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CHAPTER 28

Rectification Models in Cochlear Transduction

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INTRODUCTION

A broad variety of experiments in the auditory system leave little doubt that a nonlinearity is part of the signal processing apparatus in the auditory periphery. It is not known precisely where the locus of this nonlinearity lies, nor whether it comprises a single mechanism or multiple mechanisms. It has been customary to treat it as a static half-wave rectifier of the acoustic signal (Rose, Brugge, Anderson and Hind, 1967; Greenwood, 1986). In many models, this memoryless element is appended to a linear oscillator (linear filter) which provides the tuning for the system (Geisler and Greenberg, 1986; Geisler, 1987).

Although this simple approach provides agreement with some experimental data, there are many instances where it is clearly inadequate. In this paper we extend this model by considering clipping at an *arbitrary* level rather than at the half-way point.

STATIC RECTIFIER WITH ARBITRARY CLIPPING LEVEL

The static rectifier clips an input signal of unity amplitude at a predetermined level c ($0 \leq c \leq 1$). The value $c=0$ represents the absence of clipping altogether so that the shape of the input signal is unaltered. The value $c=1$, on the other hand, represents total clipping so that nothing passes through. Intermediate values of c represent partial rectification. The output signal is shifted upward or downward such that its flat portion sits at zero (see Fig. 1). Because this rectifier does not represent the response of a dynamical system, the energy contained in the discarded portion of the waveform is lost.

Consider a unity-magnitude cosinusoidal input signal x(t)

$$x(t) = \cos(2\pi f_s t) = \cos(2\pi t/T_s), \tag{1}$$

where f_s the frequency of the input signal, $T_s = 1/f_s$ is its period, and t is time. Passing this signal through the static rectifier changes its waveform and therefore alters its spectrum. Mathematically, the time waveform at the output of the rectifier $x_0(t)$ may be obtained by subtracting a constant level (that is dependent on c) from x(t) and then by multiplying the remainder by a unity-amplitude periodic square wave of the same repetition frequency and phase as the original signal, but with each square pulse having a full width t_0. The output waveform $x_0(t)$ is then given by

$$x_0(t) = \cos(2\pi f_s t) - \cos(\pi t_0/T_s), \tag{2}$$

in the region $-t_0/2 + jT \leq t + jT \leq t_0/2 + jT$; elsewhere it is zero. The quantity j is an integer representing successive cycles of the waveform. It is convenient to define a fractional time width $\alpha = \pi t_0/T_s$ which is related to the clipping level c by

$$c = (1 + \cos\alpha)/2. \tag{3}$$

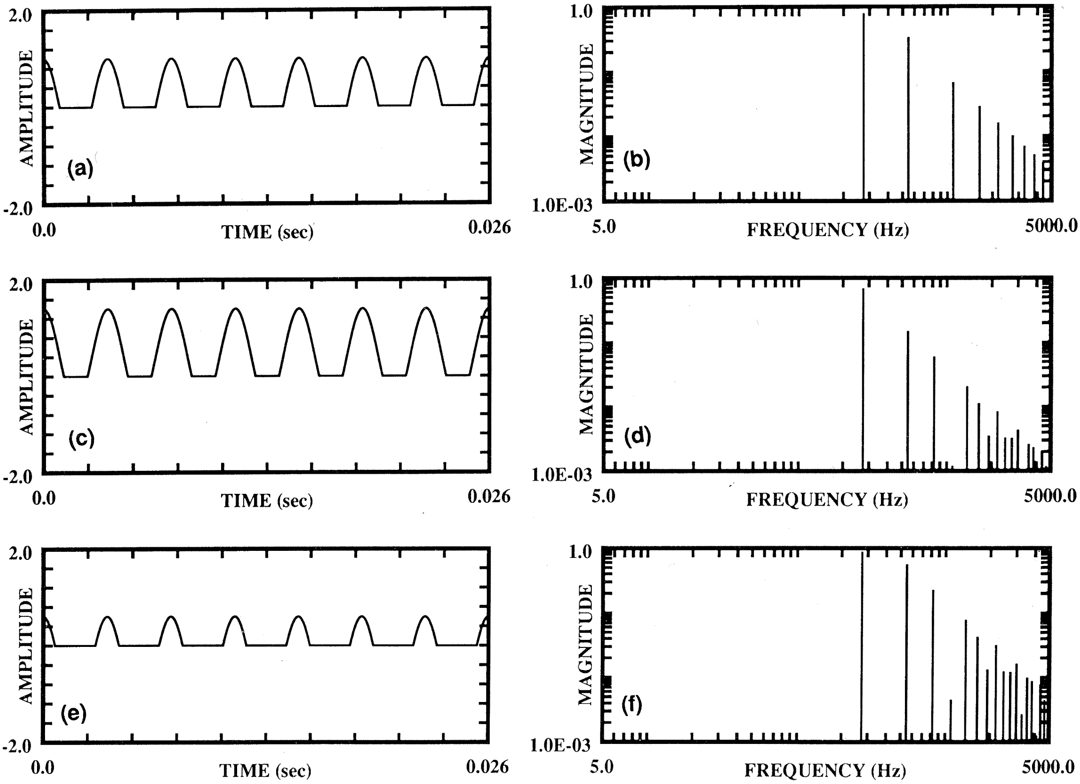

Fig. 1. Fractionally clipped cosine signals and their Fourier components for $f_s=280$ Hz. The time waveform for half-wave clipping ($c=1/2$) is shown in (a) and its Fourier components are shown in (b). Analogus curves for a signal clipped below the half-way level ($c<1/2$) are shown in (c) and (d); for a signal clipped above the half-way level ($c>1/2$) the results are shown in (e) and (f). Odd harmonics of the fundamental frequency, which are absent for $c=1/2$, contribute as the clipping level deviates from 1/2 in either direction.

For half-wave rectification $t_0=T_s/2$ whereupon $\alpha=\pi/2$ and $c=1/2$. Two other special cases are of interest: $t_0=o$ corresponds to $\alpha=o$ and $c=1$, indicating that the entire signal is clipped and nothing remains; $t=T_s$ corresponds to $\alpha=\pi$ and $c=0$ which represents no rectification and a dc offset applied to the signal.

The Fourier components are readily calculated by expressing Eq. (2) as a Fourier series in the harmonics of the signal frequency

$$x_0(t) = a_0/T+(2/T)\sum_{n=1}^{\infty} a_n\cos(2\pi n f_s t)+b_n\sin(2\pi n f_s t), \tag{4}$$

where the a's are Fourier coefficients and the b's are zero because $x_0(t)$ is an even function. Using the standard methods of calculating Fourier coefficients, the magnitudes of the spectral components at dc, at the fundamental, and at higher harmonics turn out to be

$$a_0 = (2/\pi)(\sin\alpha-\alpha\cos\alpha), \tag{5a}$$

$$a_1 = (1/\pi)(\alpha-\sin\alpha\cos\alpha), \tag{5b}$$

$$a_n = [\sin(n\alpha)\cos\alpha-n\sin\alpha\cos(n\alpha)](1-\cos\alpha)/[n(n^2-1)(\sin\alpha-\alpha\cos\alpha)], \tag{5c}$$

respectively.

Three fractionally clipped cosine signals and their Fourier components are illustrated in Fig. 1, for different clipping levels, when f_s=280 Hz. The time waveform for half-wave clipping ($c=1/2$) is shown in Fig. 1(a). As is well known, it has Fourier components only at the fundamental frequency and at its even harmonics, as illustrated in Fig. 1(b). The time waveform and its Fourier transform for a signal clipped below the half-way level ($c<1/2$) are shown in Figs. 1(c) and 1(d), respectively. Clipping above the half-way level ($c>1/2$) yields the results illustrated in Figs. 1(e) and 1(f). Odd harmonics of the fundamental frequency, which were absent when $c=1/2$, begin to contribute as the clipping level deviates from 1/2 in either direction. A dc component (at $f_s=0$) is present in all cases; its magnitude is normalized to unity in the Fourier magnitude plot.

The static rectifier alone exhibits no frequency selectivity. The shape of the output waveform depends only on the parameter c and is independent of the signal frequency; its magnitude simply scales linearly with the magnitude of the input signal. Thus, the ratio of any pair of spectral component magnitudes depends on c but is independent of f_s. In particular, the ratio of the fundamental component magnitude to the dc value depends only on c and not on the signal frequency.

LINEAR-FILTER TUNING CHARACTERISTICS

A tuning characteristic may be introduced into the model by introducing an independent linear oscillator before the rectifier. The forced linear oscillator has an equation of motion

$$mx''(t)+rx'(t)+kx(t)=f(t), \tag{6}$$

where the constants m, r, and k represent the oscillator mass, resistance, and stiffness, respectively. The primes represent differentiation with respect to time, and f(t) is the driving signal.

For a sinusoidal driving signal, the steady-state displacement x(t) is sinusoidal. In the frequency domain, it can be characterized by a transfer function $|H_x(f_s)|$, which represents the ratio of the displacement amplitude at the signal frequency f_s to the displacement amplitude at the center or resonance frequency (CF) of the oscillator, $f_0\sim(k/m)^{1/2}$. The function $|H_x(f_s)|$, which has a maximum value of unity, depends on f_s in accordance with the normalized displacement resonance curve (filter function)

$$|H_x(f_s)| = f_0r/[m^2(f_0^2-f_s^2)^2+f_s^2r^2]^{1/2} = f_0^2/[Q^2(f_0^2-f_s^2)^2+f_0^2f_s^2]^{1/2}. \tag{7}$$

This resonance curve is shown in Fig. 2(a) for a linear oscillator with f_0=280 Hz. As is evident from the figure, the displacement amplitude achievable in the linear oscillator is maximal when the signal frequency is at its CF, and decreases for either lower or higher signal frequencies. The linear oscillator behaves as a tuned circuit. Far below the resonance frequency ($f_s<<f_0$), $|H_x(f_s)|$ is independent of f_s whereas for above the resonance frequency ($f_s>>f_0$), $|H_x(f_s)|$ falls off as f_s^{-2}. A measure of the sharpness of tuning in the system is provided by the parameter $Q=2\pi mf_0/r$; the larger the value of Q, the sharper the tuning.

We may, alternatively, consider the normalized velocity resonance curve $|H_v(f_s)|$, which is the ratio of the velocity amplitude at the signal frequency f_s to the velocity amplitude at CF. In this case

$$|H_v(f_s)| = f_sr/[m^2(f_0^2-f_s^2)^2+f_s^2r^2]^{1/2} = f_0f_s/[Q^2(f_0^2-f_s^2)^2+f_0^2f_s^2]^{1/2}, \tag{8}$$

which is plotted in Fig. 2(b). Maximal velocity is attained for signals applied at the CF. In

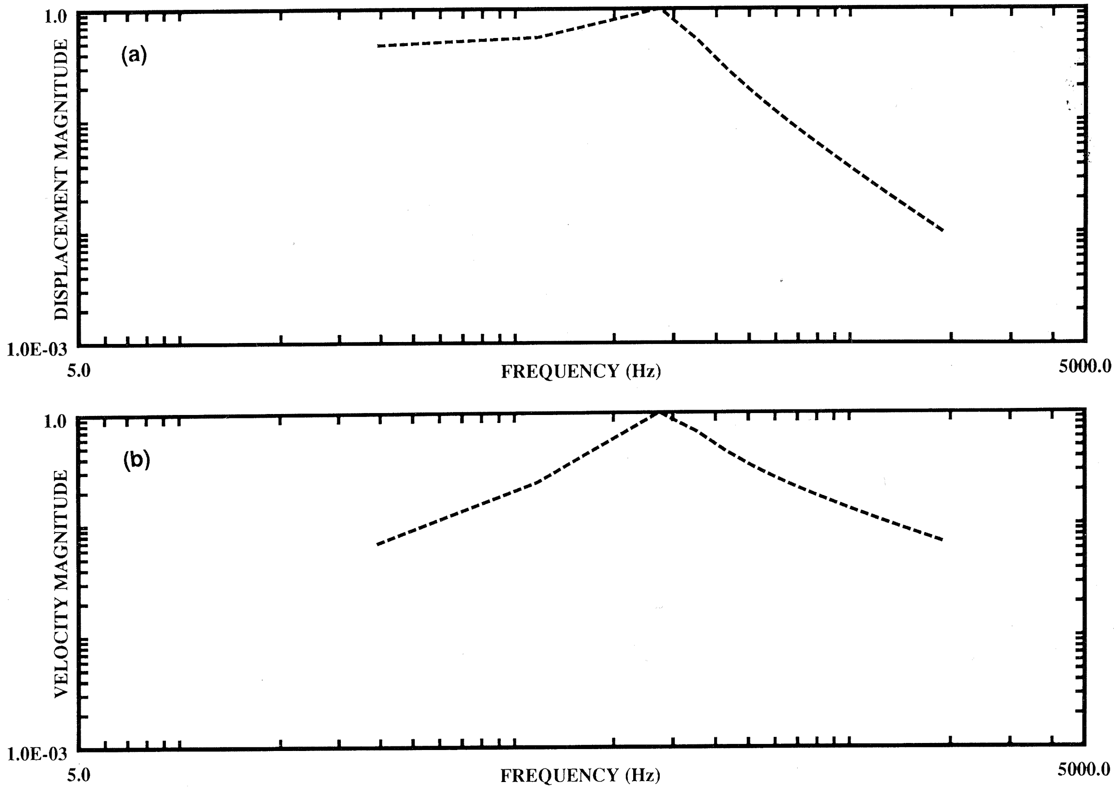

Fig. 2. Tuning curves for the linear filter. The ratio of the displacement amplitude at the signal frequency f_s to the displacement amplitude at the center or resonance frequency (CF=280 Hz) of the oscillator, versus signal frequency, is shown in (a). The ratio of the velocity amplitude at the signal frequency f_s to the velocity amplitude at the center or resonance frequency (CF) of the oscillator, versus signal frequency, is shown in (b).

this case, however, $|H_v(f_s)|$ increases linearly with the signal frequency f_s well below the resonance frequency and decreases linearly with f_s well above f_0.

Although we have used x to represent displacement, it is clear that it may stand for any appropriate variable such as receptor current or cell potential.

CASCADE OF A LINEAR FILTER AND A STATIC RECTIFIER

We now consider a sinewave signal of frequency f_s passing through the linear-filter/static-rectifier combination. The sinewave at the output of the filter is fractionally clipped at level c (e.g., if $c=1/2$ the waveform is half-wave rectified, whatever its magnitude). The first four Fourier components at the output of the cascade are shown in Figs. 3(a), 3(b), and 3(c), for $c=0.5$, 0.3, and 0.7 respectively. These are the same values of c used in Fig. 1. For the half-wave rectification case shown in Fig. 3(a), the curves of successively decreasing magnitude represent dc, fundamental, second harmonic, and fourth harmonic. The third harmonic appears in Figs. 3(b) and 3(c) when $c \neq 1/2$.

The various harmonics follow each other precisely and peak at the CF. This is because the filter first reduces the magnitude of the response in accord with its tuning curve, and the rectifier then clips at a constant fraction of the reduced output range. Although this is the most plausible way in which a linear-filter may be combined with a static rectifier, we

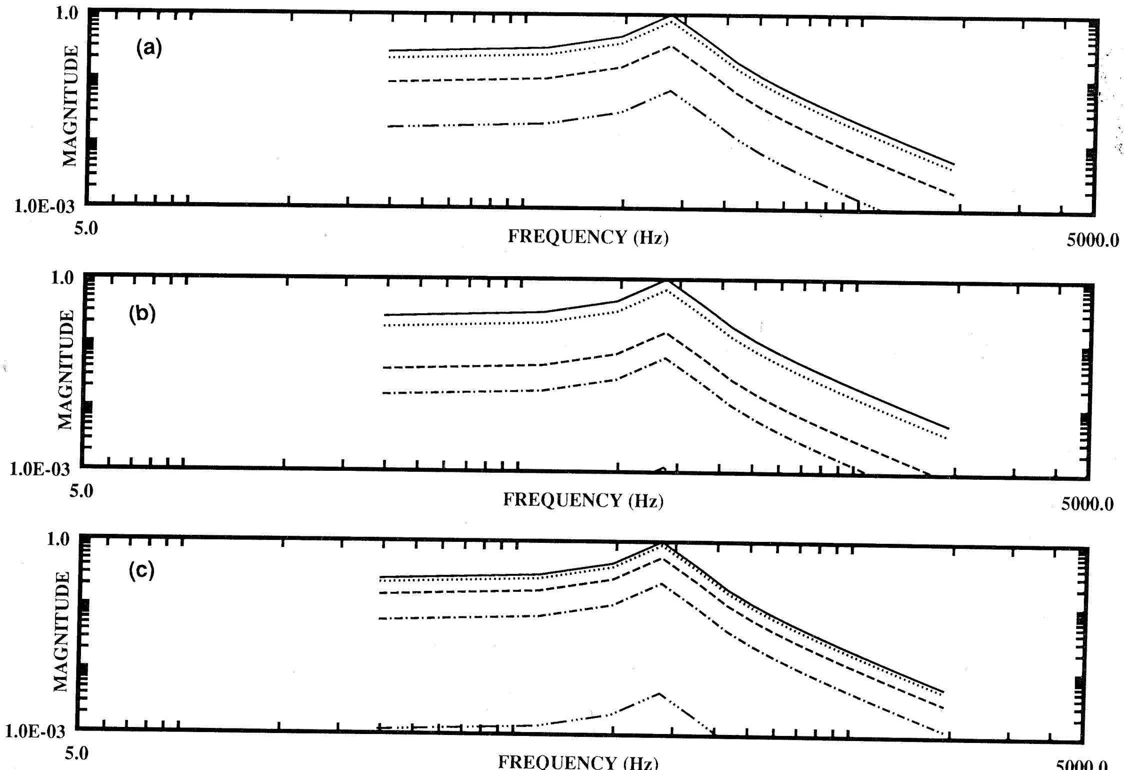

Fig. 3. Tuning curves for the linear-filter/static-rectifier combination. The first four Fourier components at the output of the cascade are shown in (a), (b), and (c), for $c=0.5$, 0.3, and 0.7 respectively. These are the same values of c used in Fig. 1. For the half-wave rectification case shown in (a), the curves of successively decreasing magnitude represent dc (solid curve), fundamental (dotted curve), second harmonic (dashed curve), and fourth harmonic (dash-three-dots curve). The third harmonic (dashed-dot curve) appears in (b) and (c) when $c \neq 1/2$. The various harmonics follow each other precisely and peak at the CF.

note that implementation of this scheme would require a rather complex processor that ascertains the range of the waveform before clipping at a constant fraction of its peak value.

DISCUSSION

The cascade of a linear filter and a static rectifying element (with arbitrary clipping level) gives rise to tuning curves that precisely follow each other. These tuning curves differ substantially from those obtained using simple dynamic nonlinear-oscillator models developed in the following papers (Keilson, Teich and Khanna, 1989a, b; Teich, Keilson and Khanna, 1989) and from those observed in cellular structures in the organ of Corti (Teich, Khanna and Keilson, 1989). This may be seen by comparing Fig. 3 with Figs. 5 and 10 in the paper by Teich, Khanna and Keilson (1989). This is because the cochlear response to an acoustic signal exhibits nonlinear *dynamic* characteristics.

ACKNOWLEDGEMENTS

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