Profit-Robust Policies for Dynamic Sharing of Radio Spectrum

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***Abstract*—We investigate profitability from secondary spectrum provision under unknown relationships between price charged for spectrum use and demand drawn at the given price. We show that profitability is governed by the applied admission policy and the price charged to secondary users. We explicitly identify a critical price (market entry price) such that if secondary demand is charged below that price, the licensee endures losses from spectrum provision, regardless of the applied admission policy. Furthermore, we show that an admission policy that admits secondary demand only when no channel is occupied is profitable for any price that exceeds the critical price. We prove that this policy is profit-robust to variations in secondary demand, i.e., if the policy is profitable for a certain price, it will be profitable for any secondary demand that the price generates, as long as the price generates demand. We also investigate profitability from a set of policies that allow more secondary users to access spectrum by defining the number of users that can be concurrently served. Our results demonstrate profit-robustness of these policies and explicitly characterize profitable prices. We provide a numerical study to verify our theoretical findings.**

I. Introduction

The emergence of secondary spectrum markets is driven by strong beliefs that current practices in spectrum allocation, whether carried via auctioning, lottery, or comparative hearing, do not ensure efficient utilization of available spectrum bands. Secondary provision of spectrum bears tremendous potential in this regard by creating mechanisms for spectrum licensees to allow the excess of their capacities, endowed via initial allocation, to float in response to varying supply and demand conditions. See [1–3] for a comprehensive discussion on this topic. It is believed that spectrum bands of high utilization efficiency are of diminishing value without some sort of an outlet for secondary provision [4].

Secondary provision of spectrum has an economic incentive for spectrum licensees as it helps generate revenue from underutilized bands. The *private commons* model for spectrum rights, as introduced by the FCC [5], is one example in this regard. Namely, licensees are allowed to maintain ownership of their spectrum bands while provisioning the surplus of their capacities to secondary users to extend their subscriber pools and collect more revenue [6].

Amid these promises, spectrum provision should be wisely orchestrated so that the process does not lead to excessive blocking of primary subscribers to the licensee and causes a net loss in revenue, especially if these subscribers are more rewarding to the licensee. Therefore, two factors have strong impact on economic feasibility of spectrum provision; (i) price for secondary access of spectrum and (ii) admission policies to share spectrum between primary and secondary users.

In this regard, obtaining a revenue-maximizing price and a policy can be achieved via solving a classical optimization problem. However, the reality is more subtle as relationships between price and demand follow market dynamics which can be hard to characterize, and thus it becomes hard to precisely identify such a price and a policy. While in some applications a price-demand relationship can be estimated via measurements (see for example [7] and [8]), or via trial and error, such procedures may not be received positively in situations that entail a one-shot strategic decision, such as opening a network for secondary access.

A. Main Contribution

The literature on economic viability of spectrum provision is mainly regulatory in nature and involves market scenarios to promote spectrum trading. See for example [9] for a study on conditions for market liquidity in exchange-based trading. This paper follows an alternative approach to establish the economic incentive for secondary spectrum provision via providing licensees with *analytical* guidelines for achieving profitability, and not necessarily maximizing it, without appealing to specific models of price-demand relationships. It turns out that solid guidelines can be defined in this regard. Precisely, a minimum price for secondary access and an admission policy are identified and shown to be profitable regardless of the underlying price-demand relationship. This paper can be considered as an extension to our work in [10] with primary focus on isolated systems that involve no spatial reuse of spectrum bands. This leads to more definitive results as will be shown in the context of the paper.

The key idea is to use a policy improvement argument to improve over a policy that caters only to primary demand. This argument will be shown to help establish a critical price below which no profit is achieved from secondary provision. It will be also shown that the argument helps identify an admission policy that is profitable for any price that exceeds the critical price. The policy is referred to as *"feasible"* policy and it is identified as a policy that accepts secondary demand only when no channel is occupied by primary users. Furthermore, the policy associated with this price shows robustness to variations in secondary demand, i.e., the price is profitable regardless of the demand it generates, as long as the price generates demand.

We also study profitability from a set of well known spectrum sharing policies that we refer to as *threshold* policies. The value of this study is twofold: First, threshold policies help put a cap on the number of secondary users that can be served concurrently, and thus they are desirable from an implementation point of view. Second, they provide better quality of service for secondary users compared to the feasible policy. Namely, consider the case for a threshold policy when only one secondary user can be admitted at a time, i.e., a secondary user is admitted as long as no secondary user is in service. In comparison with the feasible policy, where a secondary user is admitted only if no user (primary or secondary) is in service, the former policy improves the odds of secondary users being served and therefore improves the quality of service.

Threshold policies lend themselves to an analytical framework that helps establish profitability guidelines for these policies. In particular, we establish that profitability from threshold policies is robust to variations in secondary demand if price is chosen above a certain level that is exactly identified in the paper. The framework is shown to extend to cover policies that exert no control on admitting secondary demand; or what is referred to as *complete sharing* policies.

B. Paper Organization

The paper is organized as follows: In Section II, we provide an economic model for profitability from secondary spectrum provision and give basic definitions that will be used in the paper. In Section III, we compute the critical price and the feasible policy followed by a discussion on profit robustness of this policy. In Section IV, threshold policies are discussed and profitability of these policies is characterized including the complete sharing policy. The paper concludes in Section V.

II. Economic Model for Spectrum Sharing

Consider a wireless system equipped with C orthogonal channels. This can resemble, for example, an OFDM system operating on a certain frequency band. Assume there are two types of channel requests; primary and secondary arriving independently as Poisson processes of rate λ_1 and λ_2, respectively. If a request is admitted then it holds the channel for a random time, during which the channel is not available. We assume that holding times are independent and identically distributed exponential random variables; and without loss of generality we shall take the mean holding time as one unit.

The spectrum licensee is rewarded from the system according to the following scheme: An admitted request generates revenue r_1 if it is primary, and revenue r_2 if it is secondary. These values may reflect deterministic charges per admitted request or average charges if, for example, an admitted request is charged based on usage.

We identify primary requests with legacy subscribers to the licensee where they are admitted whenever possible. Secondary requests, on the other hand, represent opportunity to increase revenue beyond what can be obtained from the primary requests. Towards that end, secondary requests may be selectively admitted according to a certain admission policy.

Consider the system when it caters only to primary demand without any form of admission control. Let us coin this regime as the *lockout policy* since it arises if all secondary requests are blocked. Let n denote channel occupancy of the system under this policy. It can be shown that n evolves as a Markov process and the probability distribution of channel occupancy can be obtained by solving the detailed balance equations for the Markov process. The rate of revenue from the lockout policy is given by

$$R_{LO} = r_1 \lambda_1 (1 - E(\lambda_1, C))$$

where

$$E(\lambda_1, C) = \frac{\lambda_1^C / C!}{(\sum_{i=0}^{C} \lambda_1^i / i!)} \tag{1}$$

is the Erlang blocking function.

Definition 2.1: An admission policy for secondary demand is profitable for the price-demand pair (r_2, λ_2) if the rate of revenue from this sharing policy, denoted here by $R_{SP}(r_2, \lambda_2)$, exceeds the rate of revenue from the lockout policy, i.e.,

$$R_{SP}(r_2, \lambda_2) > R_{LO}.$$

The set of (r_2, λ_2) pairs that are profitable under this policy identify the profitability region of the policy.

The profitability region thus can be visualized as the part of the secondary price-demand plane (r_2, λ_2) that includes all profitable (r_2, λ_2) pairs. Note that it can be directly deduced that if an admission policy is profitable for some secondary price, it will be profitable for a higher price. Thus, profitability region for a given policy is delineated by (r_2, λ_2) pairs that form the curve

$$R_{SP}(r_2, \lambda_2) = R_{LO}.$$

III. Critical Price for Profitability

First we aim to identify for each value of secondary demand λ_2 a price such that if secondary price r_2 is chosen to be less than or equal to that price, there exists no profitable admission policy from admitting secondary demand. In other words, we identify the curve that delineates the largest profitability region for spectrum sharing. An admission policy that achieves such a region is referred to as *feasible* policy since it is profitable for all prices the exceed that price.

Given $\alpha \in (0, 1)$, let $V_\alpha(n)$ denote the mean discounted revenue given that the channel occupancy process has initial state n. Namely,

$$V_\alpha(n) \doteq E[\int_0^\infty \alpha^t r_1 dA(t) \mid n(0) = n],$$

where $(A(t) : t \geq 0)$ is the counting process that counts the number of admissions in the system (hence the above integral increases by $\alpha^t r_1$ in response to a primary request that is admitted at time t). Also let

$$h(n) \doteq \lim_{\alpha \to 1} V_\alpha(n) - V_\alpha(0).$$

The limit above exists and $h(n)$ is the well-known differential reward that arises in problems of infinite-horizon average-cost optimization [11].

We use a policy improvement argument to characterize the largest profitability region as follows: $h(n) - h(n+1)$ can be interpreted as the difference in the rate of revenue of the lockout policy if the channel occupancy process is started from state n rather than state $n+1$. This represents the cost of admitting a request when the process is in state n. Thus, if the arriving request is of secondary type, then the profitability principle implies that a secondary request is admitted if the revenue generated from that request exceeds the implied cost of admitting the request, i.e.,

$$r_2 > h(n) - h(n+1).$$

The previous condition identifies a price such that accepting secondary requests at prices equal to or below that price when the process is in state n is not profitable. The minimum price over all possible states is identified as

$$r_2^* = \min_{n=0,\cdots,C-1} h(n) - h(n+1). \tag{2}$$

We refer to r_2^* as the *critical price* since the licensee shall be profitable from secondary spectrum provision only if r_2 is chosen above r_2^*. Critical price resembles a balance to the opportunity cost that results from excessive blocking of primary demand which could be serviced in the absence of secondary demand. Thus, if secondary demand is priced below the critical price, the licensee will endure losses from secondary spectrum provision. Furthermore, an important observation can be drawn from (2) which will be summarized in the following theorem:

Theorem 3.1: (Robustness of the feasible policy) Critical price r_2^* does not depend on λ_2 and thus the largest profitability region is delineated by a straight vertical line in the (r_2, λ_2) plane. This implies that the policy that achieves this region (the feasible policy) is robust to secondary price-demand curves, i.e., the policy will be profitable for any secondary price $r_2 > r_2^*$ regardless of the demand the price generates, as long as the price generates demand.

This result is illustrated in Figure 1. The area that lies to the right of the vertical line $r_2 = r_2^*$ comprises the largest profitability region for admitting secondary demand. This region is achieved via what we refer to as the feasible policy. A demand function represents a relationship between secondary price and secondary demand generated as a function of that price. Demand functions follow market conditions and thus, apart from the assumption that demand decreases as price increases, they cannot be precisely characterized. Here in the figure two demand functions are considered. If secondary price is chosen above r_2^*, then the price is profitable under the two demand functions; *Demand Function 1* and *Demand Function 2*. Now assume that the largest profitability region is not delineated by a vertical line as shown by the hypothetical curve in the figure. In this case, price $\hat{r_2}$ will be profitable under *Demand Function 1* but not under *Demand Function 2*. This illustrates robustness of the alluded feasible policy.

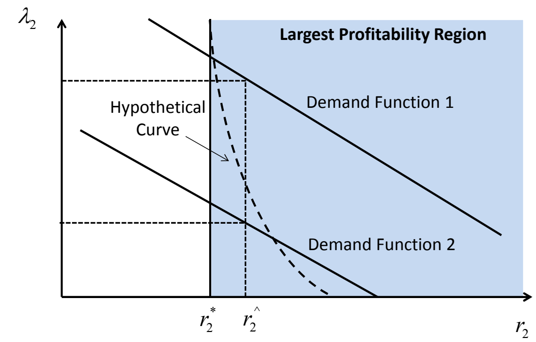

Fig. 1. Illustration of profit-robustness of the feasible policy. The policy has the largest profitability region that lies to the right of the straight line $r_2 = r_2^*$ and thus the policy is profitable for any price that exceeds the critical price.

The analysis that led to Formula (2), however, provides coarse description of the critical price and the feasible policy. An approach to compute differences in h was given in [12] where an involved argument was used to compute all the differences. Here we use an alternative approach. Namely, explicit characterization of the critical price and the feasible policy will be provided in the following theorem:

Theorem 3.2: (Identifying the critical price and the feasible policy) The critical price for profiting from secondary demand can be identified as follows

$$r_2^* = r_1 E(\lambda_1, C). \tag{3}$$

where $E(\lambda_1, C)$ is given by (1). Furthermore, the feasible policy is a policy that admits secondary demand only at the idle state, i.e., when no channel is occupied.

Proof: Note that the largest profitability region is delineated by a straight vertical line, as illustrated in Theorem 3.1, and therefore, if (r_2, λ_2) is profitable then so is (r_2, ∞). Also note that a secondary request is admitted and allocated a channel based on the number of occupied channels upon arrival. Let K be the largest of these eligible numbers. By setting $\lambda_2 = \infty$, $K+1$ channels are always occupied since a new secondary request is admitted immediately following a departure at occupancy level $K+1$. The state space of the channel occupancy process thus reduces to $K+1, K+2, \cdots, C$. The occupancy process under such a policy is illustrated in Figure 2 and the equilibrium distribution π_o is given as

$$\pi_o(i) = \begin{cases} \frac{\lambda_1^i / i!}{G_o} & i = K+1, K+2, \cdots, C \\ 0 & \text{otherwise,} \end{cases}$$

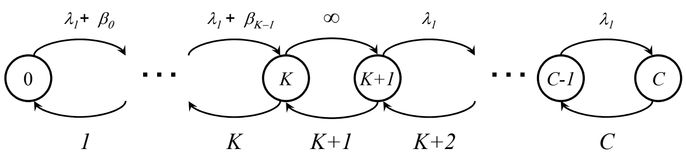

Fig. 2. Occupancy process of C channels when secondary demand $\lambda_2 \to \infty$. If the system is eligible to admit a secondary request at state i then the system leaves the state with rate ∞, i.e. $\beta_i = \infty$. Otherwise $\beta_i = 0$ for that state. If K is the largest possible occupancy underwhich a secondary request is admitted, then at least $K+1$ channels are always occupied.

where G_o is a normalizing constant given as

$$G_o = \sum_{i=K+1}^{C} \frac{\lambda_1^i}{i!}.$$

Revenue from secondary demand is generated when $K+1$ channels are occupied. A departure at this state will be directly followed by an arrival of a secondary request since $\lambda_2 \to \infty$. An admitted request will generate a revenue of r_2 and will hold the channel for an average time of one unit. Thus, $K+1$ secondary requests are served per unit time on average and the rate of revenue from secondary demand is given by $(K+1)r_2\pi_o(K+1)$. A primary request, however, is admitted if upon arrival less than C channels are occupied. The rate of revenue from this demand is therefore $r_1\lambda_1(1-\pi(C))$. The policy is profitable if the rate of revenue exceeds the rate of revenue from the lockout policy, i.e.,

$$(K+1)r_2\pi_o(K+1) + \lambda_1 r_1(1-\pi_o(C)) \quad > \quad \lambda_1 r_1(1-E(\lambda_1, C)).$$

Algebraic manipulation of this inequality yields that it is equivalent to $r_2 > r_1\frac{E(\lambda_1,C)}{E(\lambda_1,K)}$ and the minimum is achieved at $K=0$. This recovers form (3) along with the feasible policy. ∎

IV. Profitability from Threshold Policies

The feasible policy given in Theorem 3.2 admits a secondary request in the system if and only if no channel is occupied at the time of arrival of the request. In other words, $C-1$ channels are exclusively reserved for primary demand which implies that secondary demand can experience high blocking rates, especially when λ_1/C is high. In this section, we appeal to admission policies where decisions are based on number of secondary users in service and not on number of occupied channels.

We study profitability from a set of admission policies that are referred to as threshold policies. These policies enjoy two principal features: (i) Their equilibrium distribution has a closed form expression and thus they are analytically tractable and (ii) their analytical framework covers the complete sharing policy where no control is exerted on admitting secondary demand. We aim here to understand profitability from these policies and their robustness to variations in secondary demand.

Consider an admission policy that puts a threshold $0 < T \leq C$ on the number of channels that can be concurrently occupied by secondary users. We refer to such a policy as threshold policy T. Formally, let $n_1(t)$ and $n_2(t)$ denote, respectively, the number of channels occupied by primary and secondary users at time t. Under threshold policy T, a primary request at time t is admitted if its inclusion preserves:

$$n_1(t) + n_2(t) \leq C \tag{4}$$

while a secondary requests is admitted if its inclusion preserves, besides (4), the condition

$$n_2(t) \leq T. \tag{5}$$

The state space is thus identified by the set

$$S = \{(n_1, n_2) : n_1 + n_2 \leq C \quad \text{and} \quad n_2 \leq T\}.$$

Note that if $T = C$, then the policy admits without control secondary requests as long as a channel is available. This policy is referred to as the complete sharing policy.

The load under threshold policy T, $(n_1(t), n_2(t))$, evolves as a reversible Markov process and the equilibrium distribution can be obtained by solving the corresponding detailed balance equations [13] to obtain the probability vector $\pi = \{\pi(n_1, n_2) : (n_1, n_2) \in S\}$. Namely, it can be shown that

$$\pi(n_1, n_2) = \frac{\frac{\lambda_1^{n_1}}{n_1!}\frac{\lambda_2^{n_2}}{n_2!}}{G}$$

where G is a normalizing constant given as

$$G = \sum_{(n_1,n_2)\in S} \frac{\lambda_1^{n_1}}{n_1!}\frac{\lambda_2^{n_2}}{n_2!}.$$

By the PASTA property, blocking probability for primary requests is given as

$$B_1(\lambda_1, \lambda_2, C, T) = \sum_{(n_1,n_2):n_1+n_2=C} \pi(n_1, n_2) \tag{6}$$

and for secondary requests is given as

$$B_2(\lambda_1, \lambda_2, C, T) = \sum_{(n_1,n_2):n_2=T||n_1+n_2=C}^{C} \pi(n_1, n_2). \tag{7}$$

The rate of revenue from the system can be thus computed by the formula

$$R_T(\lambda_2, r_2) = \sum_{i=1,2} r_i\lambda_i(1 - B_i(\lambda_1, \lambda_2, C, T)).$$

We define profitability from a threshold policy via Definition 2.1. Namely, a policy T is profitable for the pair (r_2, λ_2) if it satisfies

$$R_T(\lambda_2, r_2) > R_{LO},$$

which leads to the conclusion that a price r_2 is profitable under this policy if and only if

$$r_2 > \left(\frac{r_1\lambda_1}{\lambda_2}\right)\left(\frac{B_1(\lambda_1,\lambda_2,C,T)-E(\lambda_1,C)}{1-B_2(\lambda_1,\lambda_2,C,T)}\right). \quad (8)$$

Inequality (8) forms the basis for investigating profitability for different threshold policies. However, solid results in this regard can be obtained for the policy $T = 1$. Namely, profitability under this policy shows robustness with respect to secondary price-demand relationships. This result is formalized in the following theorem:

Theorem 4.1: For the policy $T = 1$, if the pair (r_2, λ_2) is profitable under this policy, then it is profitable for all the pairs (r_2, λ_2) where $\lambda_2 > 0$. Furthermore, a price r_2 is profitable under this policy if and only if

$$r_2 > r_1\lambda_1\left(E(\lambda_1, C-1) - E(\lambda_1, C)\right). \quad (9)$$

Proof: Consider the state space for the policy $T = 1$ given by the set

$$S = \{(n_1, n_2) : n_1 + n_2 \le C \quad \textit{and} \quad n_2 = 0, 1\}.$$

Note that following condition (5), there are two possibilities for n_2 where n_2 can be either 0 or 1. Thus, blocking probabilities for primary and secondary requests, as given respectively by (6) and (7), can be written in the following forms

$$B_1(\lambda_1,\lambda_2,C,1) = \frac{\frac{\lambda_1^C}{C!} + \lambda_2\frac{\lambda_1^{C-1}}{(C-1)!}}{G}$$

and

$$B_2(\lambda_1,\lambda_2,C,1) = \frac{\frac{\lambda_1^C}{C!} + \lambda_2\sum_{n=0}^{C-1}\frac{\lambda_1^n}{n!}}{G}$$

where

$$G = \sum_{n=0}^{C}\frac{\lambda_1^n}{n!} + \lambda_2\sum_{n=0}^{C-1}\frac{\lambda_1^n}{n!}.$$

Note that $B_1(\lambda_1,\lambda_2,C,T) - E(\lambda_1,C)$ can be written as

$$\lambda_2\frac{\left(\frac{\lambda_1^{C-1}}{(C-1)!}\sum_{n=0}^{C}\frac{\lambda_1^n}{n!} - \frac{\lambda_1^C}{C!}\sum_{n=0}^{C-1}\frac{\lambda_1^n}{n!}\right)}{G\sum_{n=0}^{C}\frac{\lambda_1^n}{n!}}. \quad (10)$$

Also note that $1 - B_2(\lambda_1,\lambda_2,C,T)$ can be written as

$$\frac{\sum_{n=0}^{C-1}\frac{\lambda_1^n}{n!}}{G}. \quad (11)$$

The result follows by substituting (10) and (11) in (8). ■

Threshold policy $T = 1$ shares some similarities with the feasible policy introduced in Theorem 3.2 as both policies have profitability regions delineated by straight vertical lines, and thus they are both robust to variations in secondary demand. However, There is no guarantee that the policy $T = 1$ achieves the profitability region of the feasible policy. In fact, by consulting (3) and (8), the gap between the cut-off price for the policy $T = 1$ and the critical price is given by

$$r_{gap} = r_1\left(\lambda_1 E(\lambda_1, C-1) - (1+\lambda_1)E(\lambda_1, C)\right). \quad (12)$$

A trivial case for $r_{gap} = 0$ is when $C = 1$. In this case, threshold policy $T = 1$ is in fact the feasible policy as can be verified via (12).

Figure 3 shows profitability regions for the different admission policies studied in the paper. The figure is prepared for the following setting: number of channels $C = 5$, primary arrival rate $\lambda_1 = 5.0$, and primary price $r_1 = 1.0$. Profitability region for each policy lies to the right of the curve corresponding to that policy. The line $r_2 = 0.285$ delineates the largest profitability region (the region achieved by the feasible policy). Thus, a price at or below 0.285 is not profitable under any admission policy.

Consider the curves that delineate profitability regions of the different threshold policies $T = 1, \cdots, 5$. Note that the policy $T = 1$ is delineated by the straight line $r_2 = 0.567$. The equation of the line can be verified via (9). Among the threshold policies, the policy with the largest profitability region is $T = 1$ while the policy with the smallest region is $T = 5$. This latter policy corresponds to a complete sharing policy with no admission control on secondary demand.

Interestingly, the curves that delineate profitability regions of the threshold policies show monotonicity. In particular, each threshold policy is delineated by a curve that asymptotically increases as secondary price increases. Thus, if secondary price is chosen above the asymptote, then the policy will be profitable at that price for any secondary demand value as long as the price generates demand. Therefore, a threshold policy shows robustness to variations in secondary demand if prices are chosen to be larger than the asymptote.

Asymptote for threshold policy T can be computed using the following argument: Let λ_2 be large enough such that $\lambda_2 \to \infty$. In this case, a channel abandoned by a secondary request after service will be directly occupied by another secondary request since λ_2 is very large. This breaks the set of channels into two separate subsets: (i) T channels that are always occupied by secondary requests and (ii) $C - T$ channels that serve only primary requests arriving at rate λ_1. Furthermore, the mean holding time of channels is one unit and thus the rate of revenue from the first subset is r_2T while the rate of revenue from the second subset is $r_1\lambda_1(1 - E(\lambda_1, C - T))$. The asymptote can be computed by solving for r_2 in the equation

$$r_1\lambda_1(1 - E(\lambda_1, C-T)) + r_2T = R_{LO},$$

which leads to

$$r_2 = \frac{r_1\lambda_1}{T}(E(\lambda_1, C-T) - E(\lambda_1, C)).$$

This formula can be directly verified for $T = 1$ as it reduces to (9). The figure shows asymptotes of all threshold policies.

Finally, as discussed earlier, threshold polices provide lower blocking rates for secondary demand compared to the feasible policy. However, this implies higher cut-off prices to guarantee profitability as illustrated in Figure 3. Thus, if such prices are too high to generate demand then neither would higher prices. Assume that a critical price always resembles a reasonable price to generate demand. Reasonability of profitable prices

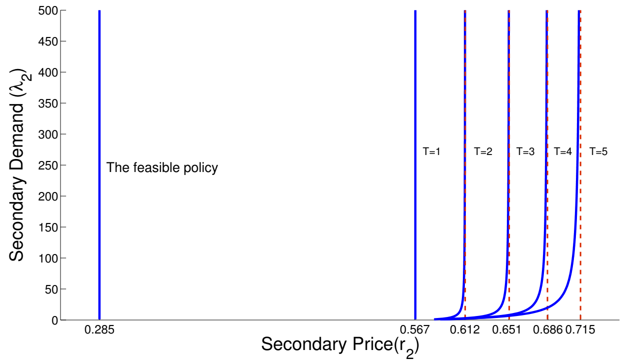

Fig. 3. A figure characterizing profitability regions for the policies discussed in the paper. Profitability region for each policy covers the area that lies to the right of the curve corresponding to that policy. Threshold policy $T = 1$ is profit-robust for values that exceed 0.567 while threshold policies $T = 2, 3, 4, 5$ are robust for price values that exceed their asymptotes.

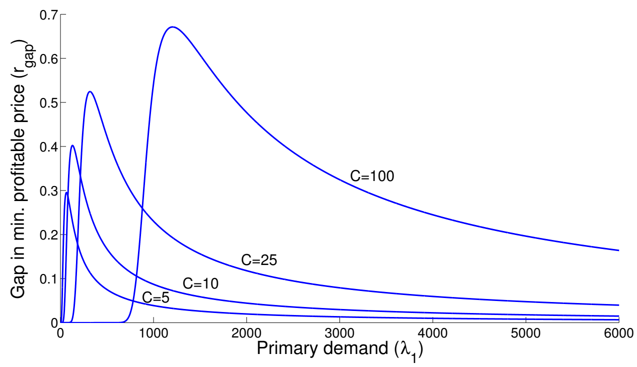

Fig. 4. Relationship between r_{gap} and primary demand λ_1 computed via formula (12). Each curve corresponds to a different value of C. The figure shows that r_{gap} is unimodal in λ_1. It also shows that as C increases the peak is higher and is achieved at a larger value of λ_1.

under threshold policies can be measured via their gap from the critical price. To understand the impact of system parameters on this gap, consider for example threshold policy $T = 1$. Formula (12) shows the difference between the cut-off price under this policy and the critical price. It can be seen from the formula that r_{gap} is directly proportional to r_1. However, the impact of λ_1 and C seems hard to characterize.

Figure 4 can help in this regard as it shows r_{gap} for different values of λ_1 obtained when $r_1 = 1.0$. Each curve corresponds to a different number of channels C. The figure illustrates unimodality of r_{gap} in λ_1. Precisely, r_{gap} increases as λ_1 increases up to a certain level and starts decreasing. Higher peaks are obtained at larger values of C at a larger λ_1. The figure shows also that r_{gap} tends to 0 as λ_1 tends to ∞ with a rate of decay that goes down for larger values of C.

V. Conclusion and Final Remarks

In this work, we provided an analytical study on profitability from secondary spectrum provision. We clearly identified a critical price for profitability below which licensees will endure losses from such a provision. Furthermore, we identified a feasible policy that achieves profitability for any price that exceeds the critical price. Our results show that the policy admits robustness to variations in secondary demand and thus can achieve profitability in light of lack of knowledge of the price-demand function.

We also investigated profitability from threshold policies. We proved that a threshold policy that allows secondary users to use at most one channel admits the same robustness property of the feasible policy, but typically at a higher price. For other threshold policies, we showed that robustness can be achieved if prices are chosen above certain levels identified via asymptotic analysis.

Perhaps one question that the paper leaves open is the question of which admission policy among the studied policies yields the highest revenue and thus is maximally profitable for a price demand pair. However, there is no guarantee that a maximally profitable policy for one pair is maximally profitable for all price-demand pairs in its profitability region. This implies that identifying a maximally profitable policy can become dependent on the price-demand function. A definitive answer to this question is not within the scope of this paper and it is left for future research.

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