Computer simulation of superposed coherent and chaotic radiation

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A computer simulation technique useful for generating superposed coherent and chaotic radiation of arbitrary spectral shape is described. Its advantages over other techniques include flexibility and ease of implementation, as well as the capability of incorporating spectral characteristics that cannot be generated by other methods. We discuss the implementation of the technique and present results to demonstrate its validity. The technique can be used to obtain numerical solutions to photon statistics problems through computer simulation. We furthermore argue that experiments involving photon statistics can be carried out using a wideband source in place of an amplitude-stabilized source whenever the spectral characteristics of the source are not important. Experimental results that corroborate the argument are presented.

I. Introduction

The photon statistics generated by thermal and chaotic light sources have evoked substantial interest ever since the well-known experiment by Hanbury Brown and Twiss[1,2] demonstrated the occurrence of photon correlation in light generated by natural sources. With most natural sources, however, the time scale over which photon correlation is observable is typically very short and is often virtually negligible; indeed very sensitive and carefully designed experiments are usually required in order to observe such correlation. Correspondingly, the coherence time of the associated electromagnetic field is very short for such sources, as the coherence time provides a measure of the time scale over which variations occur in the intensity of the field.[3]

To circumvent this difficulty, a number of researchers have devised a variety of techniques to generate simulated chaotic light with an increased coherence time in the laboratory. Most of these techniques require a laser as the primary source of light, since they make use of the narrow bandwidth and high degree of coherence of laser radiation. They generate chaotic light by simulating, to different degrees, the physical characteristics of natural sources of chaotic light. One such method[4,5] consists of scattering laser light from particles suspended in a fluid, each particle acting as an independent radiator whose frequency is the laser frequency, Doppler-shifted by the Brownian motion of the particle. In this configuration, the spectral bandwidth and shape can be adjusted to some extent by manipulating the statistical distribution of the size of the particles. Another method consists of scattering laser light from a moving rough surface, such as a rotating ground-glass screen[6,7] or a rotating roughened wheel[8]; here the different surface elements act as independent radiators, and the bandwidth of the light can be adjusted by varying the speed of motion of the surface. Reference 9 provides a survey of the properties of scattered light in general.

If only the intensity variations of chaotic light are of interest (as is the case for the statistics of photon arrivals), it is possible to use the method suggested by Ruggieri *et al.*[10] to simulate a superposition of coherent and chaotic radiation. This method uses two independent generators of Gaussian random noise to simulate the in-phase and quadrature components of the radiation field. The sum of the squares of these components yields the simulated light intensity, which is then used to modulate the output of a laser source. It should be noted that this method is less physical than the others cited above, since only the intensity (amplitude) variations of a chaotic radiation field are reproduced (the phase of the field depends solely on the characteristics of the primary laser source). Indeed, the narrow bandwidth and high degree of coherence of laser radiation are not necessary in this type of simulation, since the laser is used only to generate photons with the statistics of a Poisson point process whose rate can be

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controlled by modulating the intensity of the radiation. An arbitrary source yielding photons with the statistics of a Poisson point process with constant rate can be used in place of the laser to obtain the same simulated photon statistics (doubly stochastic Poisson process[3]) as discussed in Sec. V. Furthermore, it should be noted that with independent generators, only a limited class of power spectra can be reproduced.

We show here how a small laboratory computer can be used to generate the two random processes needed for the particular type of simulation discussed in this paper. The computer can then drive the modulator directly through a digital-to-analog converter. The advantages of using the computer include increased flexibility since an arbitrary spectral shape can be easily simulated. Increased accuracy and simplicity in the experimental setup are other benefits since critical adjustments, noise filtering, and matching of the two independent noise generators are not necessary.

II. Polarized Chaotic Radiation

A source of chaotic radiation can be modeled as a collection of a large number of independent radiators, each emitting radiation at a certain frequency, with a certain phase, a certain amplitude, and a certain plane of polarization. If we assume that the phase is uniformly distributed between 0 and 2π rad and restrict ourselves to a well-defined state of polarization, the contribution to the radiation field in that state of polarization from all the radiators emitting at a given angular frequency ω_k may be expressed as

$$A_k(t) = a_k \cos\omega_k t + b_k \sin\omega_k t, \tag{1}$$

where a_k and b_k are random variables. If the number of independent radiators emitting at ω_k is large, a_k and b_k (by virtue of the central limit theorem and the uniform distribution of phase) are independent identically distributed zero-mean Gaussian random variables whose variance is proportional to the number of radiators at that frequency.

If we assume that radiators exist only for discrete frequencies, the over-all radiation field may be expressed as

$$A(t) = \sum_{k=-\infty}^{\infty} a_k \cos\omega_k t + b_k \sin\omega_k t, \tag{2}$$

where

$$\omega_k = \omega_0 + k\Delta\omega. \tag{3}$$

In the limit where $\Delta\omega \to 0$ this is equivalent to considering a continuum of frequencies. As indicated above, the a_k and b_k coefficients are independent zero-mean Gaussian random variables whose variance is a function of ω_k; i.e.,

$$\mathrm{E}[a_k b_l] = 0 \text{ for any } k,l$$
$$\mathrm{E}[a_k a_l] = \mathrm{E}[b_k b_l] = \begin{cases} 0 & \text{for } k \neq l \\ \Delta\omega\sigma^2(\omega_k) & \text{for } k = l. \end{cases} \tag{4}$$

Here $\sigma^2(\omega)$ is a continuous function corresponding to the power spectrum of the radiation. By substituting Eq. (3) into Eq. (2) we obtain

$$A(t) = \left[\sum_{k=-\infty}^{\infty} (a_k \cos k\Delta\omega t + b_k \sin k\Delta\omega t)\right] \cos\omega_0 t + \left[\sum_{k=-\infty}^{\infty} (b_k \cos k\Delta\omega t - a_k \sin k\Delta\omega t)\right] \sin\omega_0 t \tag{5}$$

or

$$A(t) = x(t)\cos\omega_0 t + y(t)\sin\omega_0 t, \tag{6}$$

where

$$\begin{cases} x(t) = \sum_{k=-\infty}^{\infty} (a_k \cos k\Delta\omega t + b_k \sin k\Delta\omega t) \\ y(t) = \sum_{k=-\infty}^{\infty} (b_k \cos k\Delta\omega t - a_k \sin k\Delta\omega t) \end{cases} \tag{7}$$

are two jointly Gaussian random processes corresponding to the in-phase and quadrature components of the radiation field. The two random processes have the same marginal statistics.

Their autocorrelation can be easily evaluated with the help of Eq. (4):

$$\begin{aligned} R_{yy}(\tau) = R_{xx}(\tau) &= \mathrm{E}[x(t)x(t+\tau)] \\ &= \sum_{k=-\infty}^{\infty} \{\mathrm{E}[a_k^2]\cos[k\Delta\omega t]\cos[k\Delta\omega(t+\tau)] \\ &\quad + \mathrm{E}[b_k^2]\sin[k\Delta\omega t]\sin[k\Delta\omega(t+\tau)]\} \\ &= \sum_{k=-\infty}^{\infty} \Delta\omega\sigma^2(\omega_0 + k\Delta\omega)\cos k\Delta\omega\tau. \end{aligned} \tag{8}$$

In the limit as $\Delta\omega \to 0$, Eq. (8) becomes

$$R_{yy}(\tau) = R_{xx}(\tau) = \int_{-\infty}^{\infty} \sigma^2(\omega_0 + \omega)\cos\omega\tau d\omega. \tag{9}$$

Using similar algebraic manipulations, the cross-correlation between the two random processes may be written as

$$R_{xy}(\tau) = \mathrm{E}[x(t+\tau)y(t)] = \int_{-\infty}^{\infty} \sigma^2(\omega_0 + \omega)\sin\omega\tau d\omega. \tag{10}$$

We see that the two random processes are independent if the function $\sigma^2(\omega)$ is symmetric around ω_0 (since the sine function is antisymmetric), but in general, for an arbitrary nonsymmetric power spectrum, the two components will not be independent.

For narrowband radiation, if ω_0 denotes the center frequency, $x(t)$ and $y(t)$ are slowly varying with respect to ω_0, and the intensity of the radiation will be given by

$$I(t) = x^2(t) + y^2(t). \tag{11}$$

Using standard expressions for the moments of jointly Gaussian random variables, we can evaluate the autocorrelation of $I(t)$:

$$\begin{aligned} R_{II}(\tau) &= \mathrm{E}[(x^2(t) + y^2(t))(x^2(t+\tau) + y^2(t+\tau))] \\ &= 4[R_{xx}^2(0) + R_{xx}^2(\tau) + R_{xy}^2(\tau)]. \end{aligned} \tag{12}$$

From the foregoing discussion, it is clear that in order to simulate the intensity variations of chaotic light of a given power spectrum $\sigma^2(\omega)$, it is sufficient to generate two Gaussian random processes with the appropriate autocorrelation and cross-correlation prescribed by Eqs. (9) and (10). The two are then individually squared and added, and the result is used to modulate the output of a suitable light source (see Sec. V). The statistics of photon arrivals from such a source will be indistinguishable from chaotic light with the desired spectrum.

The basic notion for this type of simulation was first suggested by Lachs *et al.*,[10,11] who used it with a laser source to simulate a superposition of chaotic and coherent radiation; the chaotic component possessed a Lorentzian spectrum and was superposed on a single centrally located coherent mode. The prescribed superposed power spectrum has the form

$$\sigma_L^2(\omega) = \frac{A}{(\omega-\omega_0)^2+\Gamma^2} + B\delta(\omega-\omega_0), \tag{13}$$

where A and B are constants, 2Γ is the half-power radian bandwidth, and δ is the Dirac delta function. Since this spectrum is symmetric about ω_0, Eq. (10) provides that $x(t)$ and $y(t)$ are statistically independent. Accordingly, it was possible to simulate this light with two independent laboratory noise generators. The experimental results obtained by Ruggieri *et al.*[10] are in very good agreement with the theory.

In general, for an arbitrary (nonsymmetric) power spectrum, it will not be possible to use two independent sources to generate $x(t)$ and $y(t)$. Computer simulation provides a solution to this problem by generating $x(t)$ and $y(t)$ with the appropriate joint statistics for any specified power spectrum.

III. Computer Simulation

The well-known fast Fourier transform (FFT) algorithm[12,13] is a fast algorithm for the evaluation of the discrete Fourier transform (DFT). If we consider a sequence of N (where N is even) complex numbers Z_k, $k = -N/2, \ldots, N/2 - 1$, the DFT of this sequence will be another sequence of N complex numbers A_l defined as

$$A_l = \sum_{k=-N/2}^{N/2-1} Z_k W^{lk}, \tag{14}$$

where $W = \exp(2\pi i/N)$. Equivalently,

$$A_l = \sum_{k=-N/2}^{N/2-1} Z_k \left(\cos k\frac{2\pi}{N} l + i \sin k \frac{2\pi}{N} l\right), \tag{15}$$

from which we obtain

$$\begin{cases} \mathrm{Im}A_l = \sum_{k=-N/2}^{N/2-1} \left(\mathrm{Im}Z_k \cos k\frac{2\pi}{N} l + \mathrm{Re}Z_k \sin k\frac{2\pi}{N} l\right), \\ \mathrm{Re}A_l = \sum_{k=-N/2}^{N/2-1} \left(\mathrm{Re}Z_k \cos k\frac{2\pi}{N} l - \mathrm{Im}Z_k \sin k\frac{2\pi}{N} l\right). \end{cases} \tag{16}$$

If we consider band-limited light, the infinite summations in Eq. (7) can be replaced by summations over a finite number N of frequency terms, so that

$$\begin{cases} x(t) = \sum_{k=-N/2}^{N/2-1} (a_k \cos k\Delta\omega t + b_k \sin k\Delta\omega t) \\ y(t) = \sum_{k=-N/2}^{N/2-1} (b_k \cos k\Delta\omega t - a_k \sin k\Delta\omega t). \end{cases} \tag{17}$$

It is immediately evident that Eqs. (16) and (17) are equivalent when the following substitutions are made:

$$\begin{matrix} \mathrm{Im}A_l = x(l\Delta t) & \mathrm{Im}Z_k = a_k & \Delta\omega = 2\pi/N\Delta t \\ \mathrm{Re}A_l = y(l\Delta t) & \mathrm{Re}Z_k = b_k & t = l\Delta t. \end{matrix} \tag{18}$$

Stated differently, if the real and imaginary parts of the Z_k's are independent Gaussian random variables whose variance is an appropriate function of k [as given by Eq. (4)], the real and imaginary parts of the A_l's will correspond to Nyquist samples of $x(t)$ and $y(t)$ (taken at regular intervals). If we denote the interval between samples as Δt, the frequency step appearing in Eq. (17) will be $\Delta\omega = 2\pi/N\Delta t$. For implementation, the simulation program makes use of a generator of Gaussian random numbers with zero mean and given variance, which it calls $2N$ times to generate $2N$ random values for the real and imaginary parts of the Z_k's with variance $\Delta\omega\sigma^2(\omega_0 + k\Delta\omega)$ as given by Eq. (4). The simulation program then uses the FFT algorithm to evaluate the DFT of the Z_k sequence, thus obtaining the A_l sequence, whose real and imaginary parts will be Nyquist samples of the simulated $x(t)$ and $y(t)$. Therefore the Nyquist samples of the simulated light intensity are given by the sequence of absolute values squared $|A_l|^2$.

The simulation program is thus able to evaluate the simulated intensity and output the Nyquist samples through a digital-to-analog converter channel. The voltage output would be fed into a low-pass filter to obtain the correct waveform to modulate the light source.[14] In practice it is simpler to choose Δt rather smaller than the minimum necessary (the minimum is $f_{\max}/2$, where $f_{\max}$ is the largest frequency to be reproduced, as prescribed by the sampling theorem[14]), so that instead of a low-pass filter a simple one-stage RC filter will be sufficient, or even no filter at all, depending on how small Δt is. In the case of an infinite spectrum, such as the Lorentzian spectrum, choosing a small Δt means truncating the spectrum very conservatively, which also has the advantage of minimizing errors due to spectral truncation.

To simulate coherent modes superposed on the chaotic radiation it is sufficient to add a fixed (complex) constant with the appropriate value to the corresponding frequency term before evaluating the FFT. A related technique has been described in detail by Ruggieri *et al.*[10]

IV. Implementation

The simulation method described was implemented on a DEC PDP 11/03 laboratory computer in FORTRAN. The generator of Gaussian random numbers was taken from DEC's Scientific Subroutine Package; the quality of the random numbers generated by it was tested by evaluating the histogram of the distribution and the autocorrelation of the sequence of numbers generated, and was found to be quite satisfactory. The FFT algorithm was provided by subroutine FFT2 from the International Mathematical and Statistical Libraries (IMSL). This subroutine accepts as input a complex

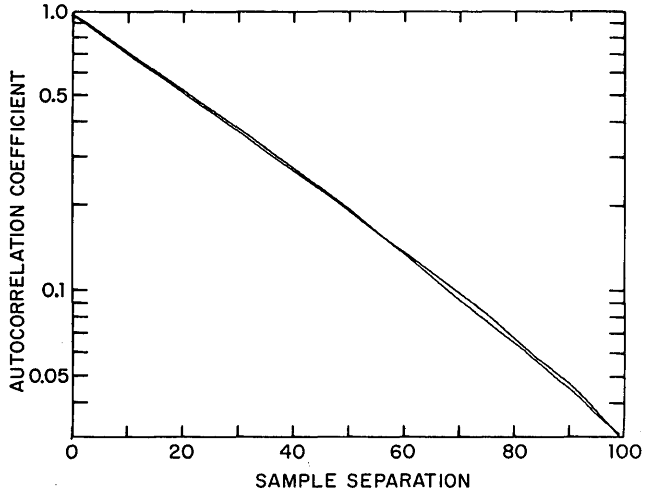

Fig. 1. Autocorrelation coefficients of the in-phase and quadrature components of computer-simulated Gaussian-Lorentzian radiation as a function of Nyquist sample separation. As expected, the two curves are very similar and appear as virtually straight lines on a log-linear plot, corresponding to an exponential falloff.

1-D array whose size N is a power of 2 and returns its DFT in the same array space. The N elements of the array are numbered from 1 to N, and the correspondence with the sequence of Z_k's appearing in Eq. (14) is as follows:

$$Z_k = \begin{cases} Z(N+k+1) & \text{for } -N/2 \le k < 0 \\ Z(k+1) & \text{for } 0 \le k < N/2, \end{cases} \tag{19}$$

where $Z(J)$ is the Jth element of the input array.

In order to simulate Gaussian-Lorentzian radiation, the elements of the input array were filled with zero-mean Gaussian random numbers with variance

$$\mathrm{E}[(\mathrm{Re}Z_k)^2] = \mathrm{E}[(\mathrm{Im}Z_k)^2] = \sigma_k^2 = A/[1 + (Bk)^2], \tag{20}$$

where A and B are constants. With the substitutions given in Eqs. (3) and (18) it can be seen to correspond to a Lorentzian spectrum with half-power radian bandwidth $2\Gamma = 4\pi/BN\Delta t$, when the rate of the Nyquist samples is $1/\Delta t$. After evaluation of the FFT, the results were tested by direct evaluation of the autocorrelation of the sequence of the real parts and of the imaginary parts of the output array, and of the cross-correlation between the two. For a Lorentzian spectrum we expect the two autocorrelations to be equal and to fall off exponentially, while the cross-correlation should be virtually zero, since the spectrum is symmetric [see Eq. (10)].

The simulation and test program was run 100 times with the parameters $N = 2^{12} = 4096$ and $\Gamma = (\pi/100)\cdot(1/\Delta t)$. Since the truncation frequency is given by $(N/2)\Delta\omega = \pi(1/\Delta t)$, we see that errors due to spectral truncation will be negligible. Furthermore the difference between consecutive samples will be very small, so that it will be possible to produce the simulated waveform by putting out the samples through a digital-to-analog converter at a regular rate, without any special filtering. This corresponds to trading off computer execution time for simplicity of implementation, which can be advantageous, since the simulation program does not have to be run in real time but can be run in advance, its output being stored on a tape or similar medium for use later in real time. The value of $\Delta\omega = 2\pi/N\Delta t = (\pi/2048)(1/\Delta t)$ is considerably less than the value of Γ, so that the shape of the Lorentzian spectrum can be accurately reproduced with discrete terms.

The average results after 100 executions of the program are presented in Figs. 1 and 2. Figure 1 shows the autocorrelation coefficient for the simulated $x(t)$ and $y(t)$ as a function of sample separation. The 100 discrete points have been connected by straight-line segments. The two curves can be seen to be very close to one another and virtually straight lines in a log-linear plot, corresponding to the expected exponential falloff. Figure 2 shows the cross-correlation coefficient as a function of sample separation. Its absolute value can be seen to be always much less than unity, indicating that the two random processes are virtually uncorrelated.

V. Poisson Statistics Using a Wideband Source

It is well known that, for an arbitrary light source, the number of photons observed during a time interval of fixed duration is a random variable whose distribution is, in general, different from Poisson. This distribution effectively reduces to Poisson under a broad range of conditions, however, so that the observation of photocounting distributions different from Poisson usually requires carefully designed experiments.[1,2] These experiments often make use of an artificial light source specifically designed to exhibit such non-Poisson behavior, such as those discussed in the introduction.[3-9]

It has been shown by Mandel[15] that the photocounting distribution for a chaotic source reduces to Poisson when the degeneracy parameter of the radiation is much less than unity. Furthermore, Troup and Lyons[16] have demonstrated that under certain conditions the photocounting distribution obtained by

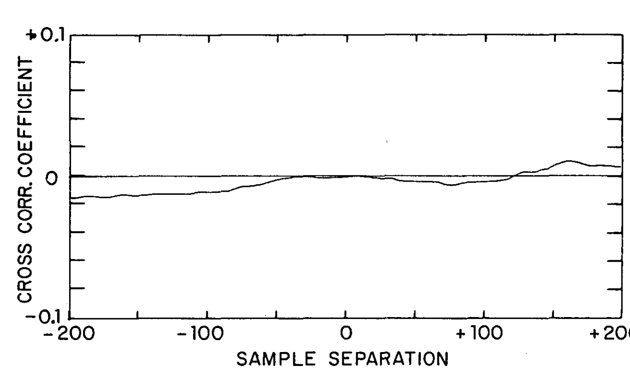

Fig. 2. Cross-correlation coefficient between the in-phase and quadrature components of computer-simulated Gaussian-Lorentzian radiation as a function of Nyquist sample separation. It can be seen that the absolute value of the coefficient is always considerably less than unity, indicating that the two components are virtually uncorrelated.

modulating a narrowband amplitude-stabilized source can also be obtained by modulating a wideband source.

We shall show that the similarity of the photocounting distributions obtained from a wideband source and from an amplitude-stabilized source is a consequence of a more fundamental similarity in the statistics of the underlying random *point processes* corresponding to photon registrations from the two types of source. We can usefully exploit this similarity by using a wideband source in place of an amplitude-stabilized source, not only in experiments where the *photocounting* distributions are important, but also in experiments where the details of the underlying *point process* are relevant.

The degeneracy parameter of an electromagnetic field is defined as the average number of photons per coherence volume.[3] In the case of a (1-D) doubly stochastic Poisson point process, the degeneracy parameter is defined as the average number of events per coherence time. When a quantum photodetector is exposed to optical radiation, quantum absorptions occur, in general, as the events of a doubly stochastic Poisson process whose coherence time corresponds to the inverse of the bandwidth of the radiation.[3] Because the coherence time provides a measure of the memory of the process, there will be correlation between events occurring closer to one another than the coherence time. Events occurring further apart will be less correlated, and events occurring several coherence times apart will be virtually uncorrelated.[17] As a consequence, if the average time interval between photons is much larger than the coherence time, it is seldom that photons occur closely enough to one another to be significantly correlated. Under these conditions, the doubly stochastic Poisson process will be very similar to a simple Poisson process with constant rate; indeed, in the limit of vanishing degeneracy parameter they will be identical.

The light from a typical light-emitting diode (LED), for example, has a coherence time of the order of 10^{-14} sec. Thus, if a photomultiplier exposed to light from an LED generates, on the average, 10^9 photon registrations (pulses)/sec, the average interval between consecutive pulses will be 10^{-9} sec. This is much larger than the coherence time. Under these conditions, for most practical purposes, the photomultiplier pulses are indistinguishable from a Poisson point process whose rate is proportional to the average intensity of the light. Since photons from a narrowband amplitude-stabilized source (ideal single-mode laser) also form a Poisson point process whose rate is proportional to the intensity of the light, the laser can be replaced by an LED (or, in general, by a wideband source). This is true for applications where the statistical behavior is of interest and where the spectral characteristics are unimportant, such as the simulation method discussed in the previous sections. It is sufficient to modulate the average intensity of the wideband source (e.g., by varying the current through the LED).

We note, furthermore, that for a typical *multimode* He–Ne laser, intensity fluctuations over a time scale of the order of 1 nsec arise from mode beating. It is thus possible to generate a Poisson point process using such a source if the light is sufficiently attenuated so that the average interval between photons is much larger than 1 nsec. If these conditions are adhered to, a multimode laser can be used in place of a single-mode laser to generate pseudothermal light through scattering as mentioned in the Introduction. An LED would not be suited to this kind of application, however, because its wider bandwidth would destroy the interference effects that are the basis for these methods.

We performed an experiment to demonstrate the validity of the discussion presented above. The light source was an inexpensive general-purpose gallium–arsenide–phosphide LED whose emission band was centered about 660 nm. The intensity of the source was maintained constant by means of a feedback system employing a Centronic OSD-50-0 silicon photodetector and an RCA CA3140 operational amplifier. The radiation was attenuated by a filter with neutral density 5 and detected by an RCA type-8575 photomultiplier tube. The output pulses from the anode of the photomultiplier tube were converted to square TTL pulses and fed into a PDP 11/03 computer. The computer was capable of recording the time of arrival of the pulses with a resolution of 1 μsec.

The times of arrival, as recorded by the computer, were processed to test whether their statistical behavior was in accord with a Poisson point process with constant rate. Figures 3 and 4 show the results of the two statistical tests performed. The experimental histogram of the distribution of the time interval between consecutive pulses is presented in Fig. 3. For a Poisson point process with constant rate, this distribution should be exponential; the solid straight line in Fig. 3 corresponds to the fitted exponential distribution for the observed mean (2.25 msec). The experimental histogram is clearly in excellent agreement with the fitted distribution.[18] In addition, for a Poisson point process with constant rate, the time interval between two consecutive pulses is a random variable that is independent of the time interval between any other set of consecutive pulses (this is a renewal process). Accordingly, the second test consisted of evaluating the correlation coefficient between interpulse intervals separated by a fixed number of intervening intervals. The results are summarized in Fig. 4. The correlation coefficient can take on values in the range from -1 to $+1$. It is clear that, as expected, the experimental values observed are virtually indistinguishable from zero, as none is larger than 0.01 in absolute value. In Fig. 4, the abscissa represents the separation between the intervals: the first data point corresponds to consecutive intervals, whereas the last data point corresponds to intervals separated by eighteen intervening intervals. The data in both figures were obtained from the observation of 32,769 photomultiplier pulses.

We also performed the same experiment with sunlight obtaining similar results.

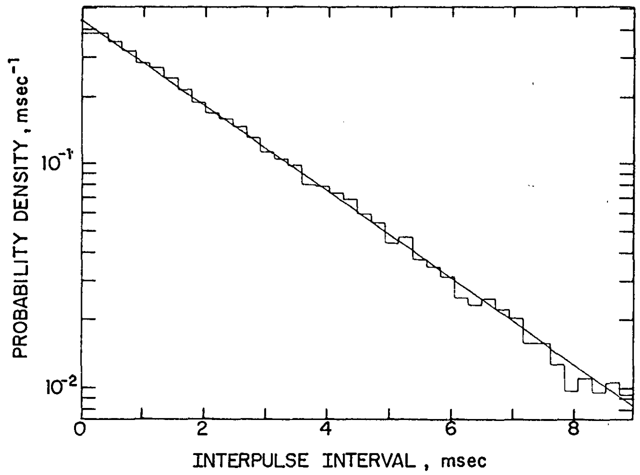

Fig. 3. Statistics of photons from a light-emitting diode (LED). Experimental histogram and theoretical distribution of the time interval between consecutive detected photons. The solid straight line represents the theoretical (exponential) probability density function for a Poisson point process with constant rate. The mean time interval is 2.25 msec.

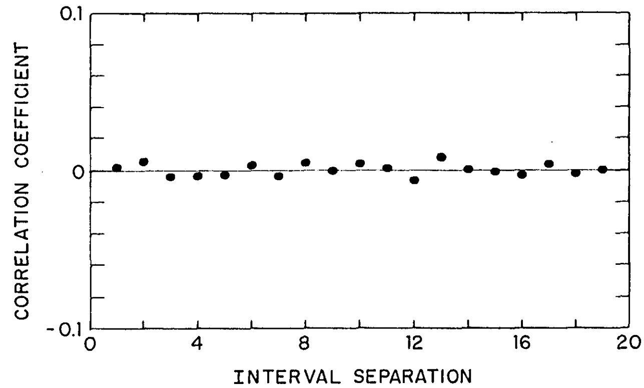

Fig. 4. Statistics of photons from a light-emitting diode (LED). Correlation between interphoton intervals for different interval separations. The first data point corresponds to consecutive intervals, whereas the last data point corresponds to intervals separated by eighteen intervening intervals. Observe that no correlation point exhibits an absolute value larger than 0.01.

VI. Conclusion

The technique presented here can be used on a small laboratory computer, together with a laser and a laser modulator, to simulate a superposition of coherent and chaotic radiation with arbitrary spectral shape. Furthermore, as indicated in Sec. V, the laser and laser modulator can usually be replaced by a wideband source such as a light-emitting diode, thus making the whole system extremely simple to set up and operate.

The same technique can be used in general for the computer generation of a Gaussian random process with a given spectrum, and in particular, it can be used to simulate lognormal intensity statistics, such as those encountered in the propagation of laser light through the turbulent atmosphere.

The most useful application of this technique is probably in conjunction with another program that simulates a Poisson point process with a given rate, to generate computer-simulated photocounting statistics under arbitrary conditions. This will be very useful for the study of systems that are not amenable to analytic solutions, such as photocounting with different types of dead time in conjunction with chaotic light of a given spectrum.[19]

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