AUDITORY SIGNAL DETECTION AND AMPLIFICATION IN A NEURAL TRANSMISSION NETWORK

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I. *INTRODUCTION*

A topic as large and as intricate as signal detection poses inevitable frustrations for both reader and writer. The general reader is entitled to clear, unambiguous prose. What he/she gets is an argument sometimes requiring word-by-word translation as though it were written in some long-lost, hieroglyphic script. The reader is often as baffled by what is omitted as by what is said.

The writer, conscious of all the limitations of research, tries to say what is true, but ends up with sentences that are either clear yet subtly wrong, or so carefully hedged as to be barely decipherable. Such dilemmas, we say, are practically unavoidable. They insure that the communication of knowledge in complex areas will always be painful. Each of us must do his utmost to minimize that pain. Accordingly, at the very beginning, we owe our readers a simple account of what we attempted, what we omitted, and why we chose as we did.

Detection theory has become a fixture of modern psychology because it provides useful quantitative tools and an easily understood vocabulary for describing relations that typically occur when a stimulus emerges in some distinctive way from a background. The ROC curve portrays expected trade-offs between false alarms and missed signals when outcomes, stimulus probabilities, or decision criteria, are varied. The detectability measure d', obtained from the ROC curve, adds quantitative precision to the meaning of discriminability.

The idea that there are trade-offs rather than fixed thresholds lies at the heart of signal detection theory. Moreover, the trade-offs are not limited to detection errors. Similar relations exist between d' and decision latencies where speed can be traded off against accuracy.

We cite these familiar landmarks because the reader will search this chapter from beginning to end without finding even one ROC curve. Discriminability measures are developed but they are not labeled d'. The time variable is central to our arguments but it appears as a *sensory integration time* , and we omit all mention of decision latencies. The reader might well ask whether anything can be left of signal detection theory if we admit to having cut its heart out from the very beginning.

The answer is that detection theory has more than one heart. It does not take precisely the same form in all its manifold applications. In this chapter we consider a particular form of the theory dealing with flows of information between the ear and the brain. When the scope of detection theory is narrowed in that way, the theory grows both more powerful and more intricate. ROC curves and the d' measure might not ever be mentioned (although they are always there ready to be computed). This is because other questions are more likely to preoccupy us at first. Such other questions involve attempts to characterize the machinery that processes signals passing through the auditory network.

If you know the energy or amplitude distribution of a signal as it enters the ear, and you also know the detectability of the signal from psychophysical experiments, can you find a consistent mechanism that will transform the one into the other? The configuration of that mechanism, the distortions it introduces, the expected changes from one class of stimuli to another: these are the first questions to which our version of detection theory addresses itself. Over the past fifty years great progress has been made toward answering a number of deep questions about the two major senses: vision and audition. We have written our chapter to give an account of the work in audition, to indicate what we now know about detection, the puzzles that remain unresolved, and finally to suggest new forms for the sought-after processing mechanism.

Ever since Green's (1960) study of intensity discrimination in white noise, together with two closely related papers by Pfafflin & Mathews (1962), and Jeffress (1964), each dealing with sinusoids masked by white gaussian noise, we have been tantalized by the knowledge that a wide range of auditory masking phenomena can be predicted in remarkable detail from the statistics of the stimulus. The discovery suggested that if auditory stimuli could be specified in just the right way, the entire detection problem might be handled without ever leaving the stimulus domain. There would then be no absolute requirement to confront any of the complexities of mechanisms processing information in the ear, the auditory pathways, or the brain itself; or so we once thought. These complexities would instead become details of a well-organized network whose functional characteristics might be better understood from the information passing through it than from meticulous study of its interconnections.

This chapter can also be described as a reluctant but necessary step back from the elusive simplicity of this pure stimulus- oriented approach to auditory detection. Stimuli provide the driving force, but there is no escaping an equivalent need to analyze or at least characterize the transmission network carrying messages away from the ear. We continue to struggle with this sought-after characterization. It has proved to be a formidable challenge although the outline of an acceptable solution is easy to state and ordinarily that is half the battle.

We seek to devise a process that extracts certain critical information from the stimulus and then operates on it in a clearly prescribed way. All inputs must be dealt with in exactly the same way, but outcomes can vary from one class of stimuli to another as dictated by the functional properties of the processing network. The problem is considered solved when the system generates outputs consistent with experimental data on signal detection, and especially when it forecasts results of experiments not yet attempted.

At least three, perhaps four, significant ideas have been suggested for characterizing the auditory detection system. They do not quite represent increasing levels of complexity, although without question recent efforts embody mechanisms not considered in earlier approaches.

A. *Ideal Observers*

The first of these major ideas involves some form of *ideal observer.* The principle is simple. The auditory system is said to extract a particular signal from a particular noise in the best possible way. It is assumed that the processing network must be structured so as to perform the extraction but the essence of the problem is mathematical, not structural. The probability distribution of signal/noise amplitudes is compared with the analogous distribution of noise alone, and a way is found to make decisions leading to the best possible separation.

The apparent simplicity of an ideal observer is illusory because the auditory system must be able to do whatever is demanded. Simplification shows itself only when a limit is imposed on the system, in other words, when the information to which the system can respond is somehow constrained.

The concept of an ideal observer traces back at least to Lawson & Uhlenbeck (1950). Their book summarized the scientific work of the wartime MIT Radiation Laboratory seeking clever ways to extract radar signals from noise and clutter. The ideal observer was a mathematical construct embodying the best possible compromise between missed signals and false alarms. It prescribed an operation on received signals and a decision mechanism that would minimize detection errors. Subsequently Peterson, Birdsall & Fox (1954) improved the idea by altering the test of optimality from minimum error to maximum likelihood ratio. In this form gains and losses from various outcomes become part of the decision calculation. They also calculated optima for a wide variety of signal and noise combinations including many typically used in auditory experiments. For example, they showed that the optimum process for detecting sine wave signals in gaussian noise involves cross-correlating the received and transmitted wave forms, whereas the optimum for detecting intensity changes in bursts of noise would be an energy detector.

Ideal observers experienced a brief blush of popularity in audition during the 1960's, but the vogue soon passed with the realization that the nature of the ideal changes along with the information available to the processing system. Marill (1956) developed a formula for the probability of a correct decision in two-alternative, forced-choice detection of a sine wave signal in gaussian noise. His calculation was based on an ideal observer lacking information on signal phase. It was evident that Marill's formula conformed more closely to auditory data than the Peterson, Birdsall & Fox optimum (although hardly perfect; Marill's psychometric function was some 5-10db better than the typical experimental data). The shape, slope, and signal-to-noise ratio properties of Marill's equation were superior to anything proposed earlier as a model for the detection system. And so for a time Marill's ideal observer (lacking signal phase information) suggested itself as the preferred solution to the detection-system puzzle.

Then it was discovered (McGill, 1968) that Marill's ideal observer is in fact a narrow-band energy detector. As soon as it became clear that an energy detector, which smears its input together without any elaborate analysis, can perform almost as well as much more complexly constructed analytical devices, the ideal observer approach to auditory signal detection was deemed expendable. Moreover, there was an even deeper problem. Any ideal device should experience no difficulty in detecting minute differences in phase-locked pure tones since they have no variance at all. But human listeners find it almost as difficult to separate small intensity differences among pure tones as among comparable bursts of noise. What manner of ideal would classify pure tones, lacking statistics altogether, with random noise?

The difficulty is fundamental. For a time advocates of the ideal observer tried to argue that pure tone generators must also contain a hidden, unmeasured gaussian noise. But Green's calculation showed that this background noise is too weak to affect detectability, and eventually the argument was abandoned (see Tanner, 1961; and Green, 1967).

B. *Neural Counting*

A second major idea for system characterization has had a long and distinguished history. It was pioneered by Hecht, Shlaer & Pirenne (1942) who sought to outline the steps transforming the energy in a flash of blue-green light into psychometric functions describing detectability of the flash by the visual system. This approach applied to audition would generate a neural counting process somewhere in the network leading back from the ear, presumably following an energy exchange in the receptor organ.

Hecht et al. originally defined their system so as to require only the tracing of lost light energy in the ocular media and in the retina itself. Transformations of the light signal other than losses incurred in its passage through the receptor were disregarded. Some years later Barlow (1956) modified the counting process to include additional events not set off by the absorption of light. These additional events were thought to be generated spontaneously in the visual system. They form a background noise level that mixes with stimulus events creating occasional confusions and false alarms. Noise of this kind, independent of the stimulus, is called *additive noise.*

Another type of noise is sometimes found in chains of events such as those occurring when information passes from the periphery to the brain via successive stages. This second type of noise arises when an event at a prior stage of the chain generates a cluster of events at a subsequent stage. Information as it passes from stage to stage increases in volume but not always in the same amount or in the same way. Hence it is said to be perturbed by a noisy process.

In many such cases the noise is traceable directly to the stimulus. It is not additive but multiplicative in nature. The easiest and most reliable way to detect *multiplicative noise* is to observe the proportionality between means and variances of events as they pass from stage to stage. Additive noise displays no such linkages. Both types of noise are currently thought to be important in sensory analysis. Hecht et al (1942) made no provision for either, limiting consideration entirely to losses in transit.

Radiant light energy is conveniently treated as a Poisson counting process. Losses in transit can then be represented as random deletions from the flow of energy, leaving the Poisson distribution intact and changing only its mean level. Evidently Hecht et al. had in mind that absorption of a critical (small) number of light quanta within a circumscribed region of the retina would set off a secondary event of some kind signaling detection provided the critical number is reached or exceeded. This implies that the huge transmission network between the eye and visual cortex can be taken to be essentially inert for detection purposes. Details of the assumed secondary event are evidently important, but not crucial to system characterization in a preliminary treatment of visual detection. Hecht et al. did not speculate on such details.

In this simple form the model has managed to withstand the assaults of time even without sanctification of several of its key points. The possible critical numbers generate successive cumulative gamma distributions establishing a template for judging visual psychometric functions on a log-energy plot. Barlow (1956), and Nachmias & Steinman (1963), showed that the experimental data are compatible with an unmeasured additive internal noise ("dark light"), as well as with critical numbers set by instructions, *i.e.* flexible high level decision criteria rather than fixed neural summation rules. Subsequently, Barlow, Levick and Yoon (1971) found evidence of a multiplicative noise — noise associated with neural cascade effects in the retina. This suggested to Teich, Prucnal, Vannucci, Breton and McGill (1982a) that weak neural signals following registration of light quanta in the eye might be multiplied (*i.e.*, amplified) as they pass to higher centers. Such stochastic multiplication generates variance or spread in the counting distribution. It is not the big bang assumed by Hecht et.al. because it also multiplies spontaneous firing not directly linked to light energy absorption. The Teich et al. result is best described as a noise typically found in amplifier devices.

These alterations do little to diminish the power of the Hecht et al. principle that analyzing visual detection requires us to begin with the energy distribution of the stimulus, and then step by step to distill out of it the detection curve of the eye. Emphasis on light *energy* as the key information variable passed to the detection network reflects a host of energy relations involving both area and time found to obtain near absolute threshold. Furthermore, the Hecht et al. counting model leads immediately to a square root law of intensity discrimination which characterizes all simple Poisson processes (McGill, 1971). This square root law, the deVries-Rose Law, is in fact found in visual brightness discrimination under suitable conditions in the intensity region just above absolute threshold (see for example, M.A. Bouman, 1961).

The unique attraction of the Hecht et al. system network is its testability, or so one might think. The psychometric function of the visual system is said to be fully determined by the statistics of the stimulus. To test the model you simply change the probability distribution of the stimulus and see whether the psychometric function conforms to the change. Unfortunately tests are easier to conjecture than to carry out. No readily available light source has yet to yield anything other than Poisson energy statistics at low output levels near the visual threshold. When lasers first came into common use as laboratory instruments their energy output was investigated and found to have a non-central negative binomial form (see, for example, Magill & Soni, 1966; Teich & McGill, 1976). It was conjectured at first that varying a laser's intensity during the flash period might permit manipulation of the energy statistics of low intensity flashes. Teich et al.(1982a) performed just such manipulations and showed alterations in the resulting psychometric function, but the statistics of the source proved to be superpositions of Poisson counting processes with different mean values.

Similar conjectures reflecting similar hopes have been voiced recently for sub-Poisson (photon-number squeezed) light sources (Teich et al., 1982b; Teich & Saleh, 1988), which cannot be represented as Poisson superpositions. But the development of reliable non-classical light stimuli as practical visual instrumentation is still a distant prospect. We shall have to see.

As with ideal observers there is also a deeper problem affecting all versions of the Hecht et al. system network. The problem in this case has to do with the ubiquity of the Poisson distribution. Hecht et al. argue that absolute visual threshold data closely approximate cumulative gamma distributions. Consequently there must be an underlying Poisson counting process and the statistics of the light source must therefore be heavily implicated. But the causal chain is not nearly as solid as Hecht et al. claimed it to be. Many additive combinations of signal and noise yield nearly the same psychometric function when plotted against signal energy. It was this phenomenon that caused Barlow (1956) to conjecture the existence of "dark light".

Moreover, several important theorems were developed after the 1950's dealing with superposition of point processes (see, for example, Cox & Smith, 1954; Drenick, 1960). These findings suggest that under suitable conditions repetitive point-like events flowing down independent channels will approach a Poisson limit when the number of channels increases and the events are smeared together, *i.e.* superposed. Comprehensive study of such superposition effects was undertaken shortly thereafter and summarized in a well-known chapter by Cinlar (1972). This more complete treatment shows that fully Poisson behavior is an abstraction not easily achieved, but quasi-Poisson processes are commonplace. The most familiar examples are the flow of traffic on a super highway or the arrival of telephone calls at a switchboard. Each of these is a completely deterministic sequence yet each exhibits Poisson behavior when the nittygritty details of the sequence are either unknown or ignored.

The important point here is the close resemblance of such superposition effects and the smearing operations characteristic of energy detection. Both imply that information on crucial details of the sequence is ignored or lost. Accordingly, virtually any stimulus, including many that are nonstatistical, can give rise to quasi-Poisson counting processes in the sensory nervous system. The chief requirement is that information be lost or subtracted. A primitive observer located somewhere in the network would see a deterministic flow of data passing to higher centers as though it were a random process. This means that Poisson-like behavior might be found nearly everywhere in sensory tracts if the observing mechanism were sufficiently ignorant of details of the information flow.

By a feat of legerdemain we have passed quickly from ideal decision-making to an extremely limited observing system lacking the ability to analyze information passing through it. Yet even this primitive mode of operation does no serious violence to the data on visual or auditory signal detection. The latter seem to require a mass flow of information instead of the structured signals that generate typical percepts. Indeed, it may be the same information looked at in two different ways. The first ten thousand digits of *pi* make an excellent random number table if we start with, say, the tenth digit. But inserting the first nine provides a clue enabling us to calculate each and every digit in turn. The sequence is deterministic, yet to a primitive observer it appears random (Wolfram, 1985). Everything turns on the discriminations available to the observer.

The fact that visual threshold data point to an underlying Poisson counting process does not tell us where the Poisson behavior is located or what its nature might be. This is the distinction that Barlow (1957) exploited in taking the system a step beyond Hecht et al.'s original prescription into neural counting only partly driven by the statistics of the stimulus flash.

C. *Differential analyzer*

A third major idea on system characterization is due to Laming (1986) whose work revived serious interest in sensory networks. Laming notes remarkable functional differences when signal and noise are delivered in separate stimulus "packages" separated by a blank interval, from cases in which the signal is added to one of two or more observation intervals marked out on a continuous noisy background. These differences eventually led Laming to consider jettisoning energy exchanges and counting models, turning instead to a cleverly constructed system that forms a running derivative of incoming stimulation.

A steady background stimulus in a sensory tract will then fade out leaving a residual zero-mean gaussian noise. This background internal noise is continuously monitored for transients marking the beginning and end of any brief stimulus. Moreover, Laming's network responds to stimulus amplitude rather than energy. When a transient appears following the onset of a stimulus, it is detected as a sudden change in the variance of the residual noise amplitude distribution. This may, and perhaps does, involve some form of counting, but omitted from Laming's formulation is any one-to-one linkage between stimulus energy (or amplitude) and a proportional counting process in sensory neurons behind the receptor.

Laming's network successfully predicts a hitherto unexplained finding thought to be critically important in auditory detection. Green's study of noise intensity discrimination (1960) displays psychometric functions having roughly double the slope generated by an equivalent energy detector. Since in all other respects Green's data on noise bandwidth and duration reflect the expected behavior of an energy detector, these slopes were regarded as an anomaly until Laming showed that they might point to the nature of the detection process, and that in fact his mechanism generated slopes consistent with the data whereas simple energy detection does not.

D. *Amplifier Networks*

The work to be reported here is a less fundamental departure from conventional detection theory than is Laming's differential analyzer. Our primary effort is aimed at the auditory system because of the latter's diversity of potential signals with widely varying statistical properties, mostly all now fairly well understood. But it is evident that we remain strongly influenced by the Hecht-Barlow visual system formulation: an energy exchange in the receptor triggering off and driving a counting process in the transmission network. Basic departures from earlier ideas will be evident in new properties assigned to the network, making the latter less of a passive transmitter, and more of an active detector.

Our transmission network is asked to amplify weak signals and at the same time to curb or diminish strong ones. Amplification is readily achieved by structuring the transmission as some form of birth-death-immigration process. The latter is a class of well-known stochastic processes in which each stage of the process may see a net increase in the number of events over prior stages. Events tend to multiply as they pass up the system. Active processes of this kind are a bit more complicated than Poisson counting, but far less sophisticated than the neural networks typically invoked to explain advanced forms of cognition (see, for example, Hopfield, 1982; Rumelhart, Smolensky, McClelland and Hinton, 1986).

Intense inputs must somehow be curbed by the network. Otherwise they would grow exponentially in the multiplicative cascade defining a birth process, leading to explosive chain reactions that could saturate and ultimately paralyze all transmission. A stochastic birth process will undergo exponential growth with any input. This outcome is delightful when inputs are small, but disastrous when they are large. Some kind of limiting action is a virtual necessity. This means that simple energy exchanges in the receptor must be ruled out. We look instead for an energy transformation, either a log or a fractional power function, or perhaps some combination of both, either at the input or perhaps at many stages of the process. Counterbalancing a transformed input signal against the exponential growth of neural activity expected from a birth-death process can generally be achieved so as to guarantee that information passes up the network essentially in linear form, but with weak signals amplified and strong ones constrained.

Testing or verifying these conceptions requires studying the output of typical birth-death-immigration networks in order to measure agreement with the psychophysical data we are trying to understand. Such conformity has been the goal of system characterization efforts from the very beginning. But ultimately we must also judge each proposed characterization by considering anatomical and neurophysiological evidence as well. Any such review would be premature until the principles governing the transmission network are reliably known. For now we observe only that the lower levels of both the auditory and visual systems seem designed to amplify weak signals, and that the envelope of activity set off by any given input signal appears to expand as messages pass to successively higher centers (see, for example, Ryan, Wolf, and Sharp, 1982; Saleh & Teich, 1985). Finally, studies of firing rates in auditory transmission generally show linearity with log intensity. This is what attracts attention to birth processes.

The proposed network, once designed, must be able to process pure tones, sine waves in noise, pure noise, so-called "frozen" noise, and a host of other inputs, operating on each in exactly the same way, and generating outcomes that correspond closely to the rich vein of experimental data guiding all current efforts to build an auditory detection theory. Such powerful constraints practically guarantee that even the cleverest characterizations will eventually fail. Our object is not to avoid failure, but rather to achieve interesting and significant failures pointing the way to new knowledge.

II. *DETECTION THEORY IN THE STIMULUS DOMAIN*

A. *Signal Energy Distributions*

We begin with a brief review of what is currently known about the energy statistics of typical auditory stimuli. Energy distributions are required as the first step of our analysis since we start with the idea that some kind of energy transformation occurs in the ear, triggering off and driving an active counting process in the transmission network leading to higher centers. Such distributions, or their close relatives, also form the heart of all stimulus-oriented detection theories.

It is startling but true that despite extensive knowledge of the statistical properties of gaussian noise developed in early studies of Brownian motion, and elaborated subsequently by S.O. Rice (1944) in his classical papers on mathematical analysis of random noise, very little information had been codified prior to 1960 on the energy distribution of acoustic sine waves in noise. All the necessary information existed, but physicists and engineers displayed little interest in the relatively wide bandwidths and long durations typical of auditory stimuli. Hence, signal detection theorists were forced to work out the answers for themselves. The principal results can be found in Green (1960), Jeffress (1964), McGill (1967), Green & McGill (1970), and Teich & McGill (1976). Important details of the proofs cited in this section, as well as in section III, can also be found in these papers.

The energy level of a pure tone is found by integrating the squared displacement of a sine or cosine waveform over its duration. If the duration is taken as T this works out to be:

$$Energy = \frac{1}{2} \, Amplitude^{\,2} . T , \tag{1}$$

where amplitude is the distance from zero displacement to the peak of the waveform. Basic measurements of this sort are usually difficult to perform. We generally measure energy or power (energy/time) relative to some convenient standard. Of course, a pure tone of center frequency ω_0 and fixed duration is no longer pure. The energy spectrum of a short duration sinusoid has the form

$$sinc^{\,2}\,(\omega - \omega_0)\,T = \frac{\sin^2 (\omega - \omega_0)\,T}{[(\omega - \omega_0)\,T]^2},$$

where ω is the frequency variable and the zero points of the spectrum are displaced from the center frequency at successive multiples of $\pm 1/T$. This spread of energy across frequency is infinite but falls off very rapidly when the duration of the signal is sufficiently long. Hence, the spread may be conveniently represented by its *equivalent rectangular bandwidth, i.e.*, a rectangular spectrum having the same peak energy as the $sinc^2$ spectrum and a bandwidth of 1/T. About 77 percent of the total energy in the $sinc^2$ spectrum is found within these limits, but the equivalent rectangular band has an energy content identical to that of the entire sinusoid. The reasoning that leads us to define things this way becomes clearer when we consider gaussian noise. It will turn out to be the case that equivalent rectangular bands are mutually orthogonal and thus completely additive. Hence it is a simple matter to construct a wideband noise out of such narrowband units. Notice that pure tones having the same amplitude and duration will always have exactly the same energy content. There is no probability distribution.

In the case of a pure tone embedded in a narrowband gaussian noise, *i.e.*, noise having a rectangular bandwidth as defined above, there will be a probability distribution of the signal/noise combination. The energy distribution is given by its density function:

$$f_s(x) = \frac{1}{N_0} \exp\left[-\frac{E+x}{N_0}\right] \cdot I_0\left[2\,(Ex)^{\frac{1}{2}}/N_0\right], \qquad o \leq x \leq \infty. \tag{2}$$

In this somewhat forbidding expression x is the energy in the signal/noise combination, E is the energy of the added sine wave signal, and N_o is the average noise energy in the unit rectangular band. The symbol $I_o[\cdot]$ represents a zero-order Bessel function with purely imaginary argument. This notation was originally devised by Rice (1944) and is found in many engineering textbooks on signal detection. We reproduce it here out of deference to such history, but it is cumbersome. As we shall show in the argument leading to equation (8), equation (2) is purely real. In fact, it turns out to be a linear transform of non-central chi-square. Rice derived the signal/noise process as an amplitude distribution. We change the variable to energy according to the rule in equation (1).

The analogous narrowband energy distribution of pure noise is obtained by setting E=O in equation (2) and noting that $I_o(O)$=1:

$$f_0(x) = \frac{1}{N_0} \exp\left[-\frac{x}{N_0}\right]. \tag{3}$$

This is the famous Rayleigh energy distribution of narrow band (pure) noise. Strictly speaking, equation (3) is not a Rayleigh distribution (which describes amplitude fluctuations in a narrow-band noise) but rather an exponential distribution. It is the distribution of the square of a Rayleigh variable. They are close relatives, each one determining the other. Suitably standardized equation (3) is a central chi-square distribution with 2 degrees of freedom. The gaussian distribution of noise perturbations produces a chi-square distribution of noise energies. The latter has two degrees of freedom because samples of noise extending over several cycles of the waveform are constructed out of two independent sinusoidal components (two quadratures).

B. *Forced Choice Psychometric Functions*

The pair of energy distributions, signal in noise and pure noise, can now be used to define a psychometric function in two-alternative forced choice (2AFC) procedure. To do it we need only calculate the probability that the energy in the signal/noise combination exceeds the energy of the pure noise. An energy detector will always choose the larger sample as the signal. Hence the probability of a correct judgment must be:

$$P\ (S > N) = \int_0^{\infty} f_0(x) \left[1 - F_s\ (x)\right] dx. \tag{4a}$$

Equation (4a) states a simple probability argument. First choose a noise and compute the probability that the signal/noise energy combination is greater. Then integrate over all possible noise samples. The outcome is a remarkably simple formula that proves to be Marill's (1956) ideal observer for the case in which signal phase information is missing:

$$P\ (S > N) = 1 - \frac{1}{2} \exp\left(-E/2N_0\right) \cdot \tag{4}$$

Our argument, of course, establishes Marill's observer to be an energy detector. The results are useful for displaying our methods in simple form. They also have considerable theoretical interest, but as a practical matter not much has been done with narrowband noise. Ronken (1969) constructed narrowband noise stimuli by computer techniques and studied the listener's 2AFC psychometric function. His data are interesting chiefly because they suggest an internal noise of undetermined origin shifting the psychometric function above the energy detector, *i.e.*, something less than ideal performance. But the data do not permit any critical test of the energy detection hypothesis.

C. *Wide-Band Noise*

To accomplish the latter we need to go to wideband noise. We have already noted that it is easy to construct such noise from narrowband components since equivalent rectangular bands are mutually orthogonal and additive. First compute the moment generating function of equation (2):

$$M_x(\theta) = \int_0^{\infty} e^{\theta x} f_s(x)\, dx, \tag{5}$$

$$= \frac{\exp\,(E\ \theta/1-N_0\,\theta)}{(1-N_0\,\theta)}\,.$$

Then go to the convolution of ν such contiguous narrowband signal-noise processes:

$$M_x(\theta) = \frac{\exp\,[\,(E_1+E_2+E_3+\cdots\ E_\nu)\,\theta\,/\,1-N_0\,\theta]}{(1-N_0\,\theta)\,(1-N_0\,\theta)\,(1-N_0\,\theta)\ \cdots\ (1-N_0\,\theta)}\,, \tag{6}$$

$$= \frac{\exp\,(E\,\theta/1-N_0\,\theta)}{(1-N_0\,\theta)^{\nu}}\,.$$

The term E now represents the sum of the signal energies in each of the narrowband components, a result that clearly establishes the smearing or superposition action of an energy detector. In theory the placement of the signal within the noise band is irrelevant. In practice it is usually added to a single component at the center of the noise band. This is because the mathematical model is only a rough approximation of the actual spectrum. We get our best approximation when the signal is at the center of the noise band. The width of the latter is now constructed as an equivalent rectangular band W in which each component unit is 1/T Hz wide. Hence:

$$W = \nu \cdot 1/T, \tag{7}$$

$$\nu = W\ T, \qquad \textit{where}\ \nu = 1,2,3\cdots$$

We have in fact constructed an orthogonal partition of the energy spectrum of a band limited noise. Inverting the generating function in equation (6) produces the density function of that noise:

$$f_s(x) = \sum_{i=0}^{i=\infty} \left[w_{E/N_0(i)} \right] \cdot \left[\frac{\frac{1}{N_0}\left[\frac{x}{N_0}\right]^{i+\nu-1} \exp\left[-\frac{x}{N_0}\right]}{\Gamma(i+\nu)} \right] . \tag{8}$$

In equation (8) Bessel function notation has been abandoned and we introduce instead Poisson weights operating on classical gamma distributions:

$$w_{E/N_0}(i) = \frac{(E/N_0)^i \exp(-E/N_0)}{i!} .$$

Equation (2) can be expressed in this form by setting $\nu=1$ in (8) above. The notation appears different but in fact portrays an identity. The representation in equation (2) is found in the engineering literature. Equation (8) is typical of the statistical literature. We show the identity because it is sometimes necessary to translate results in one notation into the counterpart form in order to follow a complicated argument.

The probability density of wideband pure noise energy is found by setting $E=O$ in equation (8):

$$f_0(x) = \frac{\frac{1}{N_0}\left[\frac{x}{N_0}\right]^{\nu-1} \exp\left[-\frac{x}{N_0}\right]}{\Gamma(\nu)} . \tag{9}$$

This is the equivalent of the Rayleigh noise energy distribution but for a wideband process with 2ν degrees of freedom. Using equations (8) and (9) we can then construct the 2AFC psychometric function for the wide band signal/noise combination against pure noise of the same bandwidth. It proves to be:

$$P(S>N) = 1 - \sum_{i=0}^{i=\nu-1} \left[w_{E/2N_o}(i) \right] \left[\sum_{j=0}^{j=\nu-i-1} \begin{bmatrix} 2\nu-1 \\ j \end{bmatrix} (1/2)^{2\nu-1} \right] . \tag{10}$$

Again, when $\nu=1$, equation (10) reduces to Marill's formula. Notice that the infinite sum in equation (8) has disappeared. The summation trick by which this is done is explained in McGill (1968b, p.373).

With wideband pure noise, equation (9) also leads to a similar 2AFC comparison of a noise level N_s containing a slight energy increment, against a background level N_o in which the increment is omitted.

The psychometric function based on the prescription established in equation (4a) is a partial binomial sum:

$$P\ (N_s > N_o) = \sum_{j=0}^{j=\nu-1} \begin{bmatrix} 2\nu-1 \\ j \end{bmatrix} \left[\frac{N_o}{N_o+N_s} \right]^j \left[\frac{N_s}{N_o+N_s} \right]^{2\nu-1-j} . \tag{11}$$

In principle, equations (10) and (11) are exact psychometric functions. They make it easy to construct templates determined by successive values of ν. The calculations are programmed on a desktop computer or a handheld programmable calculator. Our computations for the energy distributions involving sine waves in bandlimited noise and wideband noise intensity discrimination, are given in Figures 1 and 2. These are templates, similar to those provided by Hecht et al (1942) for noiseless visual detection enabling the researcher to estimate key parameters from data. On the basis of such comparisons typical psychometric functions for signals in noise lie near ν=10 whereas for noise intensity discrimination ν is estimated to be roughly 100. Original versions of these templates were developed by Green & McGill (1970) using a variety of approximations. Modern computing methods encourage precise calculations, and these are embodied in the updated figures.

D. *Estimated Bandwidth*

Equations (10) and (11) both obey Weber's law. If the degrees of freedom parameter remains constant, fixing the signal to noise ratio also fixes detectability---Weber's law. We rarely know much about the degrees of freedom parameter. Ordinarily it is determined by the ear's internal bandwidth and integration time, and is generally estimated from threshold detectability at the seventy-fifth percentile point of the 2AFC psychometric function. Thus for sine wave signals in wideband noise:

$$\hat{\nu} = 1.099 \left[E_{75/N_o} \right]^2 - \left[E_{75/N_o} \right], \tag{12}$$

where E_{75} / N_o is the threshold signal-to-noise energy ratio and $\hat{\nu}$ is the estimated degrees of freedom parameter. Jeffress (1968) obtained values for ν ranging from 5-10 by working in this way.

The small size of the parameter indicates a relatively narrow bandwidth (see equation (7)). This takes us directly into calculation of the ear's critical bandwidth. It is not a simple problem because signal durations with which listeners feel most comfortable (500-1000 msc) are somewhat longer than the effective stimulus determined by the ear's tonal integration time. We can (and often do) estimate it (using values of the order of 100-200 msc), but the threshold variable we typically measure, stimulus power, and the parameter we typically estimate, ν , are insufficient to pin down the ear's critical bandwidth. Estimates of the latter based on the best fit of the experimental data to the template in Figure 1 typically run from 50HZ-150HZ at a center frequency of 1000HZ. Bandwidth estimates will increase systematically with assumed integration time for a fixed value of threshold power. Their product determines ν and so the entire system is seen to be heavily constrained.

The fact that the energy statistics of signals in noise turn out to have so much explanatory impact in good agreement with the experimental data is simply remarkable. Clearly we are onto something, and the reader can now appreciate the excitement among auditory researchers when these relations were first discovered.

Similarly, in pure noise intensity discrimination:

$$\hat{\nu} = \left[\frac{N_{75}}{N_o} - 1 \right]^{-2} . \tag{13}$$

The typical measurement of N_{75} / N_o at threshold in studies of noise masking proves to be approximately 1.10 (see Miller, 1947; Green, 1960). This number inserted into equation (13) generates an estimate of ν of the order of 100. The bandwidth is much wider, as might have been expected, but perhaps not wide enough to make us really comfortable. The masking band for noise should be wide in order to produce sharp discriminations. The wider the band, the more acute is the discrimination if the stimulus model is accepted at face value. Moreover, the experimental slope of the psychometric function in noise intensity discrimination is steeper, roughly double that of the template at $\nu = 100$ in Figure 2. Green's (1960) measurements found this slope anomaly, and Laming's (1986) theoretical work successfully explained it. The ear's processing of pure noise behaves in many ways as we might expect if we are dealing with an energy detector, but the outcomes are not quite right, at least when we restrict ourselves to the simplest forms of energy detection requiring no transformations.

Such problems are multiplied when we turn to pure tones and "frozen" (non-stochastic) noise. Neither class of stimuli exhibits any statistics at all. One is narrowband, the other broadband. Yet, listeners have clear difficulties with both, and in effect treat frozen noise as though it were fully stochastic when masking another noise (Raab & Goldberg, 1975). In fact, if off-frequency listening is controlled, even pure tones seem to follow Weber's law in the region 30db or more above absolute threshold (Viemeister, 1972; Moore & Raab, 1974). Evidently Weber's law emerges from the dynamics of the detection process in these important test cases.

III. *DETECTION THEORY IN A PASSIVE POISSON NETWORK*

The mixed bag of successes and nettlesome problems resulting from stimulus-defined treatments of auditory energy detection, led us some time ago to consider the properties of primitive transmission networks. The first attempt (McGill, 1967) employed stimulus energy to control the mean rate of a Poisson counting process linked to its driver by a linear connection somewhere in the ear. The attempt produced a number of interesting results, including one or two that turned up in other contexts, and it generated one important insight.

We found that the Poisson transmission line is relatively transparent to signal/noise distributions of driving energy. The counting distribution, though a discrete point process, has essentially the same detection properties as the driver (except, of course, for pure tones and other non-stochastic stimuli). Hence, Poisson transmission inherits most of the successes and most of the headaches of signal detection based on energy analysis of the stimulus. Our expectation was that a Poisson transmission line would also help us deal with a number of vexing problems posed by non-stochastic stimuli. It did that but in the end it could not cope with new data on pure tone intensity discrimination. Specifically, we expected that a Poisson transmission line would introduce statistical fluctuations into pure tones and thus help to explain pure tone intensity discrimination. When experiments on the latter were done, the theory looked reasonably good, but the masking curves did not have the slope forecast by a Poisson process.

A. *Noncentral Negative Binomial Distribution*

In order to clarify these points, consider what happens when the wideband signal/noise energy distribution (equation (8)) drives a Poisson counting process. First, the m.g.f. of a Poisson with a mean count of ax:

$$M_{ax}(\theta) = \exp\left[ax\,(e^{\theta} - 1)\right].$$

Here x is a particular (driving) energy and a is a constant matching energy units to counting. There is no real transformation in the system, just proportional counting. Then drive this Poisson process by the signal/noise energy distribution in equation (8):

$$M_j(\theta) = \int_0^{\infty} f_s(x)\, M_{ax}(\theta)\, dx\ , \tag{14}$$

where j will now take on discrete values $0,1,2,\ldots,\infty$; and $M_j(\theta)$ is the generating function of a doubly stochastic counting distribution (*i.e.* two random processes in sequence, the first driving the mean of the second). After suitable manipulation we arrive at a new generating function constructed as a weighted sum of negative binomial m.g.f.s. With this insight we conclude that the sought-after distribution may be a Poisson-weighted sum of negative binomials - a discrete form of the stimulus energy distribution. The conclusion is tested and checks out. Accordingly we find for the counting probability $p_s(j)$ that exactly j counts are recorded during the ear's integration time:

$$p_s(j) = \sum_{i=0}^{i=\infty} \left[w_{E/N_o}(i)\right] \cdot \left[\binom{i+j+\nu-1}{j} \left[\frac{1}{1+aN_o}\right]^{i+\nu} \left[\frac{aN_o}{1+aN_o}\right]^{j} \right] . \tag{15}$$

The distribution in equation (15) may be described as a non-central negative binomial (Ong & Lee, 1979). It was also developed by Perina (1967) to depict laser-flash energy (ie. photon counting) distributions when a sinusoidal (coherent) signal is embedded in a broadband gaussian noise of the same center frequency (see Teich & McGill, 1976). In such physical applications equation (15) is called the Poisson transform of equation (8).

Equation (15) is no idle curiosity. If E is set equal to zero we get the counting distribution of pure noise under the assumptions we have imposed. Moreover, given these two expressions we can then find the detection law of the counting process in the transmission line. It proves to be (McGill, 1967):

$$\frac{1}{v^{1/2}} \cdot \frac{E_{75}}{N_o} \approx c \tag{16}$$

at threshold, where c is a constant not far from unity. This is Weber's law, and virtually the same result as generated directly from stimulus energy.

When the driving energy distribution in equation (14) is replaced by a delta function, the counting distribution of any fixed-energy stimulus is seen to be the Poisson superposition distribution of the transmission network. So if the latter were an appropriate model of auditory transmission, pure tones and sine waves in frozen noise would have essentially the same square-root detectability as the deVries - Rose Law of visual intensity discrimination (see McGill, 1971). This led to considerable experimentation in the 1970's, producing clear-cut results that rejected either linear coupling from energy to counting, or the whole idea of a Poisson network; and, in all likelihood, both.

B. *Failure of the Passive Poisson Network*

Research showed that when the energy in a pure tone, E_s, just discriminably different from a weaker tone, E_o, is plotted against the latter in log coordinates over a range of E_o from about 30db above threshold to 80db, the result is a straight line with a slope of about .9 in log coordinates. It is certainly not .5 as a Poisson process would require, but neither is it Weber's law as, for example, in equation (16) (see

McGill & Goldberg, 1968 a & b). This oddity set off another flurry of research to determine just what might be going on. In the outcome, experiments revealed that when spread of excitation in pure tones is controlled by using weak masking noise with a notch or gap surrounding the test frequency, the near-miss disappears and unit slope emerges. (Viemeister, 1972; Moore & Raab, 1974). Masking with frozen noise generates Weber's law purely and simply (Raab & Goldberg, 1975). Nothing in the theory of an energy-driven Poisson transmission line prepares us for such results.

The failure of the theory could hardly be clearer or more interesting because it suggests that the auditory transmission network must somehow be imposing a Weber's law format on non-stochastic input signals. Step by step the interplay of theory and experiment has pushed us into withdrawal from the simple auditory adaptation of the Hecht-Barlow mechanism with which we began. Sensory psychology's classical dictum, which holds that all problems reduce ultimately to a proper definition of the stimulus, grows increasingly difficult to defend in auditory detection unless we are prepared to search for the stimulus in the heart of the transmission network. Alternatively, we might proceed by redefining the transmission system to allow for some form of amplification. The latter will sometimes generate Weber's law detectability even when non-stochastic stimuli provide the inputs to the network. In the next section we show how this happens.

IV. *DETECTION THEORY IN AN AMPLIFYING NEURAL TRANSMISSION NETWORK*

A. *Birth-Death Stochastic Process*

Consider the ear and its transmission network as a mechanism for detecting weak signals by cascade multiplication or amplification over an undetermined number of stages. Multiplication is achieved via a stochastic birth-process, one which at each stage gives birth to some multiple >1 of the neural events it receives as inputs. At each stage there are also losses proportional to the ongoing level of activity. Thus, as the signal message passes through the system it is continuously transformed, growing within the network by the algebraic sum of such gains and losses.

The network is designed to amplify weak signals and limit intense signals. Otherwise the latter would grow exponentially paralyzing transmission in the multiplicative cascade. The trick here will be to counterbalance a transform of the driving stimulus against the expected exponential growth of the neural message in the network, so that the average size of the message remains roughly linear with stimulus energy; or remains some predetermined function of stimulus energy.

In a classical birth-death stochastic process, the probability of a specific count is defined by a well-known differential equation (see Bharucha-Reid, 1960 p.87):

$$P'_n(t) = -(\lambda_n+\mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t), \qquad n = 1, 2, 3, \cdots \tag{17}$$

$$P'_o(t) = -\lambda_o P_o(t) + \mu_1 P_1(t),$$

The subscript n refers to the count at time t while λ_n and μ_n are the gain and loss parameters associated with state n. The derivative in (17) is taken with respect to time. We are saying that if the count is n at time $t+\Delta t$, it got there either because it was already there at time t and failed to change during Δt (first term); or it was a step below in state n-1 and went up (second term); or it was a step above in state n+1 and came down (third term).

These constraints portray a narrowband stimulus message whose magnitude performs a random walk ending at time $t+\Delta t$ in a count that may be above or below its value an instant earlier.

To simplify matters further, and to introduce the idea of stage by stage multiplication, we further define:

$$\lambda_n = n \cdot \lambda, \tag{18}$$

$$\mu_n = n \cdot \mu.$$

Equation (18) implies that the tendency to multiply increases linearly with the count (not time), and losses are proportional to the count. This is stochastic multiplication typical of amplifier networks. We also take state zero, *i.e.* a count of zero, to be an *absorbing state*. Transitions up from zero are forbidden:

$$\lambda_o = 0 \,, \tag{19}$$

$$P'_o\,(t) = \mu\,P_1\,(t)\,.$$

Our last set of restrictions embodies the idea that a flux of neural events set off in the ear can die out. If the counting process should ever drift into state zero, the message is lost. Finally, we put a subscript x on λ and μ to indicate that the initial size of gains and losses in transmission are functions of the input intensity x. The multiplication rate (or loss rate) depends both on the driving energy and the current state of the count.

Driving energy is labeled x. It may be either stochastic (narrowband noise), or deterministic (tones; frozen noise). The effective level at the ear is $a\,x$ where a is taken to be constant. The central idea of a counting model is that effective energy drives an impulse record of the stimulus. The relation between stimulus energy and expected impulse count is taken to be:

$$ln\;(1{+}ax) = (\lambda_x - \mu_x)\cdot t \tag{20}$$

This is an energy transformation. The variable x measures stimulus energy and thus has a time dimension built into it. The parameter t on the right hand side of equation (20) is the running time of the counting process. Generally it is longer than the duration of the stimulus. Energy accumulates on the left hand side of the equation and a counting record of the energy accumulates on the right hand side. The log transform keeps the network linear despite its tendency to amplify whatever it sees. Parameters λ_x and μ_x are the gain and loss rates of the auditory network when the driving energy is x.

In classical branching theory t is allowed to increase without limit and a fundamental theorem of branching states:

$$\lim_{t\to\infty}\left\{\begin{matrix}\textit{state zero}\\ \textit{probability}\end{matrix}\right\} \Rightarrow \mu_x \,/\, \lambda_x = \pi\,(x)\,, \tag{21}$$

where $\pi(x)$ is the so-called "extinction" probability (message lost). We do not in fact operate at this limit because the auditory system is thought to have a fixed, relatively short integration time. Accordingly, the extinction probability appears as a parameter of the network.

If the stimulus is weak, a substantial likelihood exists that the message will fail to get through whereas intense signals should be detected with a probability close to unity. Thus $o \le \pi(x) \le 1$.. The limit will be near 1 when stimulus energy approaches zero, and near zero when x grows very large.

B. *Linear Birth-Death Process*

With these definitions and restrictions we can now write down the counting probability driven by an input of intensity x.

$$P(0) = \frac{\pi(x)\cdot\gamma(x)}{1+\gamma(x)}, \tag{22}$$

$$P(j) = [1-P(0)]\cdot\left[\frac{1}{1+\gamma(x)}\right]\left[\frac{\gamma(x)}{1+\gamma(x)}\right]^{j-1}, \qquad j=1,2,3\cdots .$$

In equation (22) $\pi(x) = \mu_x / \lambda_x, \gamma(x) = \dfrac{ax}{1-\pi(x)}$. Evidently this type of linear birth-death process gives rise to a modified (because of P(o)) Bose-Einstein counting distribution (see Feller, 1957 p.59). In fact, if μ_x is taken to be zero, then $\pi(x) = o$, and we have a pure birth process in which

$$P(j) => \left[\frac{1}{1+ax}\right]\left[\frac{ax}{1+ax}\right]^{j-1}.$$

This expression and equation (22) from which it is produced are *shifted* Bose-Einstein distributions spanning only the non-zero values of the count and reflecting the role of the zero count as an absorbing state.

A Bose-Einstein (B-E) distribution has classical geometric form. It is built from a single positive parameter, in this instance $\gamma(x)$, generally a measurement of magnitude. The tail of the B-E converges exponentially to zero. In this respect it resembles Rayleigh narrowband noise (see equation (3)).

Consequently, even at a level of considerable generality, before any particular linkage between λ_x, μ_x, and x has been specified, a birth-death cascade will generate multiplicative noise resembling acoustic noise. Perhaps this may explain why the detectability of signals in acoustic noise is so easy to forecast from the statistics of the stimulus.

The general form of the birth-death counting distribution, equation (22), suggests that the process has two distinct outcomes: a spike of probability at count zero indicating lost signals, and a Bose-Einstein counting distribution over the non-zero states reflecting signal messages that pass through the network successfully.

C. *Connection to Driving Energy*

Now return to:

$$\ln(1+ax) = (\lambda_x - \mu_x)\, t.$$

The log will have non-negative values when $x \geq o$ which implies that $\lambda_x \geq \mu_x$. The force driving the transmission is the birth component, $\lambda_x\, t$ of the process. Some part of that force is dissipated in losses that become incorporated into the death component, $\mu_x\, t$. We might also define an "immigration" component representing outside neural activity that somehow wanders into the network becoming a source of additive noise. Such intrusions are almost inevitable. They can be incorporated into the model by allowing the gain parameter to approach an arbitrary limit as stimulus energy approaches zero. A major simplification occurs when the gain and loss parameters are set as follows:

$$\lambda_x\, t = (1+ax)\ln(1+ax)\,/\,ax\ , \tag{23}$$

$$\mu_x\, t = \lambda_x\, t\ /\ 1+ax.$$

Equation (23) meets all prior specifications. The process will amplify weak stimuli and simultaneously limit intense stimuli. The gain parameter is equal to or greater than the loss parameter. Both parameters approach unity as stimulus energy approaches zero. Hence it is easy to introduce an arbitrary internal noise by altering this limit if we choose to do so. We keep the limit at unity here to avoid further complications.

The loss parameter, $\mu_x\, t$, starts out equivalent to the system gain, guaranteeing a threshold below which no messages can be generated. As signal energy grows, the loss parameter falls off. Under very intense stimulation the transmission system becomes a pure birth process driven by the log intensity of the stimulus.

The main reason for the additional steps involved in constructing equation (23), however, is that they cause everything in the network to fall neatly into place, generating a particularly simple counting distribution:

$$\pi(x) = 1/1{+}ax \;, \gamma(x) = 1{+}ax \;; \tag{24}$$

$$P(j) = \left[\frac{1}{2{+}ax}\right]\left[\frac{1{+}ax}{2{+}ax}\right]^j, \qquad j{=}0,1,2,3\cdots$$

The counting distribution is now explicitly Bose-Einstein with parameter *1+ax*. Our definitions have eliminated the separate spike of probability in state zero. The distribution now ranges down to a count of zero despite the latter's role as an absorbing state. As driving energy approaches zero, a residual internal noise is found, creating occasional false alarms at absolute threshold. If we set the critical number for detection at *n*, the probability of a false alarm can be found from the tail of the Bose-Einstein distribution in equation (24) with x approaching zero:

$$Pr\ (FalseAlarm) = [\frac{1}{2}]^n \tag{25}$$

Accurate detection of a stimulus against this internal noise requires a critical number large enough to render the false alarms negligible.

We label equation (24) an *amplifier network counting distribution*. It constitutes our first tentative step beyond passive Poisson transmission. As observed earlier, the Bose-Einstein bears a striking resemblance to narrowband acoustic noise. We may expect the amplifier network to impose such properties on information passing through it. Amplifier network counting distributions may thus be expected to have higher tails and larger variance than Poisson distributions. In the main this outcome is desirable since it suggests that Weber's law may be built into the transmission network. But along with such advantages come new problems. In particular, normal approximations which perform so well at the stimulus level for estimating detectability, even with narrowband noise, are no longer guaranteed to work. The high tails of a Rayleigh driven Bose-Einstein counting process can sometimes mislead us. There are other problems as well.

V. *AUDITORY OUTCOMES IN AN AMPLIFYING NEURAL TRANSMISSION NETWORK*

The final sections of this chapter study the output of an amplifier network driven by typical auditory stimuli. We consider pure-tone intensity discrimination, and wide-band noise masking. Generally speaking, the outcomes are quite satisfactory. They yield solid (though imperfect) agreement with experimental data. Hence, a birth-death process (amplifier) network can serve as a useful guide in auditory and neurophysiological research, providing a new organizing principle for interpreting masses of data acquired along auditory pathways.

A. *Pure Tones*

The geometric distribution proves easy to work with in two-alternative forced-choice. Suppose we drive the amplifier network with two distinct pure tones, identical in frequency but differing slightly in energy content. Call the weaker tone E_o and its counterpart, containing an energy increment, E_s. Hence $E_s \geq E_o$. For the probability that the signal tone generates a count exceeding the reference tone, we have:

$$P\ (S > N) = \sum_{k=0}^{k=\infty} \left[\frac{1}{2+aE_0} \right] \left[\frac{1+aE_0}{2+aE_0} \right]^k \cdot \left[\frac{1+aE_s}{2+aE_s} \right]^{k+1} .$$

Similarly, the probability that the counts are exactly equal proves to be:

$$P\ (S=N)=\sum_{k=0}^{k=\infty}\left[\frac{1}{2+aE_0}\right]\left[\frac{1}{2+aE_s}\right]\left[\frac{1+aE_0}{2+aE_0}\cdot\frac{1+aE_s}{2+aE_s}\right]^k .$$

When the counts are exactly equal, we are forced to choose one of the two alternatives as the signal. The reasonable course is to flip an unbiased coin. Hence the probability of a correct decision must be given by:

$$P\ (c)=P\ (S>N)+\frac{1}{2}P\ (S=N) \tag{26}$$

Insert the Bose-Einstein counting outcomes into equation (26). We find:

$$P\ (c)=\frac{3+2aE_s}{6+2aE_s+2aE_0} . \tag{27}$$

This is the psychometric function for pure tone intensity discrimination if decisions are based upon counts recorded in an amplifier network. When $E_s=E_o$, the probability of a correct choice is .5. As E_s grows relative to E_o, P(c) approaches unity. In fact, suppose the reference tone E_o has sizable energy content. Then:

$$P\ (c)\Rightarrow\frac{E_s}{E_s+E_0} \tag{27a}$$

Equation (27a) for the psychometric function proves to be identical with that governing intensity discrimination of narrowband Rayleigh noise (see Green & McGill, 1970, equation (9a); equation (11) this presentation). Amplifier network transmission has converted the pure tone counting record to narrowband noise, and this network noise is indistinguishable from acoustic noise.

The threshold value of $E_s\ /\ E_o$, with the probability of a correct choice set at .75, is found to be 3; much larger than the experimental value of about 1.1. The narrow-band process is evidently incorrect. Excitation must spread into adjacent transmission channels from a focus at the stimulus frequency. In view of the apparent conversion of pure tones to noise in the counting record, we can then invoke the stimulus theory of wideband noise to illuminate the counting record for pure tones in an amplifier network (Green & McGill, 1970, equation (10); see also equation (13) in this presentation):

$$\nu^{1/2}\left[\frac{E_{75}}{E_0} - 1\right] = 1 \,. \tag{28}$$

The result is based on a wideband normal approximation to the difference distribution of two independent noise energy samples having identical bandwidth. (There are no problems with the approximation in these circumstances.) Notice that as the degrees of freedom parameter ν (measuring spread of excitation) increases, the ratio of pure tone intensities must decrease at threshold. If spread is fixed we ought to see Weber's law, and experiments confirm that notched or band-stop noise produces just this outcome. If spread is uncontrolled, as it is in pure tone intensity discrimination, the effect of increasing spread, as depicted in equation (28), is an increasing depression of the log-linear relation between E_s and E_o, below unit slope when both intensities are measured in db.

While these outcomes accord quite well with what we know of the data on pure tone intensity discrimination, actual estimates of spread are fairly large-- perhaps too large. Moreover, good data exist showing the near miss to Weber's law to be relatively impervious to center frequency and signal duration. It is difficult to square such findings with other measures of excitatory spread; critical bands, for example. But considering the rough and ready qualities of our model (orthogonal rectangular bands, uniform spread of excitation), its performance with pure tones must be accounted a considerable success.

B. *Noise Intensity Discrimination*

Wideband noise intensity discrimination takes us to an entirely new conceptual realm when the locus of detection is an amplifier network rather than the noise stimulus itself. Our results, however, are remarkably similar to those found via stimulus energy analysis. The single notable change is an increased estimate of bandwidth caused by the noise associated with stochastic multiplication in the network.

In this instance we begin with a Rayleigh (narrowband) noise driving the Bose-Einstein distribution in equation (24). The zero state probability P(o) is then formulated as follows:

$$P\ (0) = \int_0^\infty \frac{f\ (x)}{2+ax}\, dx\ ,$$

Where

$$f\ (x) = \frac{1}{N_0}\, e^{-\frac{x}{N_0}}\ .$$

Hence

$$P\ (0) = \frac{1}{2} \int_0^\infty \left[\frac{\frac{2}{aN_0}\, e^{-\frac{x}{N_o}}}{\frac{2}{a} + x} \right] dx\ .$$

This last expression is one of the many varieties of the exponential integral. Specifically:

$$P\ (0) = \frac{1}{2}\,(\, z\, e^z\, E_1\,(z)\,) = \frac{1}{2}\, \phi\,(z)\ , \tag{29}$$

where $z = 2\ /\ an_o$. The exponential integral $E_1\,(z)$ is tabled (c.f. Abramowitz & Stegun, 1966: pp 238-248), as indeed are certain values of the function $\phi\,(z)$. Extensive use of these relations necessitates preparing one's own tables. Fortunately it is a fairly easy task with a desktop computer. Such analysis reveals:

$$0 \le \phi\,(z) \le 1\ , \qquad as\ o\ \le z\ \le \infty.$$

$$0 \le P\ (0) \le \frac{1}{2}\ ,$$

The upper limit on P(0) occurs when the driving energy x approaches zero, an already familiar fact. More generally:

$$\int_0^\infty f\ (x)\ \frac{(1+ax)^{k-1}}{(2+ax)^{k+1}}\ dx = P\ (k-1) - P\ (k)\ , \tag{30}$$

$$= \frac{1}{kaN_0}\left[1-(\frac{1}{2})^k - \sum_{j=0}^{j=k-1} P\ (j)\right].$$

This relation and the definition of *P(0)* in equation (29) permit iterative computation of successive values of the counting probability *P(k)* driven by narrowband Rayleigh noise. A typical example is shown in Figure 3 which illustrates the high tail of the noise-driven amplifier-network counting distribution. The RDBE distribution is identical to the K_o Bessel function-driven Poisson distribution found in a number of applications in biology and optics. All these applications (including our amplifier network) suggest closely related stochastic processes (see Teich and Diament, 1989).

Having established the general appearance of the distribution, we now compute its mean and variance:

$$m_j = 1+aN_0\ , \tag{31}$$

$$\sigma_j^2 = 2+3\ (aN_0)\ (1+aN_0).$$

The variance is quite large, much larger than its Poisson counterpart. This forecasts not only a further retreat from ideal energy detection, but an auditory bandwidth estimate likely to be considerably wider than that calculated via pure stimulus analysis. Both consequences are viewed here as highly desirable. They illustrate divergent interpretations of the same data based upon models that differ at key points. Our early successes with pure stimulus analysis of masking noise in auditory detection may have led us astray, lulling healthy skepticism by offering interpretations that were really very good despite many indications that they were also not quite on target.

Narrowband noise transformed by a birth-death transmission network in the manner shown by Figure 3, is evidently non-gaussian even when differences are formed between pairs of independent samples. The high tail causes serious trouble.

Wideband noise similarly transformed is another story. The convolution of many orthogonal rectangular noise bands invokes the central limit theorem and eventually swamps the high tails of individual narrowband noise distributions. Accordingly at threshold we can set up a standard normal deviate for $P\ (S \geq N) = .75$ in two alternative forced-choice:

$$\frac{0 - [\nu\,(1+aN_s) - \nu\,(1+aN_0)]}{\nu^{\frac{1}{2}}\ [2+3aN_0\,(1+aN_0) + 2+3aN_s\ (1+aN_s)]^{\frac{1}{2}}} \approx -\,.6745$$

If we now let $\Delta N = N_s - N_o$, and discard terms of the form $2/aN_0$, the deviate boils down to:

$$\frac{\nu^{\frac{1}{2}}\ \Delta N\ /\ N_0}{[3+3\ (1+\Delta N\ /\ N_0)^2]^{\frac{1}{2}}} = .6745$$

It is here that the central limit theorem helps out. If ν is sufficiently large, so that $\dfrac{\Delta N}{N_0} << 1$, we can neglect terms of the order of $\dfrac{\Delta N^2}{{N_0}^2}$. Accordingly, since $\dfrac{\Delta N}{N_0}$ is approximately .1, we come finally to the detection law for intensity discrimination of wideband noise when the latter drives a birth-death amplifier network:

$$(\nu/3)^{\frac{1}{2}} \cdot \frac{\Delta N}{N_0} \approx 1\ . \tag{32}$$

The outcome is remarkable in that analysis of the amplifier network leads to a result very similar to the one obtained directly from the stimulus (see equation (13)). The only real difference is a new estimate of the degrees of freedom parameter, now three times larger than its stimulus counterpart and reasonably close to Green's (1960) empirical detectability formula.

Typical experimental data show $\frac{\Delta N}{N_0}$ to be near .10. Hence ν is estimated to be 300 in the psychometric function illustrated in Figure 4. The slope of the latter is roughly half the slope found by Green (1960), and subsequently established on theoretical grounds by Laming (1986). Moreover, the estimated noise bandwidth of the amplifier network, assuming an integration time of the order of 200 milliseconds, cannot be much wider than 1500Hz -- still a bit narrow.

Laming's running - derivative approach differs fundamentally from the amplifier network concepts presented here. The slope and bandwidth problems just cited would appear to give an advantage to Lamings methods and to cast at least a shadow of doubt on the energy transformations we have used.

But the amplifier network no longer requires us to remain strictly wedded to pure stimulus energy detection. Hence, a variety of transforms are open for consideration. For example, an additional power function operating on equation (23), defining the birth and death parameters, will in fact steepen our psychometric function and also increase the apparent bandwidth. Hence, we can come arbitrarily close to the performance we want, but until we have a better idea of the overall performance of amplifier networks, it is perhaps wise to keep things simple and to record discrepancies as they appear. This is one such. The slope of the pure-noise psychometric function resulting from equation (23) is too shallow and the estimated bandwidth is still too narrow. Everything else improves upon earlier versions.

C. *Sine Waves In Noise*

An analysis, parallel to our study of pure noise intensity discrimination, has been carried out also for an amplifier network driven by a pure tone embedded in band-limited noise. Many details are omitted here because we remain dissatisfied with the argument.

Psychometric functions developed from differences between independent Rayleigh-driven Bose-Einstein counting distributions, pose unique difficulties if the critical masking band is insufficiently wide. The high tails of the counting distributions produce a normal approximation that limits the psychometric function on values less than unity. This peculiar asymptotic behavior is a property of the approximation, not of the counting process itself. Thus, when normal approximations become necessary in a signal/noise driven amplifier network, a correction must be made so as to eliminate the spurious asymptote.

We have managed to subvert the entire problem when signals are embedded in a narrow-band noise. The amplifier network then generates an analogue of Marill's equation. But this exact solution has been developed only for a unit rectangular band (ν=1). As we have seen, stimulus data demand equivalent rectangular masking bands measured by $\nu=5-10$ (see Jeffress, 1968). This is in the low range of degrees of freedom, beyond our current reach for an exact solution, and yet not quite up to numbers where the central limit theorem can be guaranteed; (but see Teich & Diament (1989), table II, the BI distribution. Evidently an exact solution is reasonably close.)

Given this caveat, we can produce an approximate psychometric function using what seems to us to be an appropriate correction of the normal approximation with relatively few degrees of freedom. The method involves boot-strapping from the exact narrowband solution. First find a version of the normal approximation yielding close agreement with the exact psychometric function in the narrow-band case. Then apply this amended or corrected approximation to stimuli of moderate bandwidth. The resulting expression for the standard normal deviate corresponding to a tabled percentage of correct pairwise comparisons is the following:

$$\frac{E/N_0}{[(E/N_o)^{1.5} + 6(E/N_0) + 6\nu]^{\frac{1}{2}}} \approx d \ , \qquad (33)$$

where d is a standard normal deviate running from zero to infinity. Equation (33) can be computed by systematically increasing values of E/N_o for a fixed ν. The result is then inserted into a program instruction converting normal deviates to probabilities, and the psychometric function is constructed easily on a PC or programmable calculator.

A well-controlled threshold measurement and a normal ear listening to a pure tone at 1000Hz masked by wideband noise, will begin to detect at signal-to-noise power ratios somewhere near 25. If we take the ear's tonal integration time to be about 150 milliseconds, the threshold energy ratio calculates out at 25 .(.15) = 3.75. In pure stimulus analysis this threshold yields ν=12 for the corresponding degrees of freedom parameter. Here, the same parameter (ν=12) leads via equation (33) to a threshold energy ratio of approximately E/N_o=8.08. Evidently the multiplicative noise in our amplifier network requires more energy for detection than an ideal energy detector would need. Hence, relatively low energy ratios found at threshold in signal detection experiments suggest even narrower bandwidths than those calculated from pure stimulus analysis.

D. *Final Observations on Amplifier Networks*

These results imply that an amplifier network will generate wider internal noise bandwidths in noise intensity discrimination and narrower bandwidths in signal detection than those computed from direct analysis of stimulus energy. This outcome is very desirable, and a strong endorsement of the type of transmission system we have been studying. It does seem, nevertheless, that our new estimates of critical bandwidth are perhaps a bit too narrow; just as our estimates of noise bandwidth from intensity discrimination are not quite wide enough.

All things considered, the indications of progress are obvious. We have been able to characterize an active detection network based upon the principle of stochastic multiplication. We have managed to do so without getting hopelessly lost among the myriad complexities of the auditory pathways. Abandoning total dependence on the stimulus, and constructing the functional outline of an amplifier network (a hybrid of stimulus, receptor, and transmission mechanisms) generates solutions for a variety of problems deemed either intractable or overwhelmingly complex when approached in the conventional stimulus-oriented way. Time will tell whether this system is the one we have been seeking.

References

1. Barlow, H., Retinal noise and absolute threshold, *Journal of the Optical Society of America*, 1956, *46*, 634-639.
2. Barlow, H., Increment thresholds at low intensities considered as signal/noise discriminations, *Journal of General Physiology*, 1957, *136*, 469-488.
3. Barlow, H. B., Levick, W. R., and Yoon, M., Responses to single quanta of light in retinal ganglion cells of the cat, *Vision Research*, 1971, *11*, *Suppl. 3*, 87-101.
4. Bharucha-Reid, A., *Elements of the Theory of Markov Processes and Their Applications*, McGraw-Hill, New York: 1960.
5. Bouman, M., History and present status of quantum theory in vision, in *Sensory Communication*, edited by W. Rosenblith, pp. 377-401, Cambridge, MA: MIT Press and John Wiley, 1961. Chapter 21
6. Cinlar, E., Superposition of point processes, in *Stochastic Point Processes: Statistical Analysis, Theory, and Applications*, edited by P. Lewis, pp. 549-606, New York: John Wiley, 1972.
7. Cox, D. and Smith, W., On the superposition of renewal processes, *Biometrika*, 1954, *41*, 91-99.
8. Drenick, R., The failure law of complex equipment, *Journal of the Society of Industrial and Applied Mathematics*, 1960, *8*, 680-690.
9. Feller, W., *An Introduction to Probability Theory and Its Applications*, (1), John Wiley, New York: 1957. 2nd Edition
10. Green, D., Auditory detection of a noise signal, *Journal of the Acoustical Society of America*, 1960, *32*, 121-131.
11. Green, D., Additivity of masking, *Journal of the Acoustical Society of America*, 1967, *41*, 1517-1525.
12. Green, D. and McGill, W., On the equivalence of detection probabilities and well-known statistical quantities, *Psychological Review*, 1970, *77*, 294-301.
13. Hecht, S., Shlaer, S., and Pirenne, M.H., Energy, quanta and vision, *Journal of General Physiology*, 1942, *25*, 819-840.
14. Hopfield, J., Neural networks and physical systems with emergent collective computational abilities, *Proceedings of the National Academy of Science*, 1982, *79*, 2254-2558.
15. Jeffress, L., Stimulus-oriented approach to detection, *Journal of the Acoustical Society of America*, 1964, *36*, 766-774.
16. Jeffress, L., Mathematical and electrical models of auditory detection, *Journal of the Acoustical Society of America*, 1968, *44*, 187-203.
17. Laming, D., *Sensory Analysis*, Academic Press (HBJ), London: 1986.
18. Lawson, J.L. and Uhlenbeck, G.E., *Threshold Signals*, McGraw-Hill, New York: 1950.
19. Magill, J. and Soni, R., Photoelectric counting distributions for a noise-modulated system, *Physical Review Letters*, 1966, *16*, 911-913.
20. Marill, T., Detection theory and psychophysics, *Technical Report No. 319*, October 30, 1956, Massachusetts Institute of Technology Research Laboratory of Electronics, Cambridge, MA.

21. McGill, W., Neural counting mechanisms and energy detection in audition, *Journal of Mathematical Psychology*, 1967, *4*, 351-375.

22. McGill, W., Variations on Marill's detection formula, *Journal of the Acoustical Society of America*, 1968a, *43*, 70-73.

23. McGill, W., Polymonial psychometric functions in audition, *Journal of Mathematical Psychology* , 1968b, *5*, 369-376.

24. McGill, W., Poisson counting and detection in sensory systems, in *Concepts of Communication*, edited by E. Beckenback and C. Tompkins, pp. 257-281, New York: John Wiley, 1971. Chapter 9

25. McGill, W. and Goldberg, J., Pure tone intensity discrimination and energy detection, *Journal of the Acoustical Society of America*, 1968a, *44*, 576-581.

26. McGill, W. and Goldberg, J., A study of the near-miss involving Weber's law and pure-tone intensity discrimination, *Perception and Psychophysics*, 1968b, *4*, 105-109.

27. Miller, G., Sensitivity to changes in the intensity of white noise and its relation to masking and loudness, *Journal of the Acoustical Society of America*, 1947, *19*, 609-619.

28. Moore, B. and Raab, D., Pure tone intensity discrimination: some experiments relating to the "near miss" to Weber's law, *Journal of the Acoustical Society of America*, 1974, *55*, 1049-1054.

29. Nachmias, J. and Steinman, R., Study of absolute visual detection by the rating scale method, *Journal of the Optical Society of America*, 1963, *53*, 1206-1213.

30. Ong, S. and Lee, P., The non-central negative binomial distribution, *Biometrics Journal*, 1979, *21*, 611-627.

31. Perina, J., Superposition of coherent and incoherent fields, *Physics Letters*, 1967, *24A*, 333-334.

32. Peterson, W., Birdsall, T., and Fox, W., The theory of signal detectability, *Transactions of the Professional Group on Information Theory (IRE)*, 1954, *PGIT 2-4*, 71-212.

33. Pfafflin, S. and Mathews, M., Energy detection model for monaural auditory detection, *Journal of the Acoustical Society of America*, 1962, *34*, 1842-1853.

34. Raab, D. and Goldberg, I., Auditory intensity discrimination with bursts of reproducible noise, *Journal of the Acoustical Society of America*, 1975, *57*, 437-447.

35. Rice, S.O., Mathematical analysis of random noise, *Bell System Technical Journal*, 1944, *23*, 282-332.

36. Rice, S.O., Mathematical analysis of random noise, *Bell System Technical Journal*, 1945, *24*, 46-156.

37. Rumelhart, D., Smolensky, P., McClelland, J., and Hinton, G., Schemata and sequential thought processes, in *Parallel Distributed Processing*, edited by J. McClelland and D. Rumelhart, vol. 2, pp. 7-57, Cambridge, MA: MIT Press, 1986.

38. Ryan, A., Wolf, N., and Sharp, F., Tonotopic organization in the central auditory pathway of the Mongolian gerbil: A 2-deoxyglucose study, *Journal of Comparative Neurology*, 1982, *207*, 369-380.

39. Saleh, B. and Teich, M., Multiplication and refractoriness in the cat's retinal-ganglion-cell discharge at low light levels, *Biological Cybernetics*, 1985, *52*, 101-107.

40. Tanner, W., Application of the theory of signal detectability to amplitude discrimination, *Journal of the Acoustical Society of America*, 1961, *33*, 1233-1244.

41. Teich, M. and Diament, P., Multiply stochastic representations for K distributions and their Poisson transforms, *Journal of the Optical Society of America*, 1989 , *A.6*, 80-91.

42. Teich, M. and McGill, W., Neural counting and photon counting in the presence of dead time, *Physical Review Letters*, 1976, *36*, 754-758.

43. Teich, M., Prucnal, P., Vannucci, G., Breton, M., and McGill, W., Multiplication noise in the human visual system at threshold: 1. quantum fluctuations and minimum detectable energy, *Journal of the Optical Society of America*, 1982a, *72*, 419-431.

44. Teich, M., Prucnal, P., Vannucci, G., Breton, M., and McGill, W., Multiplication noise in the human visual system at threshold: 3. the role of non-Poisson quantum fluctuations, *Biological Cybernetics*, 1982b, *44*, 157-165.

45. Teich, M. and Saleh, B., Photon bunching and antibunching, in *Progress In Optics XXVI*, edited by E. Wolf, pp. 1-104: Elsevier Science Publishers B.V., 1988.

46. Viemeister, N., Intensity discrimination of pulsed sinusoids. The effects of filtered noise, *Journal of the Acoustical Society of America*, 1972, *51*, 1265-1269.

47. Wolfram, S., Origins of randomness in physical systems, *Physical Review Letters*, 1985, *55*, 449-452.

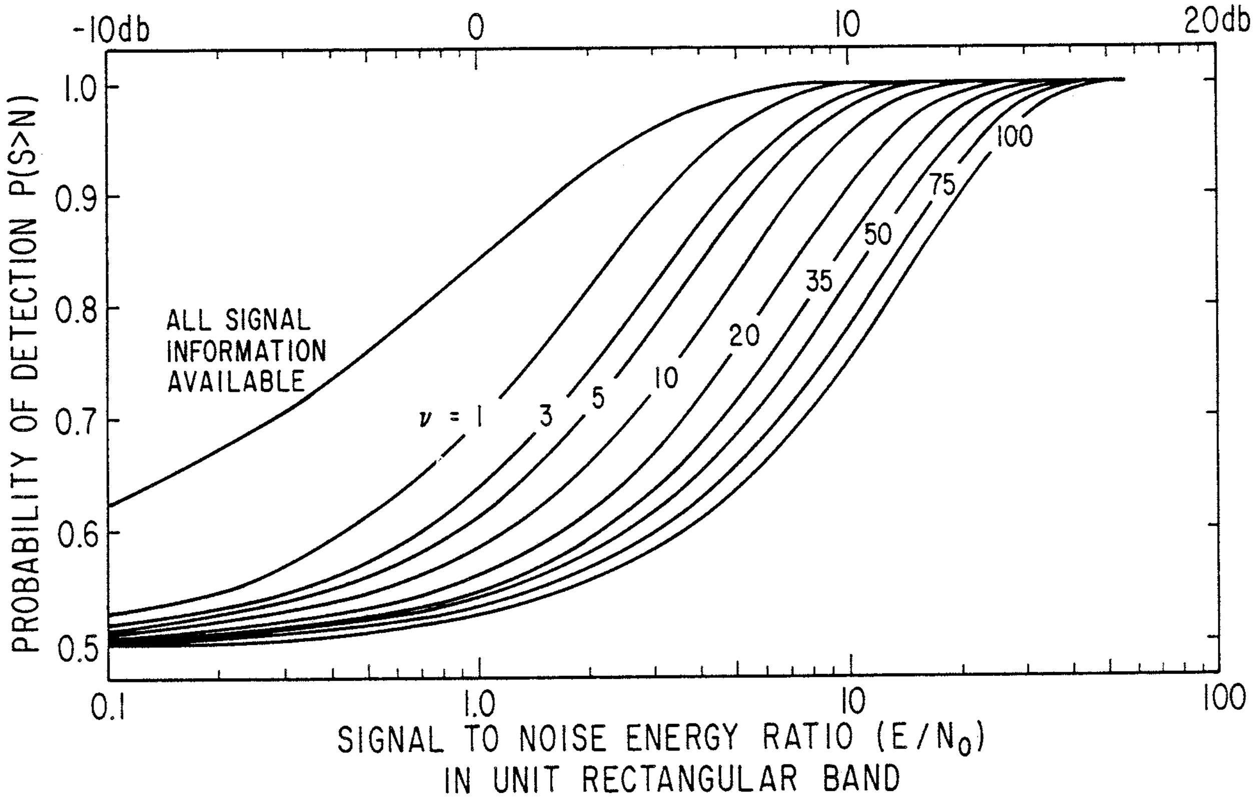

Figure 1:

Two alternative forced-choice psychometric functions governing detection of a pure tone in bandlimited gaussian noise. These stimulus-based psychometric functions are computed from the probability that the energy in the signal/noise condition exceeds the energy in the noise condition They are computed from equation (10) in the text and are exact. The parameter ν is one half the degrees of freedom of the noise band. It is based upon the ear's critical bandwidth and integration time. Psychometric functions such as these serve as templates for estimating bandwidth and integration time from typical experimental data.

The curve labeled *All Signal Information Available* is the Peterson, Birdsall, Fox (1954) optimum for the case of the signal known exactly. Similarly, the function at ν=1 is Marril's (1956) equation in which the signal is known except for phase. Neither of these ideal observers comes close to the data.

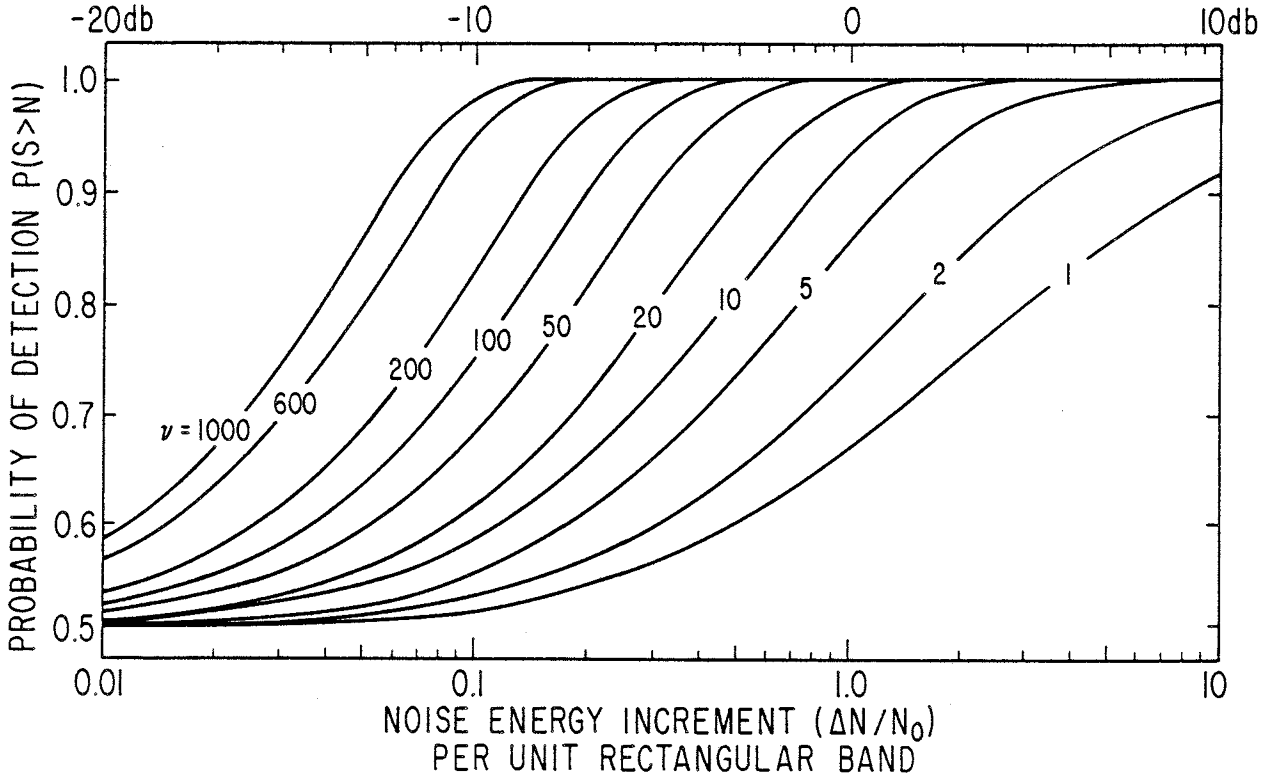

Figure 2:

Two-alternative forced-choice psychometric functions governing detection of a small increment ΔN in the average power level N_o per unit bandwidth of a bandlimited gaussian noise. The functions between ν=1−100 are computed from equation (11) in the text and are exact. For $\nu \geq 100$, our psychometric functions are based on a simple normal approximation. At ν=100 the two computations differ in the third decimal place over the operative range of the curve. For practical purposes, they too are exact. These are stimulus-based psychometric functions. The parameter ν is one half the degrees of freedom in the effective noise band. Again the functions serve as templates for estimating bandwidth and (noise) integration time from typical experimental data on noise intensity discrimination.

Note that the curves become more sensitive as the product of bandwidth and integration time increases, and that with increasing sensitivity the functions grow steeper on a log energy plot. The steepening is slight, however; not enough to account for Laming's (1986) observations on the slope of the psychometric function in noise intensity discrimination.

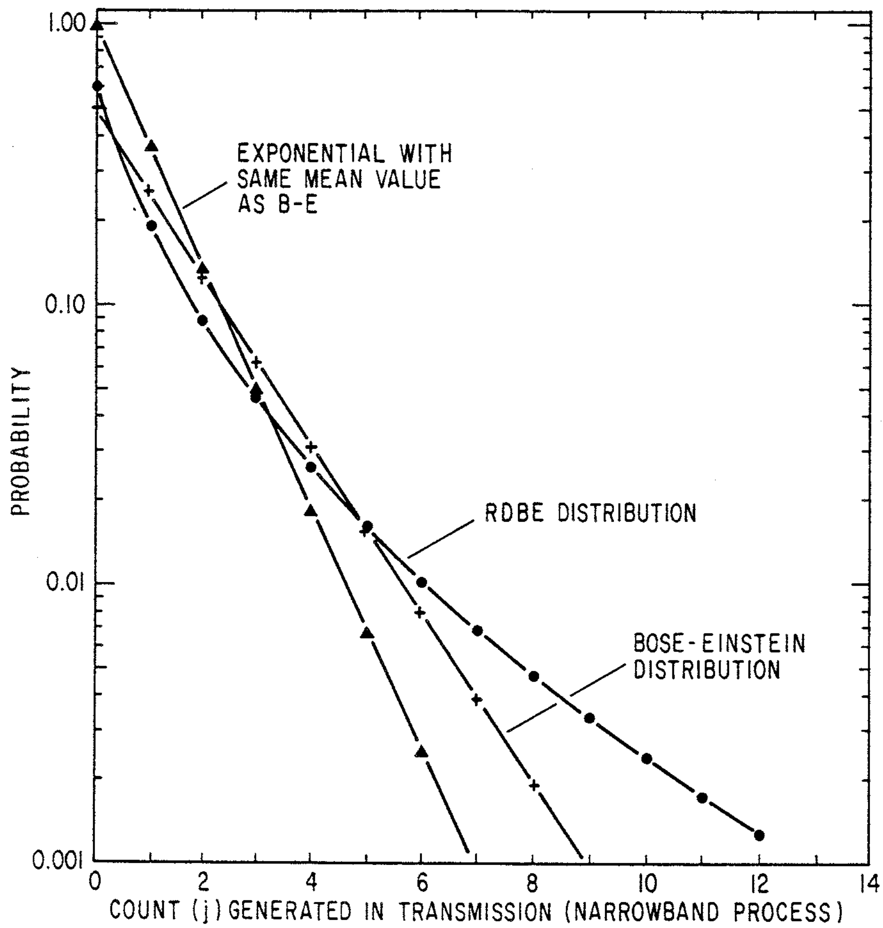

Figure 3:

The Bose-Einstein counting distribution is a typical pure birth process. The effective driving energy, aN_o is set equal to 1. The ordinate is logarithmic making the B-E linear. Also shown for comparison is an exponential (continuous) curve deriving from the Rayleigh energy distribution of narrowband acoustic noise. The curve differs only slightly. When however, the B-E is driven by this type of narrowband noise, the resulting Rayleigh Driven Bose-Einstein (RDBE) counting distribution is seen to have a much higher tail, *i.e.* larger variance, than its B-E parent. Hence the RDBE differs markedly from the B-E and its continuous equivalent, the Rayleigh energy distribution of narrowband noise.

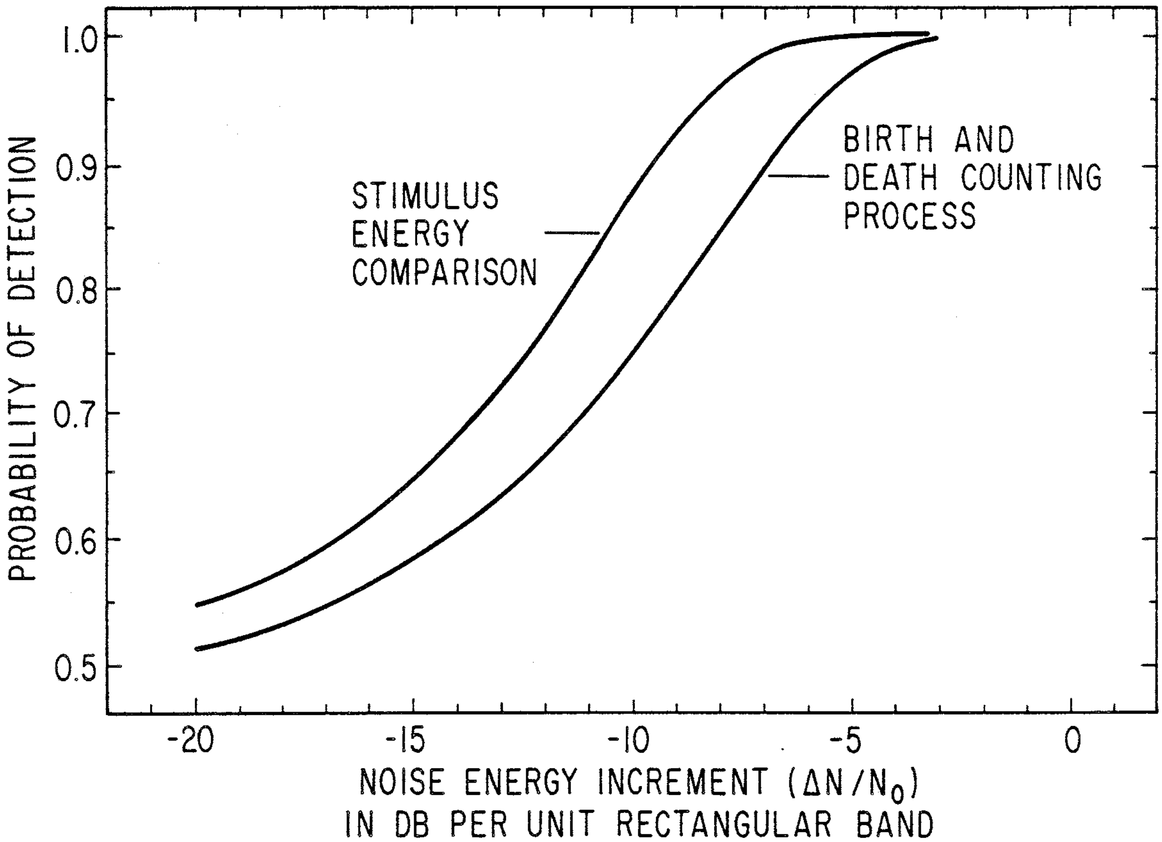

Figure 4:

Comparison of two alternative forced-choice psychometric functions emerging from stimulus analysis, and amplifier network analysis in noise intensity discrimination. The *stimulus energy* curve is based on equation (11) with $\nu = 300$. The curve for a *birth and death counting process* is obtained from equation (32) with $\nu = 300$. Actually they are nearly the same function translated by about 2.4db. In fact both curves can be found among the template psychometric functions in Figure 2. The transformations involved in the amplifier network are then read off as an apparent decrease in effective stimulus bandwidth (*i.e.* a shift to the right in Figure 2). Equation (32) enables us to reconstruct this shift and recompute the effective noise bandwidth based on a larger estimate of the degrees of freedom parameter.

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