dark. No attempts were made to optimize the junction behavior.

The conditions affecting the structural and electrical characteristics of the films and the *p-n* junctions are being investigated together with the possibility of extending this technique to other materials which do not lend themselves to standard methods of chemical doping.

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THREE-FREQUENCY HETERODYNE SYSTEM FOR ACQUISITION AND TRACKING OF RADAR AND COMMUNICATIONS SIGNALS

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The operation of a three-frequency heterodyne system for communications and radar use is discussed. The technique provides important advantages over the conventional heterodyne detector, and is applicable in the infrared, optical, and microwave. The signal-to-noise ratio and minimum detectable power for the specific case of an optical or infrared system are calculated and compared with values for the standard configuration. A practical CO_2 laser radar has been examined as a particular example of this mode of operation.

A heterodyne system[1] which transmits two signals of a small, but well-known difference frequency $\Delta\nu$, is proposed.[2] Aside from the mixer, the receiver includes a nonlinear element such as a square-law device, and provides its output signal at a frequency very close to $\Delta\nu$ *regardless of the Doppler shift of the transmitted signals*. This obviates the need for high-frequency electronics, thus improving impedance matching, system noise figure, and range of operation over the conventional case. The usual frequency scanning of either the detector or the local oscillator (LO) is eliminated, as is the necessity for a stable LO. Furthermore, it allows targets to be *continuously* observed with Doppler shifts of considerably greater magnitude and range than presently possible.

Although the system requires a two-frequency transmitter, it is in some ways simpler than systems currently in use. If the radiation power of the received signal (P_r) is large and/or the expected Doppler frequency range of the signal (Δf) is small (i.e., $P_r/\Delta f \gg h\nu/\eta$ in the visible or infrared), the signal-to-noise ratio (SNR) for the system is approximately $\eta P_r/6h\nu B$. Here, η is the quantum efficiency of the detector, $h\nu$ is the photon energy, and B is the bandwidth of the final filter centered about $\Delta\nu$ (see Fig. 1). Under certain circumstances B may be made very small, and the SNR may be greater than that obtained with a conventional system. Consideration of the opposite limit yields a minimum detectable power which is larger than that obtained by use of the standard technique by a factor of approximately $(3\Delta f/B)^{1/2}$. Thus, the greater the knowledge of Doppler shifts to be encountered, the greater the achievable SNR and the lower the minimum detectable power. The system can, of course, be switched from the three-frequency mode to the ordinary heterodyne mode of operation (with or without AFC) if Doppler information is required.

In Fig. 1(a), we present a block diagram for a radar version of the system. A transmitter emits two waves of frequencies ν_1 and ν_2 whose difference $\Delta\nu$ is known to high accuracy.[3] The waves are Doppler shifted by the moving target such that a wave of frequency ν returns with a frequency ν' given by the standard formula

$$\nu' = \nu(1 \pm 2v_{\parallel}/c) , \tag{1}$$

where $v_{\parallel}$ is the radial velocity of the target and c is the speed of light. After scattering from the target, therefore, the new frequency difference between the two waves $\Delta\nu'$ is given by

$$\Delta\nu' = \nu_1' - \nu_2' = \Delta\nu \pm (2v_{\parallel}/c)\Delta\nu . \tag{2}$$

Aside from this frequency shift resulting from the Doppler effect, there is a frequency broadening of each wave associated with the scattering by a moving target in a typical radar configuration.[4] For a rotating target, this broadening is of order $4r\omega_{\perp}/\lambda$,

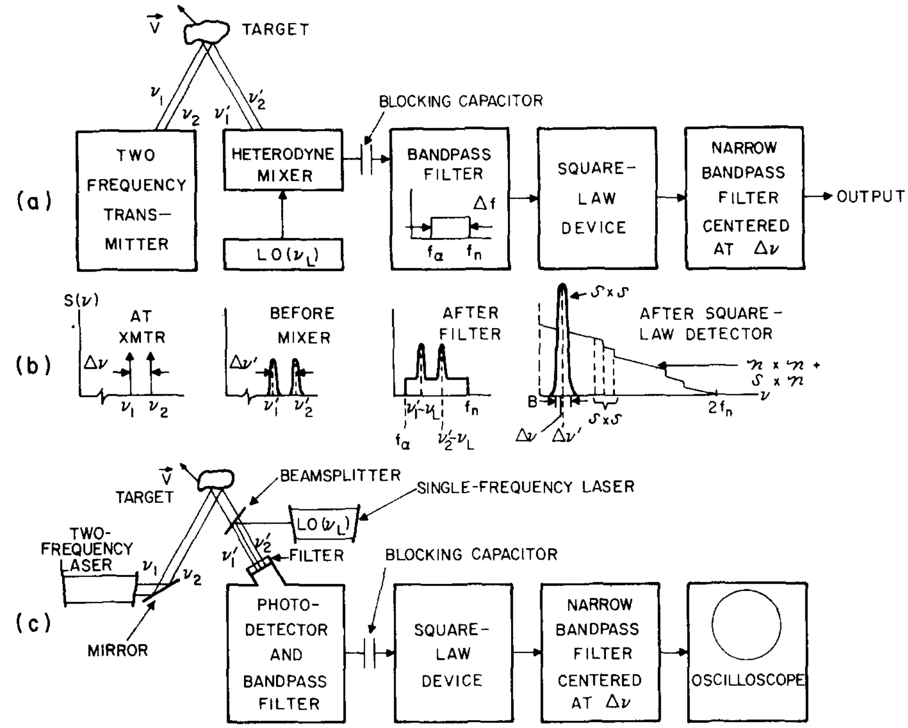

Fig. 1. (a) Block diagram of the three-frequncy mixing heterodyne system for radar application. (b) Power-spectral-density $S(\nu)$ versus frequency ν at various points in the system depicted in (a). (c) A possible infrared or optical implentation of the system.

where r is the "radius" of the target, $\omega_\perp$ is its component of angular velocity perpendicular to the beam direction, and λ is the wavelength.[5] For any practical system, as will be shown shortly, the difference $\Delta\nu' - \Delta\nu$ is much smaller than the broadening effects and may usually be neglected. In the following considerations, therefore, we take $\Delta\nu' - \Delta\nu = 0$. The power-spectral-density $S(\nu)$ for the transmitted and for the return signals from the target are shown in Fig. 1(b). The broadening of the signals is apparent.

Heterodyning with a local oscillator (LO) of frequency ν_L then provides two electrical difference-frequency signals[6,7] (centered at $\nu_1' - \nu_L$ and $\nu_2' - \nu_L$), and a dc component which is blocked. The ac output of the detector must then be broadband coupled, through a filter of bandwidth Δf, to a square-law device. The value chosen for Δf should be as small as possible in order to maximize the SNR, as will be seen later, but must encompass the difference frequencies generated in the mixer [see Fig. 1(b)]. The square-law device, which must have a response over Δf, then generates a component at the frequency $(\nu_1' - \nu_L) - (\nu_2' - \nu_L) \approx \Delta\nu$. Since the output of the square-law device is essentially independent of the Doppler[8] as well as the LO frequencies, variations in these parameters have no effect on the system ouput. A narrowband filter centered near $\Delta\nu$ and placed after the square-law device achieves a low noise bandwidth. Thus all amplifiers and other electronic detection equipment process narrowband signals at a low frequency, which provides ease of matching as well as good noise figure. This might, in turn, decrease the LO power necessary for optimum coherent detection.[5] Only the heterodyne mixer and the square-law device need have high-frequency response and be broadband coupled. Similar results would, in fact, be expected with any nonlinear device (e.g., a half-wave linear, in place of the square-law element) so that there is a wide choice for designing a heterodyne-nonlinear detector combination, perhaps in a single package.

The power-spectral-density, and therefore, the SNR at the output of the square-law device, may be calculated by the "direct method" of Davenport and Root.[9] Generalizing their treatment to two signals plus Gaussian noise (arising from the strong LO), we have obtained the power-spectral-density schematically shown in Fig. 1(b). The noise × noise ($n \times n$), plus the signal × noise ($s \times n$), contributions are shown as the solid line. In calculating these contributions, it has been assumed that $\Delta f \to f_n$, i.e., the first filter is lowpass rather than bandpass. The dotted lines represent signal × signal ($s \times s$) contributions which are not shown, except for the one of interest centered at $\Delta\nu' \approx \Delta\nu$. Calculations have been performed for two cases: (a) when the signals ν_1' and ν_2' are sinusoidal and independent, and (b) when ν_1' and ν_2' are independent

Gaussian random processes with individual power spectral densities which are also Gaussian. This appears to be a reasonable approximation to at least some experimental observations in the microwave,[10] infrared,[5] and visible.[11]

We have calculated a bound on the over-all three-frequency heterodyne system SNR for the special case of sinusoidal infrared or optical returns from a (specular) target. If the received radiation power is equally divided between beams 1 and 2, and a lowpass filter follows the heterodyne mixer ($\Delta f \to f_n$), the SNR is bounded by[12]

$$\frac{\Delta f}{4B}\left[\frac{(\eta P_r/h\nu\Delta f)^2}{1+2\eta P_r/h\nu\Delta f}\right] \le \mathrm{SNR} \le \frac{\Delta f}{2B}\left[\frac{(\eta P_r/h\nu\Delta f)^2}{1+2\eta P_r/h\nu\Delta f}\right]. \tag{3}$$

The bound arises from different possible noise levels, and depends on the relative values of $\Delta\nu$ and f_n, as may be seen from Fig. 1(b). There are two limiting cases which may be examined. For strong signals and/or small uncertainties in Doppler frequency ($P_r/\Delta f \gg h\nu/\eta$), Eq. (3) reduces to

$$\eta P_r/8h\nu B \le \mathrm{SNR} \le \eta P_r/4h\nu B \ . \tag{4}$$

For this case, the SNR ($\approx \eta P_r/6h\nu B$) is degraded over the conventional heterodyne detector by ~ -8 db *assuming equivalent output bandwidth B*. However, the independence of the three-frequency system on the LO and Doppler frequencies might, in practice, allow B to be reduced to a value well below that which could be used in the standard heterodyne configuration. Thus, a net increase in SNR might actually be possible.

In the opposite limit ($P_r/\Delta f \ll h\nu/\eta$), with Δf $\Delta f \gg B$, Eq. (3) yields the bound

$$(\eta P_r/2h\nu)^2/B\Delta f \le \mathrm{SNR} \le (\eta P_r/\sqrt{2}h\nu)^2/B\Delta f\,. \tag{5}$$

For convenience, we estimate this to be

$$\approx (\eta P_r/\sqrt{3}h\nu)^2/B\Delta f\ ,$$

which may be expressed as a minimum detectable power (SNR $\equiv$ 1) for the three-frequency mixing system ${}^3P_r$ (min). Thus,

$${}^3P_r\,(\mathrm{min}) \approx (\sqrt{3}\,h\nu/\eta)(B\Delta f)^{1/2}\ , \tag{6}$$

which is to be compared with the equivalent quantity for the conventional configuration $P_r(\mathrm{min}) = (h\nu/\eta)B$, when the bandwidths B are the same. The quantity ${}^3P_r(\mathrm{min})$ approaches $P_r(\mathrm{min})$ only when $\Delta f \to B$.[13] Since the system provides its advantage for $\Delta f \gg B$, the minimum detectable power (MDP) will generally be increased (in dB) over the equivalent amount for the usual system by 5 log($3\Delta f/B$). Nevertheless, it must be kept in mind that the three-frequency scheme may allow B to be considerably reduced in a practical system.

For the Gaussian signal case (both statistics and spectrum), the SNR results are very similar to those already presented, except for a multiplicative function containing the width and height of the spectrum, as well as the quantities B and f_n. For the special case of equal signals, with $B = \sqrt{8}\gamma$, and centered precisely about $\Delta\nu'$ (γ is the individual beam spectral width), the SNR need simply be multiplied by the quantity $[2\Phi(1) - 1] = 0.68$. This particular combination of parameters gives the simple result above in terms of the probability function Φ. The SNR is, therefore, reduced below that obtained for the sinusoidal case (delta-function spectrum) for the same bandwidth B. This is understood to arise from the condition that some signal is being excluded in the Gaussian case as compared with the delta-function case, but the noise is the same. In fact, if $\Delta\nu'$ is known to high accuracy, the best SNR for the Gaussian case is obtained in the limit as $B \to 0$, since the noise decreases faster than the signal, as B decreases. In the limit of $\gamma \to 0$, maintaining constant power, the Gaussian results reduce to the delta-function results as expected.

As a practical example of this mode of operation, we consider a CO_2 laser radar operating at 10.6 μm in the infrared[14] [see Fig. 1(c)]. If we assume that we wish to acquire and track a 1-m-radius satellite with a rotation rate of 1 rpm, the expected bandwidth (resulting from rotation) of the radar return is of order[5] $4r\omega_\perp/\lambda \sim 40$ kHz. We, therefore, choose a difference frequency $\Delta\nu$ at a (convenient) value of 1 MHz, which eliminates spectrum overlap. If the satellite has a radial velocity ≈ 10 km/sec, the Doppler frequency ν_D is ≈ 2 GHz, yielding a value $\Delta\nu' - \Delta\nu = 2v_\parallel\Delta\nu/c = (\Delta\nu/\nu)\nu_D \sim 60$ Hz. This shift is very small indeed compared with general frequency modulations in an ordinary heterodyne system, justifying the assumption that $\Delta\nu' - \Delta\nu \to 0$. Thus, assuming we have only an upper bound on the satellite velocity, i.e., its velocity may be anywhere in the range 0–10 km/sec, we choose $\Delta f \approx 2$ GHz and $B \sim 20$ kHz. The MDP for this system would, therefore, be $\sim(\sqrt{3}\,h\nu/\eta(B\Delta f)^{1/2}$ which is equivalent to the MDP obtained from a conventional setup with a bandwidth of approximately 10 MHz. If the value of ν_D is more confined, the MDP is correspondingly reduced. For strong returns, of course, the SNR will show an enhancement commensurate with the bandwidth B. Thus, the advantages of the three-frequency system may be secured with such a radar. Similar results would obtain at other frequencies; in the microwave, for example, $\Delta\nu$ may be made as small as tens of Hertz. In some cases, it may be possible to reduce clutter by the insertion of an extremely sharp notch filter at exactly $\Delta\nu$.

Use of the three-frequency method for a communications system (in which the transmitter and receiver may be moving relative to each other) is similar to the radar already described, but with the signal sent through the channel to the mixer. In this case, only one of the carrier waves (say ν_2) is modulated, and the output of the narrowband filter is fed into a demodulator. By modulating only one of the beams, the $S \times S$ component reaching the demodulator results from the convolution of a delta-function (at ν_1) with the modulated signal (centered at ν_2), which is simply the original undistorted spectral information ready for demodulation by a suitable device such as a mixer, an envelope detector, or a discriminator.

We have also considered the case of four-frequency mixing where the transmitter and receiver are identical. This would be convenient in some cases. But because of the very large signal component arising from the beating between the two strong LO beams, the $s \times n$ term becomes very large. Thus, it turns out that the SNR is multiplied by a factor $(P_\gamma/P_{LO})^2 \ll 1$, where P_γ and P_{LO} are typical signal and LO powers, respectively. Thus, four-frequency mixing in this configuration does not appear to be a useful concept.

A more complete account of this theory, along with some general cases not presented here, will be published shortly. I am grateful to E. N. Protonotarios and to R. Yen for valuable discussions.

[1] For a review of the conventional heterodyne technique in the optical and infrared, see M. C. Teich, in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic Press, Inc., New York, 1969); *Infrared Detectors*, Vol. 5 (to be published). For use of the conventional technique in microwave radars, see M. I. Skolnik, *Introduction to Radar Systems* (McGraw-Hill Book Co., New York, 1962).

[2] Multiple-frequency cw microwave radar has been used previously for *range* measurements. The receiver configuration, as well as the object of the system, is different from that considered here, however. See Ref. 1 (M. I. Skolnik, p. 106).

[3] This is particularly easy to accomplish if the transmitter is a two-mode laser, since the modes tend to drift together, keeping $\Delta\nu$ constant.

[4] Propagation through the atmosphere may also impose frequency broadening on the beams.

[5] M. C. Teich, Proc. IEEE 57, 786 (1969); 56, 37 (1968).

[6] A third signal at $\nu_1' - \nu_2'$ is very weak and may be neglected.

[7] The quantum theory of heterodyne detection predicts that only difference, and not sum- and double-frequency components, occur for $h\nu \gg kT$, where kT is the thermal excitation energy of the detector. See M. C. Teich, Appl. Phys. Letters 14, 201 (1969).

[8] The reception in this system has the additional advantage of being angle-independent, in the sense that the Doppler shift is proportional to the radial velocity $v_{\parallel}$, and, therefore, is generally a function of angle.

[9] W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise* (McGraw-Hill Book Co., New York, 1958), p. 250.

[10] M. I. Skolnik, *Introduction to Radar Systems* (McGraw-Hill Book Co., New York, 1962), p. 185.

[11] W. E. Murray, Jr., in *Optics Research* (MIT Lincoln Laboratory, Lexington, Mass., 1969), No. 1, p. 6.

[12] This result applies for a photoemitter or a reverse-biased photodiode. For a photoconductive or photovoltaic detector, the quantity η must be replaced by $\eta/2$. See Ref. 5.

[13] From Eq. (3), it may be seen that ${}^3P_\gamma(\text{min})$ is always greater than $P_\gamma(\text{min})$.

[14] H. A. Bostick, IEEE J. Quantum Electron. QE-3, 232 (1967); H. A. Bostick and L. J. Sullivan, in *Optics Research* (MIT Lincoln Laboratory, Lexington, Mass., 1969), No. 1, p. 21.

PASSIVE *Q*-SWITCHING OF A N_2O LASER USING ETHYLENE*

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Passive Q-switching of a N_2O laser operating on the $P(19)$ line at 922.37 cm^{-1} has been achieved using ethylene as the saturable absorber. Use of a grating together with an iris eliminates spurious oscillations on the other laser lines.

Passive Q-switching of the CO_2 laser has been achieved using different gases among which are sulfur hexafluoride,[1] vinyl chloride,[2] boron trichloride,[3] but only rotating mirrors Q-switching[4] of the N_2O laser has been reported to date. In this letter, we report the passive Q-switching of a N_2O laser oscillating on the $P(19)$ line at 922.37 cm^{-1}[5] by ethylene. The absorption coefficient of C_2H_4 at 922.37 cm^{-1} (10.84 μ) has been measured to be 0.12 cm^{-1} Torr^{-1}. At N_2O wavelengths other than 10.84 μ, ethylene is nearly transparent so that a laser cavity with a high Q in a narrow spectral band centered at 10.84 μ is needed to suppress cw oscillations on lines other than $P(19)$. A laser ful-

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