ and $3/2$, respectively, and for "special times" (the vacuum contribution is neglected). Dotted curves c'' and $\bar{c}''$ are for $\sigma = 3/2$ and for "average times" and "special times", respectively. They correspond to the theory with a damped signal and undamped vacuum noise.

Curves c' and $\bar{c}'$ are for $\sigma = 3/2$, and are obtained from (13) [(15)] with $\langle n_{ch} \rangle = 0$, $\langle n_d \rangle = 0$ and $\exp(-\gamma z/c) = \langle |u_0|^2 + |v_0|^2 \rangle^{-1}$. The dotted curves c'' and $\bar{c}''$ are obtained from (16) [(17)] with $p_0(n)$ a Poisson distribution. They provide the distribution $p(n, z/c)$ for a damped signal and undamped noise with $\sigma = 3/2$ and for "average times" and "special times" respectively, as discussed above. The exact curves should lie between c' and c'', $\bar{c}'$ and $\bar{c}''$ but should be closer to c' and $\bar{c}'$, as is seen in Fig. 1. Some additional shift of the peak of $p(n, z/c)$ to the left and a slight broadening may appear in connection with the approximation discussed in footnote 1, providing a better fit between the "passive" and "active" curves.

Introducing the relative variance of n, $\beta^2 = \langle(\Delta n)^2\rangle / \langle n \rangle^2$ and using (21) with $\langle n_d \rangle = 0$ for simplicity, we obtain $\beta^2 = \beta^2_{\text{quant}} + \beta^2_{\text{class}}$ in the same way as in [7,8], where $\beta^2_{\text{class}} = \langle(|u|^2 + |v|^2)^2\rangle / \langle |u|^2 + |v|^2 \rangle^2 - 1$, $\beta^2_{\text{quant}} = (\langle |u|^2 + |v|^2 \rangle \langle n_c \rangle)^{-1}$;

β^2_{quant} is equal to $1/\langle n_c \rangle$ in a passive medium, where $\langle |u|^2 + |v|^2 \rangle = 1$ and to $(\beta^2_{\mathrm{class}} + 1) \langle |u|^2 + |v|^2 \rangle / \langle n_c \rangle \langle (|u|^2 + |v|^2)^2 \rangle$ in other cases. For $\sigma \to 0$, $\beta^2_{\mathrm{class}} \to 0$ and $\beta^2 \to \beta^2_{\mathrm{quant}} = 1/\langle n_c \rangle$, which is valid for Poisson distribution. These results are comparable with those discussed in [7,8].

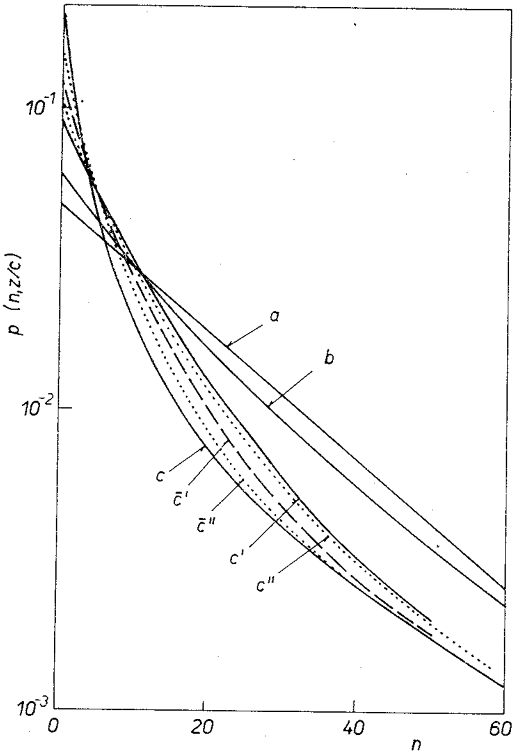

Fig. 3. The photocounting distribution $p(n, z/c)$ for chaotic single-mode radiation passing through a lossless active medium with $\langle n_{ch} \rangle = 20$. The full curve a is the Bose-Einstein distribution, while full curves b and c represent the "passive" solutions obtained in the Diament-Teich description, with $\sigma = 1/2$ and $3/2$ respectively. Full curve c' represents "active" curve for $\sigma = 3/2$ and "special times" (the vacuum contribution is neglected). Dotted curves c'' and $\bar{c}''$ are for $\sigma = 3/2$, and "average times" and "special times", respectively, assuming that the signal is damped and the vacuum noise is undamped. The "active" curves for $\sigma = 1/2$ are similar to the Bose-Einstein distribution a and are not shown.

We see that the "passive" descriptions applied to the propagation of radiation through an active lossless medium can be understood as approximately describing self-radiation, damping, and the contribution of chaotic energy from the reservoir to the field. However, there are some variations reflecting, for "special times", the presence of the self-radiation (self-radiative enhancement and the vacuum noise) and the exchange of energy between the radiation and the reservoir (i.e. damping caused by the flow of energy from the field into the reservoir and the reservoir contribution to the field) in the active medium. For "average times", there is an additional small degree of smoothing. We observe that for $\sigma = 1/2$, the main source of uncertainty is the reservoir; fluctuations in the signal are insubstantial, while for $\sigma = 3/2$ the main uncertainty arises from the signal fluctuations.

In Fig. 3 we present $p(n, z/c)$ in a lossless medium for incident single-mode chaotic radiation. The full curve a is simply the Bose-Einstein distribution with $\langle n_{ch} \rangle = 20$, the full curves b and c represent $p(n, z/c)$ in accordance with the Diament-Teich description as shown in Fig. 2, with $\sigma = 1/2$ and $3/2$ respectively. The "active" curves for $\sigma = 1/2$ are similar to the Bose-Einstein distribution a, and are not shown. The full curve c' and the broken curve $\bar{c}'$ are obtained from (13) [(15)] with $\langle n_c \rangle = 0$ for "average times" and "special times" respectively. The dotted curves c'' and $\bar{c}''$ are obtained from (16) [(17)] with $p_0(n)$ a Bose-Einstein distribution.

In the case of a superposition of coherent and chaotic radiation, the relation of the "passive" and "active" curves in a lossless medium is similar to that of Poisson radiation. The corresponding "passive" curves have been published in [9], Fig. 2b. This is clear from the factor $\exp(-\gamma z/c)$ and $\langle n_d \rangle$ for this case. The "active" curves are slightly broader and the peak of $p(n, z/c)$ is slightly shifted to the left in comparison with the Poisson case.

Of course, if the required parameters $(\langle W_0 \rangle, \sigma, \gamma, \langle n_d \rangle)$ are determined from the first several moments, the theoretical distribution $p(n, z/c)$ can be compared with the experimental distribution. The theory will describe not only a lossless medium where $p(n, z/c)$ has a peak at approximately $n = \exp(-3\sigma^2/2) \langle W_0 \rangle$ for the incident coherent state[2]), but a variety of cases $\langle n \rangle \gtreqless \langle W_0 \rangle$. In some approximation the position of the peak of $p(n, z/c)$ is given by the signal component of (21a), while the chaotic component will serve to broaden $p(n, z/c)$. Thus in lossy active media with $\langle n \rangle < \langle W_0 \rangle$, the peak of $p(n, z/c)$ will approximately lie between the origin and the peak for a lossless medium $(n \doteq \exp(-3\sigma^2/2) \langle W_0 \rangle)$, while for pumped active media exhibiting gain below threshold $\langle n \rangle > \langle W_0 \rangle$, the peak of $p(n, z/c)$ will approximately lie between the peak for the lossless medium and the peak of $p_0(n)$ (or of $p(n, z/c)$ for the non-damping medium, $\gamma = \langle n_d \rangle = 0$). For pumped active media exhibiting gain above threshold, $\gamma < 0$ and $\langle n_d \rangle < 0$ as has been shown. Thus the peak of $p(n, z/c)$ will occur at higher n than for $p_0(n)$. An increase of $\langle n_d \rangle$ generally leads to a broadening of $p(n, z/c)$. Above threshold, there is strong exponential amplification of the incident radiation, with the exception of the self-radiative enhancement; strong exponential broadening of $p(n, z/c)$ occurs because the chaotic component is equal to $\langle n_d \rangle (-1 + \exp(-\gamma z/c))$, with $\langle n_d \rangle > 0, \gamma < 0$, in agreement with the theory of parametric amplification [21].

Finally we note that fixing $\varphi = \ln \mathscr{K}$ ($\mathscr{K} = 1 + 4K'/\omega$, where K' is the coefficient of the nonlinear terms in the Hamiltonian) and considering laser radiation

[2]) This value of n determines the maximum of the lognormal distribution (4); the shift of $\bar{P}(\mathscr{K})$, which is a response to the Fock state in the "passive" descriptions, characterizes shifts of $p(n, z/c)$ for all other statistics of incident radiation.

with $\langle W_0 \rangle \gg 1$, allowing K' to be understood as a c-number classical field replacing the field operator, the theory also describes the quadratic phenomena of nonlinear optics, such as second harmonic generation. As $\langle W_0 \rangle$ is high, the vacuum contribution can be neglected and for ideal laser light $P(W_0) = \delta(W_0 - \langle W_0 \rangle)$. Thus (13) directly yields the Poisson distribution, as has been verified experimentally [20] for second harmonic generation.

The theory considered here, along with its consequences, may be useful for describing the laser and maser amplification process, particularly for long laser amplifiers and high-pressure systems, and also for the detection of astronomical optical radiation traversing a long path in a random medium with transitions at or near the radiated frequency. They may also be useful for photocounting communications in turbulent media [22, 23].

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