Mathematical Methods *through* Dance

Extrema

Kinesthetic Expressions of the Beauty and Utility of Math

Scene 1. Expressing equations and the forms of a line

A function is a mathematical equation that translates one variable, x, into another y. We can plot the function $y(x)$ for varying values of x using the cartesian coordinate system. The measure of x is plotted on the horizontal x-axis (the *abscissa*) and the measure $y(x)$ is plotted on the vertical y-axis (the *ordinate*).[1]

[1] Learn more at http://sites.bu.edu/straub/mathematical-methods-for-molecular-science/

René Descartes and the discovery of linear functions

The equation for a line $y(x) = mx + b$ defines the set of all points x that fall on the line $y(x)$ (see Figure 1). The set of all ordered pairs (x, y) are plotted on the xy-plane using cartesian coordinates. Cartesian coordinates are named for the French philosopher and mathematician *René Descartes*(1596-1650) who pioneered the use equations to define geometric shapes like our line. As a plot can be worth a thousand words, an equation can be worth a thousand plots.

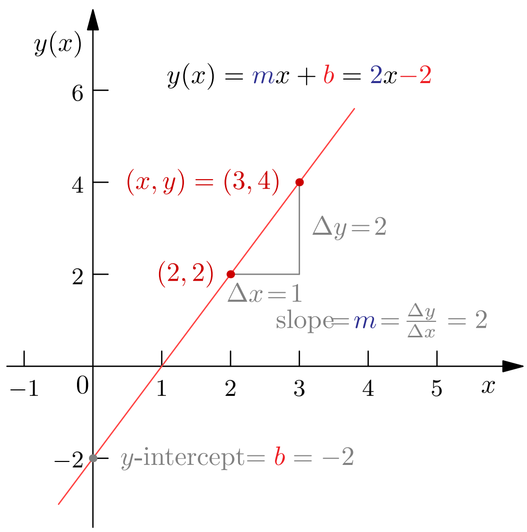

Figure 1: Like the horizontal x-axis and vertical y-axis, the red line of slope 2 extends from minus infinity ($-\infty$) to infinity (∞).

Galileo Galilei, inertia, uniform motion, and the equation of a line

Objects at rest tend to stay at rest. Objects in motion tend to stay in motion, traveling in a straight line at constant speed unless acted upon by a force. This is the property of *inertia* that was first appreciated by the Italian astronomer, physicist and engineer *Galileo Galilei* (1564-1642) in his studies of motion. A linear equation defines the distance traveled in a given time when moving at constant speed. The greater the speed, the greater the slope and the farther the distance traveled in a given time (see Figure 2).

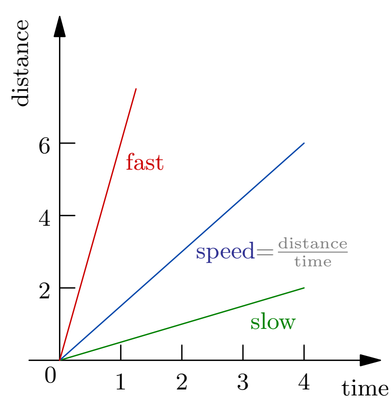

Figure 2: A linear equation defines the distance traveled in a given time when moving at constant speed.

> Dancers representing the x and y axes move from the origin towards infinity. The y-axis is distance, the x-axis is time, and the slope is speed. Points initially at the origin extend to form a line. As the points collapse on the axes, they are herded toward the origin before moving onto the plane to form a line of greater slope and faster speed. Again the points are herded by the axes before being rearranged to form the smallest slope and slowest speed.

Scene 2. Oscillating waves and an undulating line

The world is full of oscillating waves. Waves in water. Waves of sound. Waves of light. Waves of vibration of atoms and molecules. It is a miracle that all of these varied waves can be represented mathematically as a simple oscillating function known as the *sine wave*.

Two thousand years of sine waves

The sine function oscillates up and down. The wave never grows larger in y than $+1$ or smaller than -1. The wave is periodic, repeating every 2π units of x over and over again to infinity (see Figure 3). The sine function was unknown to the Greek geometer *Euclid* (*circa* 300 BCE). It was first tabulated by *Hipparchus of Nicaea* (180 – 125 BCE).

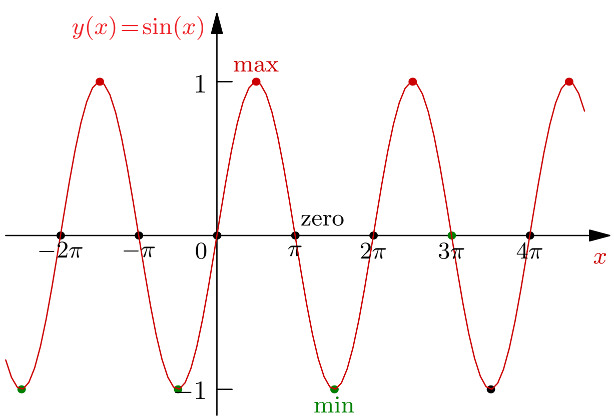

Figure 3: The sine function oscillates in the vertical y-direction between $+1$ and -1 as x varies with an infinity of maxima, minima, and zeros.

Oscillations that do not last forever

While the sine wave oscillates forever, the amplitude of water waves and sound waves diminish in time. We can model a diminishing wave by multiplying a function that oscillates by a function that grows smaller. The result is a *compound function* (see Figure 4). An oscillating function (gray) multiplied by a decreasing function (blue) results in a compound function with a *damped oscillation* (red). Rather than oscillating forever, the amplitude of the damped wave grows smaller and smaller until the oscillations disappear.

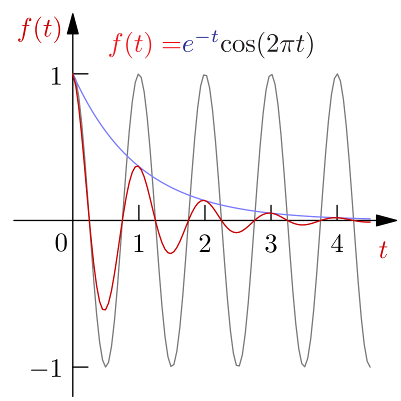

Figure 4: The amplitude of the oscillations of the wave slowly diminish in time and then all but disappear.

> Dancers form an oscillating wave, assuming the roles of maximum, minimum, zero, or thing-in-between. As the wave oscillates, maxima decrease, minima increase, and zeros stay in place. For an instant, they pass through the x-axis forming a line before assuming the form of the original wave – upside down. The oscillation continues as the points pass through the x-axis to reform the original wave. Finally, the oscillation is repeated with a twist. The amplitude of the oscillation diminishes in time and finally ends with every point a zero.

Scene 3. Expressing the form of an undulating wave

A variety of coordinate systems may be used to represent and visualize mathematical functions. Transforming between coordinate systems can lead to a dramatic transformation of the shape of the function and provide insight into the function's underlying symmetries.

Newton reimagines Descartes's xy-plane

The x and y axes of the cartesian coordinates have a "'square symmetry." They are poorly suited for drawing round functions. Fortunately, there is a coordinate system in which *round functions* are easily expressed. It is known as *plane polar coordinates* and was discovered by British physicist, mathematician, astronomer and theologian *Isaac Newton* (1642-1727). In plane polar coordinates. a point is defined as the ordered pair (r, θ). r is the radial distance from the origin. θ (pronounced *thay-tah*) is the angle of counter-clockwise from the x-axis to the line connecting the point to the origin (see Figure 5).

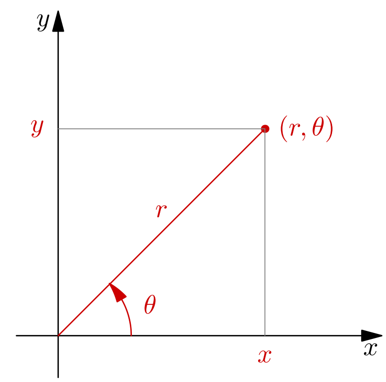

Figure 5: Any point on the infinite plane can be represented as (x, y) in cartesian coordinates or (r, θ) in polar coordinates.

Two views of an oscillating wave

In cartesian coordinates, the sine function appears as an undulating wave. The value of $y = \sin(\theta)$ oscillates between $+1$ and -1 as $x = \theta$ increases. In polar coordinates, the undulating sine wave appears as a rotation around a *unit circle* having $r = 1$ where $y = \sin(\theta)$. As the angle θ increases, the magnitude of the wave oscillates between $+1$ and -1 (see Figure 6).

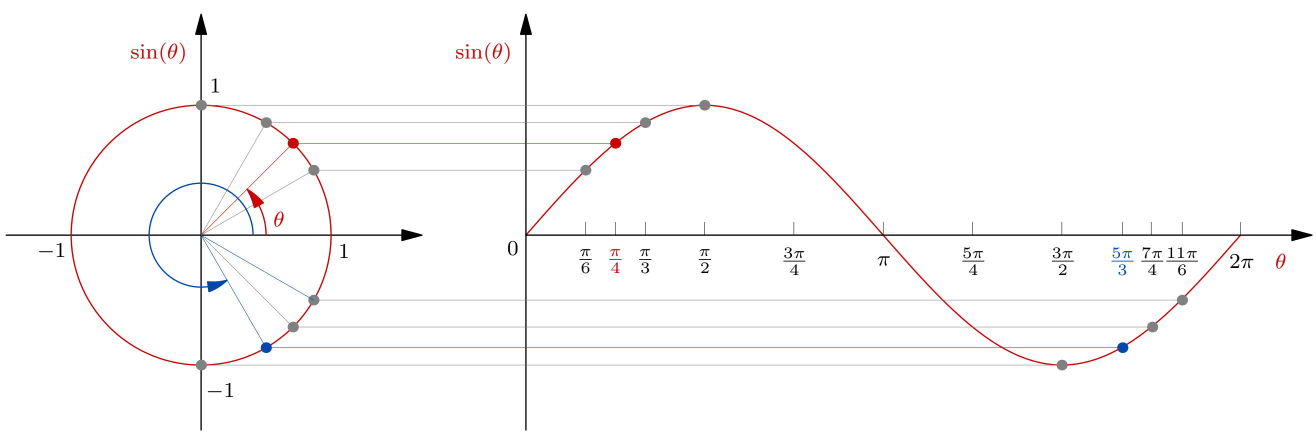

Figure 6: The sine wave oscillates up and down as it travels left and right in cartesian coordinates (right) or rotates about the unit circle in polar coordinates (left).

> Dancers again form an oscillating wave holding their positions in x while oscillating in y. After one period each point returns to its original position and the oscillating dance repeats. After several oscillations, the points leave the square *cartesian coordinates* to reform the wave in circular *polar coordinates*. The oscillations continue as a rotation around a circle. After several oscillation-rotations, the points return to cartesian coordinates to reform the familiar undulating wave.