$$H'(f'_x,f'_y) = B_1B_2\delta(f'_x,f'_y) + G_1(f'_x/2^{1/2})G_2(-f'_x/2^{1/2})\delta(f'_y). \quad (8)$$

The correlation function between g_1 and g_2 is generally represented in the frequency plane by $G_1(f)G_2*(f)$, where $G_2*(f)$ is the complex conjugate of $G_2(f)$. In the case of a real function g_2, $G_2*(f) = G_2(-f)$. Thus, considering Eq. (8), the axis x'_c now contains the correlation between g_1 and g_2, since the frequency axis for G_2 has been reversed.

Values of θ other than $\pm 45°$ provide the capability of scaling the arguments of g_1 and g_2 as well as producing either the correlation or convolution functions. The bias term B_1B_2 can be eliminated by a central stop in P_f. However, since most optical detectors are sensitive to light intensity rather than amplitude, it may be desirable to include a portion of the bias term to avoid ambiguous results.

Figure 2 shows the application of this method to cross correlation of instantaneous canine blood-pressure and blood-flow-rate signals. A partial blockage or stenosis of the blood vessel was used to create the abnormal flow pattern used in Fig. 2(b). Not surprisingly, the correlation between normal pressure and flow wave forms [Fig. 2(a)] is greater than that between normal pressure and stenotic flow [Fig. 2(b)], even though the peak-to-peak amplitudes of the flow wave forms were unchanged by the stenosis.

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Apodization and image contrast: comment

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In a recent Letter in this journal, Tschunko[1] plotted the MTF curves for an apodized optical system. He considered two typical amplitude filter functions and stated that, generally, MTF is not used in the treatment of apodization.

The purpose of this Letter is to point out that there is extensive literature dealing with the MTF in the presence of apodization. We refer below to some typical papers[2–7] directly relevant to Ref. 1. Some of these papers have already been quoted extensively in the existing literature.[8–13]

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Equivalence of threshold detection with and without dead time

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In the absence of dead time, the probability $p_0(n|W_s)$ of recording exactly n counts in a time interval T from a detector illuminated by a source of radiation with constant intensity is

$$p_0(n|W_s) = \frac{W_s^n}{n!}\exp(-W_s). \quad (1)$$

Here $W_s = (\lambda + \lambda_n)T$, where λ is the average number of counts per unit time due to the signal and λ_n is the average number of counts per unit time due to noise (with the assumption that noise presents itself as an independent Poisson point process with constant rate λ_n). This is the well-known Poisson distribution with mean W_s. In the absence of radiation the probability of recording n noise counts is again given by Eq. (1) with W_s replaced by $W_n = \lambda_n T$. If such a counter is used as a threshold detector, we seek the probability that the number of counts registered exceeds a certain threshold value n_t both in the presence and in the absence of radiation, and we denote that quantity by $P_0(n_t,W)$ where W represents either W_s or W_n. Since the events corresponding to $p_0(n_1|W)$ and $p_0(n_2|W)$ are mutually exclusive whenever $n_1 \neq n_2$, $P_0(n_t,W)$ will be given by the expression

$$P_0(n_t,W) = \sum_{n=n_t+1}^{\infty} p_0(n|W) = \sum_{n=n_t+1}^{\infty} \frac{W^n}{n!}\exp(-W) = 1 - \sum_{n=0}^{n_t} \frac{W^n}{n!}\exp(-W). \quad (2)$$

In the presence of a fixed nonparalyzable dead time τ, we can make use of the results obtained by Ricciardi and Esposito[1] and other workers,[2–7] who have provided an expression for the probability of registering exactly n counts in a time interval T, under the assumption that the counts occur as a Poisson point process with constant rate

$$p(n|W,\tau/T) = \begin{cases} \sum_{k=0}^{n} p_0(k|W[1 - n\tau/T]) - \sum_{k=0}^{n-1} p_0(k|W[1-(n-1)\tau/T]), & n < T/\tau \\ 1 - \sum_{k=0}^{n-1} p_0(k|W[1-(n-1)\tau/T]), & T/\tau \le n < T/\tau + 1 \\ 0, & n \ge T/\tau + 1 \, . \end{cases} \tag{3}$$

Again, the probability that the number of counts exceeds the threshold value n_t is

$$P(n_t,W,\tau/T) = \sum_{n=n_t+1}^{\infty} p(n|W,\tau/T)$$
$$= 1 - \sum_{n=0}^{n_t} p(n|W,\tau/T). \tag{4}$$

Substituting Eq. (3) into Eq. (4) and exchanging the order of summation, we obtain

$$P(n_t,W,\tau/T) = 1 - \sum_{k=0}^{n_t} \frac{[W(1 - n_t\tau/T)]^k}{k!} \exp\{-W(1 - n_t\tau/T)\}, \tag{5}$$

for $n_t < T/\tau$.

We observe that this expression for $P(n_t,W,\tau/T)$ is identical to the expression for $P_0(n_t,W(1 - n_t\tau/T))$ provided in Eq. (2). Thus, for a counter with nonparalyzable dead time τ and a sampling time interval T, the probability of exceeding a certain threshold number of counts n_t is the same as for a counter with no dead time that registers counts at the same rate, but during a sampling time $(T - n_t\tau)$.

This fact can be established directly, without making use of Eq. (3), through the following intuitive argument. Imagine that a Poisson point process generator is connected to the inputs of two counters in parallel, designated A and B. Counter A exhibits a nonparalyzable dead time τ after the detection of each and every pulse (i.e., it is unable to detect additional incoming pulses during a time interval of duration τ after the registration of a pulse). Counter B is constructed in such a way that it behaves precisely like counter A for the first n_t pulses, and thereafter behaves like a counter with no dead time.

It is clear that with such a system the probability of exceeding n_t counts is identical for both counters, since both reach n_t counts precisely at the same time. However, for $k > n_t$ the probability that counter B records exactly k counts can be calculated by observing that it has been dead (unable to detect pulses) for an over-all length of time $n_t\tau$, while during the remaining time interval $T - n_t\tau$ it has detected k pulses. Since the pulses are Poisson distributed, each pulse is independent of the others, and it is irrelevant in what manner the time interval $T - n_t\tau$ is chopped (as long as there is no correlation between where each dead-time interval τ is placed and the pulses that are possibly killed by it). Therefore, if we denote the rate of the Poisson process as λ, the probability of recording exactly k pulses in that length of time is given by Eq. (1) with W_s replaced by $\lambda(T - n_t\tau)$; i.e.,

$$p_B(k|\lambda T,\tau/T) = p_0(k|\lambda(T - n_t\tau))$$
$$= \frac{[\lambda T(1 - n_t\tau/T)]}{k!} \exp\{-\lambda T(1 - n_t\tau/T)\}$$
$$\text{for } k > n_t. \tag{6}$$

Note that this argument is valid only for $k > n_t$ and not for $k = n_t$; in fact, only in the former case are we certain that the counter is dead during a length of time precisely equal to $n_t\tau$. When k is equal to n_t, the dead-time interval following the last pulse might go beyond the end of the sampling time T. In conclusion, the probability that counter B registers more than n_t counts is

$$P_B(n_t,\lambda T,\tau/T) = \sum_{k=n_t+1}^{\infty} p_B(k|\lambda T,\tau/T)$$
$$= \sum_{k=n_t+1}^{\infty} \frac{[\lambda T(1 - n_t\tau/T)]^k}{k!} \exp\{-\lambda T(1 - n_t\tau/T)\}$$
$$= 1 - \sum_{k=0}^{n_t} \frac{[\lambda T(1 - n_t\tau/T)]^k}{k!} \exp\{-\lambda T(1 - n_t\tau/T)\}, \tag{7}$$

which is identical to Eq. (5). As we remarked previously, this is the same as the probability that counter A exceeds n_t counts. Equation (7) can now be used to obtain Eq. (3) by observing that

$$p(n|W,\tau/T) = P(n - 1,W,\tau/T) - P(n,W,\tau/T). \tag{8}$$

This argument therefore provides an alternate route to Eq. (3), for $n < T/\tau$.

The foregoing discussion demonstrates the equivalence of detectors with and without nonparalyzable dead time in the presence of constant-intensity radiation and additive Poisson noise. Equation (5) shows that the performance of the detector with dead time will be the same as that of a similar detector without dead time used under exactly the same conditions, with the same threshold number of counts and sampling time, and the same radiation intensity, but with a quantum efficiency reduced by the factor $(1 - n_t\tau/T)$. This result is valid in general whenever a detector with dead time is used in conjunction with a Poisson point process (e.g., neural counting[1,7] and nuclear counting[2,7]).

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