the two phenomena. Further, all electron beam spots on silicon wafers examined which indicated a rosette-type stress pattern by x-ray topography also indicated birefringence. Also, all spots which did not show a rosette pattern did not exhibit birefringence.

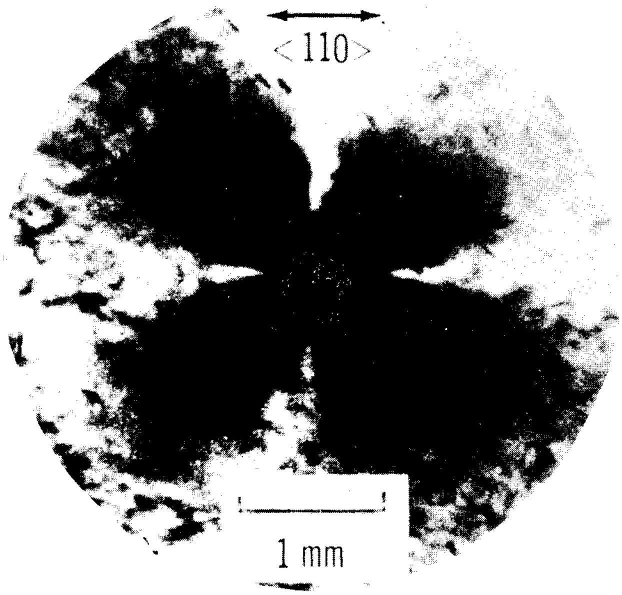

Fig. 3. X-ray topograph of electron beam spot on silicon wafer showing rosette stress pattern. {220} reflection. 12.5×

The results of this investigation on silicon indicate that not only are x-rays more sensitive than infrared radiation to thermally induced stresses but they also could be more informative in relating the stresses to the crystallography of the lattice. These characteristics of x rays have also been indicated by Goldstein[10] in his work on light scattering from elastic strains. It appears that a more intensive study of the dynamical interactions of x rays with an imperfect lattice is required before we can understand the relationship between scattering phenomena arising from various electromagnetic radiations.

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OPTIMUM HETERODYNE DETECTION AT 10.6 μm IN PHOTOCONDUCTIVE Ge:Cu

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A measurement of the signal-to-noise ratio and minimum detectable power for heterodyne detection of scattered radiation at 10.6 μm has been made. Using photoconductive Ge:Cu as the detector, the observed minimum detectable power, at a frequency of 70 kHz and in a bandwidth of 270 kHz, was 3.5×10^{-14} W. This corresponds to a minimum detectable power of 1.3×10^{-19} W in a 1-Hz bandwidth, which is within a factor of 10 of the theoretically perfect photon counter.

Using a Ge:Cu photoconductor as a heterodyne detector[1–3] at 10.6 μm, we have observed a minimum detectable power which is within a factor of 10 of the theoretical quantum limit, $h\nu B$. In the experimental arrangement used, the radiation from a CO_2-N_2-He laser, emitting approximately 10 W at 10.6 μm, was incident on a modified Michelson interferometer (see Fig. 1). One mirror of the conventional interferometer was replaced by an off-center rotating aluminum wheel with a roughened surface. The diffusely scattered radiation from the wheel provided a Doppler-shifted signal which was

*Operated with support from the U. S. Air Force.

recombined at the beam splitter with the unshifted (or local oscillator) radiation reflected from the mirror of the other interferometer leg.

Previously reported heterodyne experiments using photoconductors have been made only in the visible[4,5] and in the near infrared (1.15 μm).[3] The use of an InAs photodiode has permitted measurements to be extended to 3.5 μm.[6] Heterodyne detection measurements of scattered radiation at 6328 Å have been made by G. Gould et al.[7] and by R. D. Kroeger.[8] With the availability of the CO_2 laser, a sensitive detector at 10.6 μm is especially significant for such applications as heterodyne spectroscopy experiments as well as for systems use.

Figure 1 shows the experimental setup which, with the exception of the rotating wheel and the copper, was mounted on a granite slab supported by compressed fiberglass blocks. To further minimize the effect of acoustic vibrations, the 1.25-m-long sealed laser tube was enclosed in a shield constructed of acoustic tile. The laser was operated on a single line and mode. An uncoated Irtran II flat (of thickness 0.64 cm) served as a beam splitter, and front surface mirrors were of standard aluminum-coated glass.[9] A 2.54-cm-focal-length Irtran II lens inserted in the signal beam focused the radiation to a single point on the rim of the rotating wheel. The function of the lens was to collect sufficient scattered radiation to permit an incoherent (nonheterodyne) measurement of the scattered signal power at the detector for calibration purposes, and to insure spatial coherence of the scattered radiation at the detector.[7] Irises were used to maintain the angular alignment of the wave fronts of the two beams to ~2 mrad, which is well within the required angular tolerance for optimum photomixing ($\lambda/a \sim 5$ mrad for a detector aperture $a = 2$ mm).[2] It should be noted that this angular alignment restriction is twenty times less stringent than in the visible region of the spectrum. A Perkin-Elmer wire-grid polarizer insured that the recombined beams had a common linear polarization.

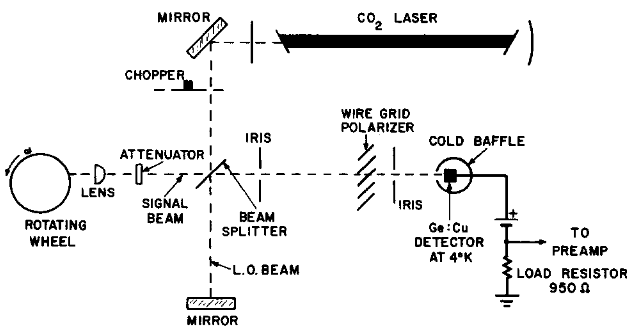

Fig. 1. Experimental arrangement. The electric field vector lies perpendicular to the plane of the paper.

The detectors were made by indiffusion of Cu into high resistivity n-type germanium host material for a period of 16 hr at 760° C. The samples, which were 2 mm × 2.2 mm × 3 mm in size, were then quenched in air. The resulting copper atom concentration was 6.8×10^{15} cm^{-3}, and the compensation by the original donors was such as to produce a free hole lifetime of about 2×10^{-9} sec at 4°K. With a bias voltage of 13.5 V on the detector, its (incoherent) low-power responsivity was found to be 0.2 A/W by calibration with a black-body source of known temperature. The heterodyne signal obtained from the load resistor was fed into a thermocouple-type rms voltmeter through a controlled-bandwidth, low-noise preamplifier. Alternately, the load resistor output was fed simultaneously to an oscilloscope and to a spectrum analyzer.

The results of a typical measurement are shown in Fig. 2. The solid line is the observed signal-to-noise power ratio, $(S/N)_{\rm power}$ of the heterodyne signal as a function of the signal beam radiation power (P_s). Only noise arising from the presence of the L. O. beam (which was the dominant contribution to the noise) is considered. Various values of P_s were obtained by inserting calibrated CaF_2 attenuators in the signal beam, while the unattenuated power was measured by chopping the signal

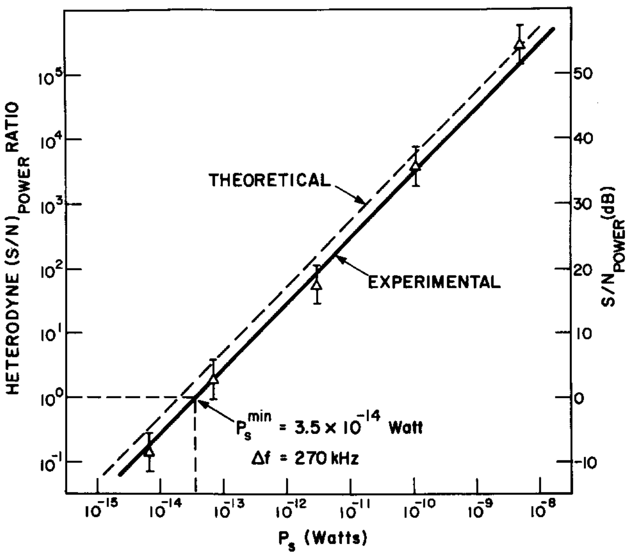

Fig. 2. The solid line is the observed signal-to-noise power ratio, $(S/N)_{\rm power}$, of the heterodyne signal as a function of the signal beam radiation power (P_s) for a typical run. The observed minimum detectable power $P_s^{\rm min}$ [defined as that signal beam power for which the heterodyne S/N is unity] corresponds, in a 1-Hz bandwidth, to 1.3×10^{-19} W.

beam in the absence of the L. O. With a heterodyne signal centered at about 70 kHz, and an amplifier bandwidth of 270 kHz, the experimentally observed minimum detectable power P_s^{min} [defined as that signal beam power for which the heterodyne S/N is unity] is seen to be 3.5×10^{-14} W. This corresponds, in a 1-Hz bandwidth, to a minimum detectable power of 1.3×10^{-19} W. Measurements were made with an L. O. power of 1.5 mW. A plot of the theoretical expression[1,3] $(S/N)_{power} = \eta P_s/2h\nu\Delta f$, which includes the effect of *g-r* (generation-recombination) noise from the photoconductor[10,11] is also shown in Fig. 2. With an assumed quantum efficiency $\eta = 1/2$, it is seen to lie above the experimental curve by a factor of approximately 2, but within the limit of experimental accuracy. Had noise from sources other than the L. O. been taken into account in computing the S/N, the experimental curve would be a factor of 3 below the theoretical. Although the signal-to-noise ratio agrees closely with theory, the observed values of signal and noise[10,11] are higher than calculated on the basis of the measured responsivity. An explanation for this observation is being sought in a possible dependence of the photoconductor gain on the frequency or on the power of the incident radiation.

Figure 3(*a*) shows a multiple-sweep display of the heterodyne signal obtained with a signal power of 1.0×10^{-8} W. The loss of definition of the waveform in the third cycle reflects the finite bandwidth of the heterodyne signal. Figure 3(*b*) shows a single trace of this signal for a longer time scale. The modulation bandwidth is caused by statistical fluctuations of the heterodyne signal arising from the moving diffuse surface of the wheel. For the experimental configuration used, where the roughness of the wheel (~10 μm) is comparable with the size of the focused spot on the wheel (~50 μm), the bandwidth of the noise modulation, B, should be approximately[7] v/d. Here, v is the velocity at which the illuminated spot traverses the surface, and d is the diameter of the focused spot on the wheel. With $v = r\omega$ and $d \sim F\lambda/D$, B is given approximately by $r\omega D/F\lambda$. Here v is the tangential velocity of the wheel (157 cm/sec), ω is its angular velocity (10 π sec^{-1}), and r its radius (5.05 cm). F represents the focal length of the lens (2.54 cm), while D is the diameter of the radiation beam at the output of the laser (~5 mm). Evaluating this, we obtain a calculated value $B \sim 30$ kHz which is comparable with values observed on the spectrum analyzer.

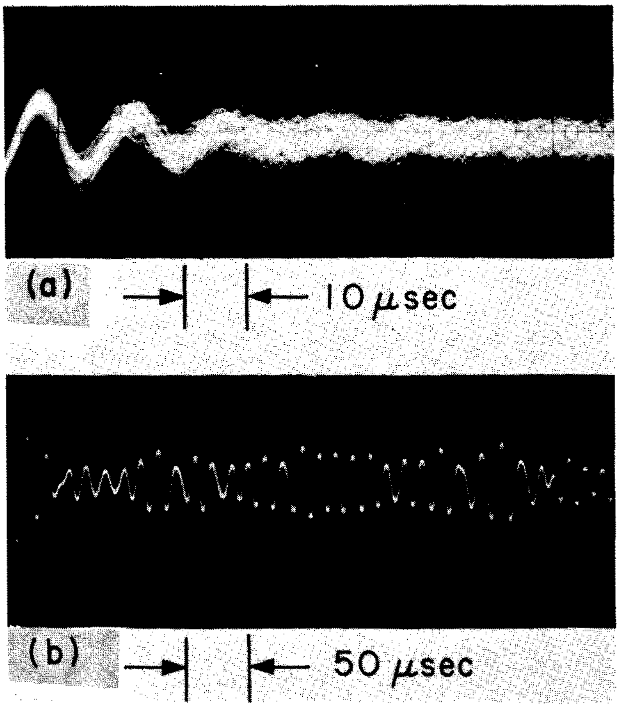

Fig. 3. (*a*) A multiple-sweep display of the heterodyne signal. The loss of definition of the waveform in the third cycle reflects the finite bandwidth of the heterodyne signal. (*b*) A single sweep of the heterodyne signal shown in (*a*), but with a longer time scale. The modulation of the signal envelope arises from the random nature of the scattering surface.

As a final note, we should like to point out that intensity fluctuations arising in the CO_2 laser were inappreciable at the (70 kHz) frequency of operation. It is expected that higher frequency operation will be limited only by the lifetime of the carriers in the detector, and by the detector capacitance. By proper compensation, Ge:Cu detectors with lifetimes as short as 10^{-12} sec have been made at this laboratory. Nonetheless, operation near the theoretical limit for higher frequencies, although expected, has not yet been experimentally demonstrated.

It is a pleasure to thank F. D. Carroll for technical assistance. We are grateful to R. J. Carbone for information and assistance in the design of the CO_2 laser, and to H. A. Bostick for helpful discussions.

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SPATIALLY RESOLVED LASER HETERODYNE MEASUREMENTS OF PLASMA DENSITIES IN WEAKLY IONIZED GASES*

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A two-wavelength He:Ne laser heterodyne system has been successfully used to measure electron number densities in the range from $4 \times 10^{12}\,\text{cm}^{-3}$ to $10^{11}\,\text{cm}^{-3}$, over a path length of 24 cm. These experiments were performed on a single-shot basis in pulsed argon discharges. The spatially resolved laser measurements compared quite favorably with integrated electron density measurements made with a microwave cavity operating in the TM_{010} mode.

In this Letter, we report spatially resolved laser heterodyne[1,2] measurements of plasma densities in a weakly ionized gas ($n_e > 5 \times 10^{10}\,\text{cm}^{-3}$) on a single-shot basis. These measurements compare quite favorably with simultaneous microwave (~5 GHz) cavity measurements, and we believe that this is the first time such independent methods have been cross-checked over a large range of plasma densities.

The plasma to be studied was placed within the cavity of one He:Ne laser (the controlled laser) which was oscillating at 0.6328 μ and 1.1523 μ. A change in the plasma's refractivity thus resulted in a slight change of the laser frequencies. This change was measured by standard optical heterodyne techniques, using a second He:Ne laser as a reference or local oscillator.

The frequency of the qth mode of the controlled laser, within the width of a Doppler-broadened transition, is given by

$$\frac{\omega}{c} d_l + \frac{\omega}{c} \Delta n_p(t) d_p + \Delta\phi(\omega) = q\pi \qquad (1)$$

where d_l (105 cm) is the length of the controlled laser cavity in which a plasma of length d_p (24 cm), with refractivity $\Delta n_p(t)$, is formed. The extra phase term $\Delta\phi(\omega)$ is due to the dispersive character of the lasering transition.[3]

Since the plasma refractivity $\Delta n_p(t)$ is significantly affected by the motion of neutral particles across the laser beam as well as by the formation and decay of free electrons in the plasma,[4,5] it is necessary to write Eq. (1) for laser operation at two different wavelengths (0.6328 μ and 1.15 μ) in order to separate out the electronic contribution to the frequency shift. The simultaneous solution of the two equations results in the following relationship between the electron number density and the frequency shifts.

$$n_e(t) = \frac{8\pi^2 cm\epsilon_0}{e^2}\left(\frac{d_l + c(\partial\Delta\phi/\partial\omega)}{d_p}\right) \times \frac{\lambda_2}{\lambda_2^2 - \lambda_1^2}\left(\Delta f_2 - \frac{\lambda_1}{\lambda_2}\Delta f_1\right) \qquad (2)$$

where $\Delta f_{1,2}$ are the frequency shifts of the corresponding laser transitions, $\lambda_{1,2}$. In the derivation of this equation, the contribution of excited states and ions to the plasma refractivity has been ignored, since a computer search revealed no argon lines

*Work supported in part by the National Aeronautics and Space Administration, Wright-Patterson Air Force Base, and Zenith Radio Corporation.