this wire suitable for 60-Hz applications, neglecting, of course, eddy current losses in copper.

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FIELD-THEORETICAL TREATMENT OF PHOTOMIXING*

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A quantum theory of photomixing is developed in which double- and sum-frequency signal components do not appear, when $h\nu \gg kT$, in distinction to the classical theory. Optimum sinusoidal photomixing occurs for a first-order coherent total incident field with stationary constituent beams. The heterodyne process may be viewed as the annihilation of a single nonmonochromatic photon. An interpretation is given to temporal interference experiments at the single-photon level recently performed by Radloff.

The phenomenon of photomixing, or heterodyning, has been observed in the optical[1] and in the infrared[2] regions of the electromagnetic spectrum. We present here a quantum theory of photomixing which differs from both the classical and the semiclassical treatments which presently exist. It is applicable to nonstationary as well as stationary fields, of an arbitrary statistical nature.

We consider an infrared or optical heterodyne receiver in which two plane parallel electromagnetic waves of frequencies ω_1 and ω_2 impinge normally on an ideal quantum-mechanical photodetector in its ground state. It is assumed that the photon energy $h\nu$ is much greater than the thermal excitation energy of the detector, kT. Glauber,[3,4] has shown that the average count rate for such a detector (to good approximation), at the space–time point $x = \mathbf{r}, t$, may be expressed as the first-order correlation function $G_{\mu\nu}{}^{(1)}(x,x)$, where

$$G_{\mu\nu}{}^{(1)}(x,x) = \text{trace}\{\rho E_\mu^-(x)\, E_\nu^+(x)\}. \tag{1}$$

Here, ρ is the density operator for the field,[4,5] and the quantities E^- and E^+ are the negative- and positive-frequency portions of the electric field operator E, respectively. The subscripts μ, ν label Cartesian components. For simplicity, only projections of the field along a single (possibly complex) unit vector are considered, so that the correlation function above may be written as a scalar quantity rather than as a tensor.

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Coherent detection experiments are frequently performed using a given beam and a time-delayed form of the same beam[6] (so-called homodyne detection), so that it is more convenient to discuss time correlations of the field relative to the radiation source rather than to the detector.[7] That is, the output of a detector illuminated by a single beam is proportional to $G^{(1)}(x',x')$, where $x' = \mathbf{r}, t'$. When illuminated by a phase-retarded form of the same beam, the output of the detector *at time* t' may be written as $G^{(1)}(x'',x'')$ where $x'' = \mathbf{r}, t''$ and $t'' > t'$. Thus, phase retardation is equivalent to time displacement at the detector. The detection of the total incident field (consisting of two component beams) at the time t is therefore expressed in terms of two individual time parameters t_1 and t_2.

For the heterodyne experiment, we may write the total electric field operator as a superposition of the operators for the constituent waves.[4] The positive-frequency component of the field present at the photodetector, $E^+(\mathbf{r},t)$, may therefore be written

$$E^+(\mathbf{r},t) = \lambda_1 E^+(\mathbf{r},t_1) + \lambda_2 E^+(\mathbf{r},t_2). \tag{2}$$

The complex coefficients λ_1 and λ_2 contain the relative strengths of the two waves, and are taken to be independent of the properties of the field. The count rate R may then be expressed as

$$R = \text{trace}\{\rho[\lambda_1^* E^-(x_1) + \lambda_2^* E^-(x_2)][\lambda_1 E^+(x_1) + \lambda_2 E^+(x_2)]\}, \tag{3}$$

with $x_j = \mathbf{r}, t_j$. Again, the quantity t_j is taken relative to the radiation source, and r represents a point on the detector surface. We now assume that the total incident radiation field would display maximum-contrast fringes[8] in a hypothetical spatial interference experiment performed with the radiation present at any two points on the detector surface, at an arbitrary time t. It is not difficult to show that in this case, the term containing the quantity $\lambda_1^* E^-(x_1)\lambda_2 E^+(x_2)$ factors into the scalar product $\lambda_1^* \mathcal{E}^*(x_1)\lambda_2 \mathcal{E}(x_2)$. Furthermore, first-order coherence of the individual beams may be shown to be a consequence of this same property of the combined total beam. Using this assumption, then, and the correlation function identity $[G^{(1)}(x_1,x_2)]^* = G^{(1)}(x_2,x_1)$, the count rate becomes

$$R = |\lambda_1|^2 G^{(1)}(x_1,x_1) + |\lambda_2|^2 G^{(1)}(x_2,x_2) + 2\mathrm{Re}\{\lambda_1^* \mathcal{E}^*(x_1)\lambda_2 \mathcal{E}(x_2)\}. \tag{4}$$

We now direct our attention to component beams which are stationary. This condition, coupled with individual first-order temporal coherence for these fields, implies monochromaticity of the individual beams. The function $\mathcal{E}^*(x_j)$ for a well-collimated, fully polarized beam of frequency ω_j may then be expressed as[4] $\mathcal{E}^*(x_j) = [G^{(1)}(x_j,x_j)]^{1/2} \exp i(\omega_j t_j - \mathbf{k}_j \cdot \mathbf{r})$, where the space–time point x_j has its usual definition. Inserting this into Eq. (4), a small amount of algebra leads to a count rate expressed as

$$R = |\lambda_1|^2 G^{(1)}(x_1,x_1) + |\lambda_2|^2 G^{(1)}(x_2,x_2) + 2[G^{(1)}(x_1,x_1)\, G^{(1)}(x_2,x_2)]^{1/2} \times \mathrm{Re}\{\lambda_1^*\lambda_2 e^{i(\omega_1-\omega_2)t_1} e^{i\omega_2\tau}\}, \tag{5}$$

where the quantity $\omega_2\tau = \omega_2(t_1 - t_2)$ may be thought of as a phase difference between the beams. The spatially dependent exponential portions of $\mathcal{E}$ and $\mathcal{E}^*$ have been suppressed in writing Eq. (5) because the first-order coherence requirement for the total field, imposed across the photodetector surface, ensures parallel constituent beams, and we have assumed normal incidence. In the quantum treatment, therefore, it is seen that the parallelicity requirement for optimum photomixing need not be stated as a separate condition.

The count rate for a restricted ensemble,[4] chosen such that the phase difference $\omega_2\tau$ is constant in time and precisely cancels the phase factors arising from λ_1^* and λ_2, and for a field possessing first-order coherence with stationary constituent beams, may therefore finally be written as

$$R = |\lambda_1|^2 G^{(1)}(x_1,x_1) + |\lambda_2|^2 G^{(1)}(x_2,x_2) + 2[|\lambda_1|^2 G^{(1)}(x_1,x_1)\,|\lambda_2|^2 G^{(1)}(x_2,x_2)]^{1/2} \times \cos(\omega_1-\omega_2)t. \tag{6}$$

The quantity t_1 has been written as t in the interference term. We note that $G^{(1)}(x_1,x_1)$ and $G^{(1)}(x_2,x_2)$ are count rates which are constant in time and do not possess any fluctuating components. In terms of the classical intensities I_1 and I_2 for the individual beams, this is equivalent to

$$R = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos(\omega_1 - \omega_2)t. \tag{7}$$

It is clear from the quantum analysis that rapidly varying sum- and double-frequency components never appear for the usual absorption detector when $h\nu \gg kT$, and therefore would not be observed even with detectors of arbitrarily small resolving time. In the low-frequency limit, where $h\nu \ll kT$, the quantum theory for the heterodyne detector reduces to the *classical* result, since in this region photons are emitted as readily as they are absorbed.

A physical interpretation of the beating process may be obtained by considering the constraints which the coherence condition places on the density operator for the radiation field. Titulaer and Glauber[9] have shown that a field which has first-order coherence may be regarded as consisting of photons of only a single (in general nonmonochromatic) variety. If the field contains solely this type of photons, it is first-order coherent, and the photomixing process may be considered as the annihilation of a *single* one of these photons. Dirac's[10] well-known comment, "…. Each photon interferes only with itself. Interference between two different photons never occurs", therefore applies to the heterodyne experiment, as it applies in the case of spatial interference experiments as recently discussed by Pfleegor and Mandel.[11] This is not surprising since we are considering a type of interference experiment which is a one-quantum process. For multiple-photon processes, such as two-quantum photodetection[7,12] or the Hanbury-Brown Twiss effect, however, this is not necessarily true.[4]

Radloff[13] has recently performed temporal interference experiments with two independent He-Ne laser beams of low intensity ($\approx 10^7$ photons/sec in each beam). He used a photoemissive detector with a quantum efficiency of 2%. The beat signal was piezoelectrically adjusted to a frequency of about 2 kHz, and the receiver bandwidth was restricted to 10 kHz by means of a low-pass filter. In his geometry, the mean interval between photons was about 70 times the photon transit time from source to detector so that, to good approximation, only one photon was received at a time. Thus, his time interference experiments are analogous to the spatial interference experiments at the single-photon level recently reported by Pfleegor and Mandel.

The observed interference in Radloff's experiment is entirely compatible with the theoretical results presented in this paper. That is, the interference is associated with the annihilation, or detection, of a *single* (nonmonochromatic) photon. Equivalently, it is not difficult to show that an application of the uncertainty principle $\Delta E \Delta\tau \gtrsim \hbar$ to the detection process makes it impossible to ascertain from which source a given photon came, in analogy with a similar result for spatial interference.[11] Radloff's interpretation of the heterodyne process, as involving more than a single photon, does not provide an appropriate description for the process. Indeed, Dirac's comment[10] cited earlier does apply to these experiments.

An important parameter in any experiment is, of course, the signal-to-noise ratio. For 2 beams of equal intensity, with a photoemissive detector (which is the configuration of Radloff's experiment), the voltage signal-to-noise ratio $(S/N)_V$ may easily be shown to be[2]

$$(S/N)_V = (\eta N/2\Delta f)^{1/2}. \qquad (8)$$

Here, η is the detector quantum efficiency, N is the number of photons per second in each beam, and Δf is the receiver bandwidth. In deriving this expression, it has been assumed that the only contribution to the noise is the shot noise; neither excess noise arising from photon bunching[14] nor noise arising from amplification has been considered. Inserting the values appropriate for Radloff's experiment, we obtain $(S/N)_V \approx (10)^{1/2}$ which, as expected, is in good agreement with the experimental observations (as seen from the upper oscilloscope trace in Fig. 1 of his paper[13]). The minimum detectable number of photons per second $N_{\min}$ (for a $S/N = 1$ and equal beam intensities) is given by

$$N_{\min} = 2\Delta f/\eta. \qquad (9)$$

This quantity could therefore be decreased far below the 10^6 sec^{-1} value obtained with Radloff's experimental parameters, by simply decreasing the receiver bandwidth Δf. Stated differently, the same experiment could be performed with beams of considerably lower intensity, provided only that Δf is decreased.

Thus, it is seen that the description of a temporal interference experiment at the single-photon level is similar to that of a spatial interference experiment at the single-photon level. The interference, in both cases, may be associated with the detection of a single photon.

A fuller account of the theory of quantum-mechanical photomixing, along with a treatment of some cases not considered here, will be published elsewhere. It is a pleasure to thank Professor R. J. Schwarz for helpful discussions.

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