Scaling in Heartbeat Rate Variability

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- Markus Feurstein, Wirtschaftsuniversität Wien
- Stefan Thurner, Allgemeines Krankenhaus Wien

Wavelet and Multifractal Analysis Summer School 2004
After Lowen & Teich, 
*Fractal-Based Point Processes*, 
FRACTAL-BASED POINT PROCESSES

- Fractal point processes
- Fractal-rate point processes


ALMOST DONE!

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INTERVAL-BASED MEASURES
CONGESTIVE HEART FAILURE

INABILITY OF HEART TO INCREASE CARDIAC OUTPUT IN PROPORTION TO METABOLIC DEMANDS

Symptom complex:
Many different presentations and etiologies

Typical symptoms:
- Shortness of breath
- Swelling in legs
- General fatigue and weakness

Clinical diagnostics:
- Ascultate heart
- Carotid pulse
- Electrocardiogram
- Chest radiograph

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\[ S_T(f) \propto f^{-\alpha_{sr}} \]

\( \alpha_{sr} \) = scaling exponent

SCALE-INDEPENDENT
TIME-SCALE ANALYSIS
DISCRETE WAVELET TRANSFORM

EXAMINES ALL SCALES
MITIGATES AGAINST NONSTATIONARITIES

\[ m = \text{scale index}; \ 2^m = \text{scale} \]

\[ W_{\psi,\tau}^{\text{wav}}(m,i) = \sum_{k} 2^{-m/2} \psi(2^{-m} k - i) \tau_k \]

\[ \sigma^2_{\text{wav}} \equiv \text{Var}[W_{\psi,\tau}^{\text{wav}}(m,i)] = 2^{-m} \sum_{k} \sum_{l} \psi(2^{-m} k - i) \psi(2^{-m} l - i) R_{\tau}(l - k) \]

\[ A_{\tau}(k) \equiv \text{Var}[W_{\psi,\tau}^{\text{wav}}(m,i)] / E^2[\tau] \]


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HEART-FAILURE

\[ \tau_k = (R-R)_k \]

\[ \tau_{k+1} = (R-R)_{k+1} \]

(a)

INTERBEAT INTERVAL \( \tau_k (\text{sec}) \)

INTERVAL NUMBER \( k \)

(b)

WAVELET COEFFICIENT \( W \)

SCALE INDEX \( m \)

INTERVAL NUMBER \( k \)

(c)

\[ \sigma_{\text{wav}} \]

\[ m = 8 \]

\[ m = 4 \]

\[ m = 2 \]

d)

SMALLER VALUES OF \( \sigma_{\text{wav}} \) THAN FOR NORMAL SUBJECTS
\[ \sigma^2_{\text{wav}}(T) \propto T^{\alpha_{At}} \]

\( \alpha_{At} \) = scaling exponent  

SCALE-INDEPENDENT

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ROBUSTNESS WITH WAVELET FORM


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SPECTRAL ANALYSIS

GENERALIZED–RATE–BASED PERIODOGRAM


\[ S_\lambda (f) \propto f^{-\alpha_s} \]
\[ \alpha_s = \text{scaling exponent} \]

SCALE-INDEPENDENT VLF

HEART FAILURE

\[ 1/f \]

NORMAL

b) SHUFFLED INTERVALS

FREQUENCY \( f \) (cycles/sec)

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DISCRETE WAVELET TRANSFORM

a) CONSTRUCTION OF NORMALIZED VARIANCE $F(T)$


b) CONSTRUCTION OF NORMALIZED HAAR-WAVELET VARIANCE $A(T)$

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$A(T) \propto T^{\alpha_A}$

$\alpha_A =$ scaling exponent

SCALE-INDEPENDENT


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IDENTIFYING PATIENTS WITH CARDIAC DYSFUNCTION

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MEASURES OF STATISTICAL SIGNIFICANCE

- **p VALUE, d’, AND VARIANTS** (rely on Gaussian assumption)

- **SENSITIVITY/SPECIFICITY**

MEASURES OF CLINICAL SIGNIFICANCE (distribution free)

- SENSITIVITY \( \equiv \) proportion of heart-failure patients that are properly identified
  
  e.g., Hypothesis that all normal patients are so identified \( \equiv 100\% \) SPECIFICITY

- **ROC CURVES & AREA UNDER ROC**

After Turcotte & Teich, 

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ROC CURVES & AREA UNDER ROC

SCALE-DEPENDENT $\sigma_{\text{wav}}$ (32)
SCALE-INDEPENDENT $\alpha_{Ar}$

HAAR WAVELET


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\[ 16 \leq 2^m \text{ (scale)} \leq 32 \]

\[ \frac{1}{32} < f \text{ (cycles/interval)} < \frac{1}{16} \]

After Heneghan, Lowen, & Teich
Proc. 1999 ICASSP (Phoenix, AZ)
paper SPTM-8.2.
M. C. Teich 2004
<table>
<thead>
<tr>
<th>Measure</th>
<th>Execution Time (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VLF, LF, HF, and LF/HF</td>
<td>330</td>
</tr>
<tr>
<td>pNN50</td>
<td>40</td>
</tr>
<tr>
<td>SDANN</td>
<td>160</td>
</tr>
<tr>
<td>(\sigma_{\text{int}})</td>
<td>190</td>
</tr>
<tr>
<td>(A(10))</td>
<td>160</td>
</tr>
<tr>
<td>(\sigma_{\text{wav}}(32))</td>
<td>20</td>
</tr>
<tr>
<td>(S_\tau(1/32))</td>
<td>60</td>
</tr>
<tr>
<td>DFA(32)</td>
<td>650,090</td>
</tr>
<tr>
<td>(\alpha_Y)</td>
<td>650,110</td>
</tr>
<tr>
<td>(\alpha_{A\tau})</td>
<td>220</td>
</tr>
<tr>
<td>(\alpha_{S\tau})</td>
<td>920</td>
</tr>
<tr>
<td>(\alpha_A)</td>
<td>610</td>
</tr>
<tr>
<td>(\alpha_U)</td>
<td>570</td>
</tr>
</tbody>
</table>


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GENERALIZED-RATE-BASED PERIODOGRAM

\[ S_\lambda(f) \]

\begin{align*}
\text{FREQUENCY } f \text{ (cycles/sec)}
\end{align*}

a) NORMAL

MODEL

DATA

b) HEART FAILURE

MODEL

DATA


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DOES THE HEARTBEAT REFLECT DETERMINISTIC CHAOS?

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GENERALIZED-RATE-BASED PHASE-SPACE RECONSTRUCTION

a) NORMAL

b) HEART FAILURE


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References


