Reservation Policies for Revenue Maximization from Secondary Spectrum Access in Cellular Networks

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Abstract

We consider the problem of providing opportunistic spectrum access to secondary users in wireless cellular networks. From the standpoint of spectrum license holders, achieving benefits of secondary access entails balancing the revenue from such access and its impact on the service of primary users. While dynamic optimization is a natural framework to pursue such a balance, spatial constraints due to interference and uncertain demand characteristics render exact solutions difficult. In this paper, we study guiding principles for spectrum license holders to accommodate secondary users via reservation-based admission policies. Using notions of dynamic optimization, we first develop the concept of average implied cost for establishing a connection in an isolated locality. The formula of the cost provides an explicit characterization of the value of spectrum access. We then generalize this concept to arbitrary topologies of interference relations and show that the generalization is justified under an analogue of the reduced load approximation judiciously adapted from the wireline to the wireless setting. An explicit characterization of this quantity demonstrates the localized nature of the relationship between overall network revenue and reservation parameters. Based on this relationship, we develop an online algorithm for computing optimal reservation parameters at the cells based on localized message passing. The results

in the paper are verified through a numerical study.

1. Introduction

The increase in demand for wireless communications, along with reported inefficiencies in radio spectrum utilization, have recently led to a global effort to reform legacy spectrum regulations [1, 2, 3, 4]. One aspect of these reforms is to grant spectrum license holders extended property rights that allow trading of spectrum in secondary markets. From the standpoint of spectrum regulators, such markets help improve spectrum utilization. From the standpoint of license holders, secondary markets provide a novel opportunity to increase revenue by expanding their subscriber pools.

In this work, we focus on wireless cellular networks and devise techniques for license holders to maximize their revenue under market agreements which attract users of secondary nature. In particular, we differentiate between two types of network users who pay the license holder pre-defined fixed prices for unit time of use of the network. The first represents primary, or original, users of the cellular network. The secondary type represents users who seek network access on opportunistic basis.

Revenue maximization can be readily studied within the framework of dynamic programming to define a decision policy for pricing and/or admission control based on user type and the state of the network [5, 6, 7, 8]. However, the choice of fixed prices in this work stems from the practical side of the problem since users of cellular networks, in general, favor fixed and simple pricing policies. Thus, we focus on admission control since it can be implemented by network operators without user involvement.

Still, it is well-known that computational complexity of dynamic approaches, in general, becomes prohibitive for even smallest nontrivial networks [9]. In particular, in a wireless setting, the effect of interference from an established connection in one cell extends beyond that cell and indirectly affects all cells in the network. For example, a connection in progress leads to a temporal reduction in utilization in its immediate neighbourhood, which may in turn help accommodate more connections in the second-tier of cells around it. Thus, an optimal dynamic solution for the problem typically entails making admission decisions based on the current state of the network (i.e., channel occupancy in each cell), and therefore its implementation is rather impractical.

Our main contribution is to provide *guiding principles* for implementing a set of policies that require less information from the network and are referred to as reservation-based admission policies. Under such policies, admission of a call request by a primary user is accepted as long as there is capacity in the network, while a request by a secondary user is granted only if the total interference at each cell stays below a fixed threshold, or reservation parameter, typically taken to be less than the capacity of the cell. This way, part of the capacity of each cell is reserved exclusively for primary traffic which has more priority and is more rewarding as well. While admission control for cellular networks is a well studied topic, see [10] for a recent survey, we consider the problem for general network topologies under full realization of the effect of interference.

The choice of reservation policies is motivated by their optimality for the isolated cell [11, 12, 13]. Thus, we consider first the isolated cell for which the "average implied cost" of an established connection can be explicitly determined. Then we extend this concept to general topologies and adapt the so-called reduced load approximation (RLA) [16], widely used in the analysis of circuit-switched networks, to compute certain network performance measures in the wireless setting and provide analytical insight. The premise behind the RLA is to assume that a decision to admit/reject a call is based on independent decisions at the different cells. This allows approximated values of call blocking probabilities to be directly calculated.

Our second contribution is to devise an adaptive mechanism for implementing the reservation-based admission policy. Namely, we provide an online algorithm to update reservation parameters at the cells in light of fluctuations in traffic rates. We exploit the fact that sensitivity of the revenue to a unit change of the reservation parameter at the cell can be locally evaluated by the base station using localized message passing techniques. We use this fact to propose an online distributed algorithm that computes optimal reservation parameters in the network. Since unimodality of the revenue function is not guaranteed in the generality of topologies, we suggest an implementation akin to simulated annealing that probabilistically improves the revenue in each step of the process. Finally, we provide a numerical study in support of the theoretical results.

The rest of this paper is organized as follows: In Section 2, we present an operational model for cellular networks under a reservation-based admission policy. We suggest an analytical framework to capture the network-wide effect of interference and use the RLA to compute blocking probabilities. In Section 3, we derive an exact expression for the implied cost in the special case of the isolated cell and then give corresponding formulas for general topologies based on the RLA. In Section 4, we present the revenue-maximizing distributed algorithm. We provide a numerical study in Section 5 and finally conclude the paper in Section 6.

2. Analytical Framework

A cellular network is considered under the following teletraffic conditions: At each cell i, connection requests of type $m = 1, 2$ arrive independently as a Poisson process with rate $\lambda_i^{(m)}$. Here, type 1 refers to requests by primary users and type 2 to those by secondary users. Once established, a connection lasts for an exponentially distributed time with unit mean, independently of the history prior to the request arrival.

We model a cellular network as a weighted graph $G = (N, E)$ where N denotes the collection of cells and edge weight $w_{ij} \geq 0$ is the interference bandwidth required from cell j by a connection established at cell i. Since a connection may generate interference on other connections in the same cell, the modeled physical situation typically implies that $w_{ii} > 0$ for each cell i. A connection can be sustained only if it experiences admissible interference, and that a new connection request cannot be honoured if it leads to premature termination of another connection that is already in progress.

To formalize this condition, let n_i denote the number of connections in progress at each cell i so that $\sum_{i \in N} n_i w_{ij}$ is the total interference acting on cell j. Given an interference capacity κ_j for cell j, a *network load* $\mathbf{n} = (n_i : i \in N)$ is *feasible* if

$$\sum_{i \in N} n_i w_{ij} \leq \kappa_j \quad \text{for each } j, \tag{1}$$

and it evolves as a time-homogeneous Markov process with state space $S = \{\mathbf{n} \in \mathbb{Z}_+^N : \mathbf{n} \text{ is feasible}\}$.

Identifying w_{ij} values depends on the underlying spectrum access mechanism employed in the network. For example, in narrowband networks, a frequency band is divided into non-overlapping channels where operational constraints prohibit the same channel to be assigned simultaneously to connections in the same cell or in any neighbouring cells. Thus, condition (1) dictates that w_{ij} is the total number of channels a connection established at cell i would lock at cell j. In wideband networks, connections share the whole frequency band and w_{ij} can be characterized by the strength of electromagnetic coupling between connections at the different cells. One approach to compute such values is via defining capacity regions based on a set of linear constraints. See for example [17] and [18] for an in-depth discussion on identifying such values.

2.1. Optimal Admission Policy

An admission policy is a decision policy that accepts or rejects a connection request based on the type of the connection and the state of the network. A necessary criterion for an admission policy is to preserve the feasibility condition defined in (1). Let $\mathbf{h}$ be an admission policy defined on the state space S. Assume that the license holder charges an admitted type 1 connection $r^{(1)}$ units currency per unit time and charges an admitted type 2 connection $r^{(2)}$ units. Let $n_i^{(m)}(t)$ be the number of connections of type m established at cell i at time t. Assuming call duration of unit mean, the long term rate of revenue generation from the network is given by

$$W(\mathbf{h}) \triangleq \lim_{\tau\to\infty} \frac{1}{\tau}\mathbf{E}_h \left[\int_0^\tau \sum_{i\in N} \left(r^{(1)} n_i^{(1)}(t) + r^{(2)} n_i^{(2)}(t) \right) dt \right].$$

An optimal policy is a policy that maximizes $W(\mathbf{h})$. A well-known approach for characterizing an optimal policy is via dynamic programming. However, the complexity of such approach becomes prohibitive for even smallest nontrivial networks. Moreover, an optimal dynamic solution typically entails making admission decisions based on the state of channel occupancy in the whole network, and therefore its implementation is rather impractical.

2.2. Reservation Policies

Consider a set of admission policies which we refer to as reservation-based policies. Each such policy will be represented by a vector $\mathbf{R} = (R_i : i \in N)$ where R_i is the reservation parameter at cell i and such that $0 \le R_i \le \kappa_i$. Under a reservation policy $\mathbf{R}$, a type 1 connection is admitted if its inclusion preserves condition (1), while a type 2 connection is admitted only if, in addition to (1) , its inclusion preserves the total interference, from type 1 and type 2 connections, at each cell i below R_i. This way, the reservation policy guarantees priority for type 1 connections by reserving $(\kappa_i - R_i)$ of the interference capacity of each cell exclusively for type 1 connections.

Note that reservation-based policies require less network information. Namely, the state of the cell is lumped into two quantities; the interference capacity and the reservation parameter. Furthermore, given the decaying nature of interference with distance, the effect of interference from cells beyond a certain distance can be neglected. This observation will be used later in

Section4 to devise an algorithm for computing reservation parameters based on a localized message passing technique.

Now, given a reservation policy $\mathbf{R}$, let $B_i^{(m)}$ denote the blocking probability of type m connections at cell i. By the PASTA property of Poisson arrivals, $B_i^{(m)}$ can be determined by the equilibrium distribution of the Markov process and the long-term revenue rate under policy $\mathbf{R}$ can be expressed as

$$W(\mathbf{R}) = \sum_{i \in N} \sum_{m=1,2} r^{(m)} \lambda_i^{(m)} (1 - B_i^{(m)}). \tag{2}$$

Explicit expression for equilibrium occupancy probabilities can be obtained by exploiting reversibility of the occupancy process, however a major difficulty arises in computation of the equilibrium distributions, and therefore of blocking probabilities, due to the computational complexity of determining normalizing constants. Even in cases where such computation is possible, the results give little insight on the relationship between the overall revenue and individual reservation parameters. Such a relationship can be alternatively pursued by adapting the RLA to the situation in hand [15].

Our starting point is to consider an isolated cell with capacity κ, reservation parameter R, and arrival rate vector $\boldsymbol{\lambda} = (\lambda^{(1)}, \lambda^{(2)})$. Assume that an established connection requires one unit capacity from the cell. Also let n denote the total number of connections in progress in the cell. The state diagram of the cell occupancy process is shown in Figure 1. The steady state probability of having a total of n connections in progress can be directly obtained by solving the detailed balance equations. Hence

$$\pi_\lambda(n) = \begin{cases} \frac{(\lambda^{(1)}+\lambda^{(2)})^n}{n!} Z & \text{if } 0 \le n < R \\ \frac{(\lambda^{(1)}+\lambda^{(2)})^R (\lambda^{(1)})^{n-R}}{n!} Z & \text{if } R \le n \le \kappa, \end{cases}$$

where Z is a normalizing constant such that $\sum_{n=0}^{\kappa} \pi_\lambda(n) = 1$. Furthermore,

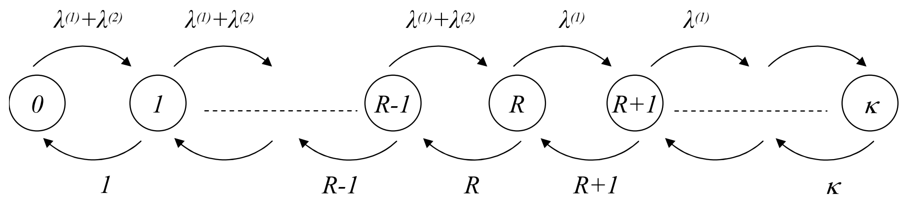


Figure 1: State transition diagram of the occupancy process for the isolated cell under a reservation policy with parameter R.

blocking probabilities for type 1 and 2 are given respectively by

$$
\begin{aligned}
B^{(1)}(\boldsymbol{\lambda}, \kappa, R) &\doteq \pi_{\lambda}(\kappa) && (3)\\
B^{(2)}(\boldsymbol{\lambda}, \kappa, R) &\doteq \sum_{n=R}^{\kappa} \pi_{\lambda}(n). && (4)
\end{aligned}
$$

For general topologies, we approximate the blocking probability for type m in cell i, $B_i^{(m)}$, by the quantity

$$\hat{B}_i^{(m)} = 1 - \prod_{j \in N} (1 - b_j^{(m)})^{w_{ij}}, \tag{5}$$

where $\{b_j^{(m)} : j \in N\}$ satisfy the fixed point relation

$$b_j^{(m)} = B^{(m)}\left(\boldsymbol{\rho}_j, \kappa_j, R_j\right), \tag{6}$$

with $\boldsymbol{\rho}_j = (\rho_j^{(1)}, \rho_j^{(2)})$ and

$$\rho_j^{(m)} = (1 - b_j^{(m)})^{-1} \sum_{i \in N} w_{ij} \lambda_i^{(m)} \prod_{l \in N} (1 - b_l^{(m)})^{w_{il}}. \tag{7}$$

The rationale behind this formula is a hypothetical situation as follows: Consider w_{ij}'s to be integer numbers obtained by properly scaling inequality (1). Thus, it is useful to interpret w_{ij} as units interference at cell j from a connection in progress at cell i. Under the reduced load approximation, any connection request is subject to independent admission/rejection decision for each unit of interference that it generates at each cell. Namely, a connection request is admitted if all units interference "sub-requests" are admitted independently at all cells. Thus, equality (6) can be interpreted as the blocking probability at cell j for one unit interference generated by a type m connection, and $\rho_j^{(m)}$ becomes the total arrival rate of units interference of type m at cell j after being thinned by other cells in the network. Finally, equality (5) gives the blocking probability of a (full) type m connection request at cell i, provided that each cell j admits a unit interference with probability $1 - b_j^{(m)}$. In view of the exact analysis of the isolated cell, equalities (6) and (7) are consistency conditions that should be satisfied by the probabilities $\{b_j^{(m)} : j \in N\}$.

One way of solving the system of equations (6) is via iteration. However, a unique solution for (5) is guaranteed only for the case when $R_j = \kappa_j$ for all j. Furthermore, for that special case, the approximation is known to be exact for an asymptotic regime when $\sum_{k=1,2} \lambda_j^{(k)}$ and κ_j increase in proportion [16]. In the next section, we employ the RLA to capture the sensitivity of revenue to a unit change in a reservation parameter. The derived expression will provide the base for the revenue-maximizing algorithm devised later in Section 4. Before that, we discuss the important concept of implied cost, which aims to capture the implications of admitting a connection in the network. In this context, we will provide an illuminating insight on this concept based on the simple case of the isolated cell.

3. Implied Cost

Subjecting a given cell to an additional unit of interference affects blocking at all the cells in the network, at various extents. For example, during the holding time of a connection, the interference generated by that connection can cause rejection of new connections arriving at neighbouring cells, which may in turn open up room for admitting new connections in other cells. Here the concept of *implied cost* that captures such effects of acceptance/rejection decisions is discussed.

3.1. Isolated Cell

Consider the isolated cell example and let $\sigma_R(n)$ denote the reduction in the long-term revenue if the system is started with $n+1$ instead of n connections in progress. Note that, if $r^{(m)}$ and $\lambda^{(m)}$ are bounded, the implied cost does not depend on the initial state of the system [5, Theorem 3.2]. Hence, $\sigma_R(n)$ can be interpreted as the implied cost of admitting a connection when the cell occupancy is n. A reservation parameter R is optimal if it dictates admission of a type m connection when $r^{(m)} > \sigma_R(n)$, i.e., when the immediate reward exceeds the implied cost of admission.

The quantity $\sigma_R(n)$ is explicitly identified in [5] for the isolated cell where for $0 \leq n < R$:

$$\sigma_R(n) = \frac{r^{(1)}\lambda^{(1)}B^{(1)}(\boldsymbol{\lambda},\kappa,R) + r^{(2)}\lambda^{(2)}B^{(2)}(\boldsymbol{\lambda},\kappa,R)}{(\lambda^{(1)}+\lambda^{(2)})B^{(1)}(\boldsymbol{\lambda},n,n)} \tag{8}$$

and for $R \leq n \leq \kappa$:

$$\begin{aligned} \sigma_R(n) &= \frac{r^{(1)}B^{(1)}(\boldsymbol{\lambda},\kappa,R)}{B^{(1)}(\boldsymbol{\lambda},n,R)} \\ &\quad + \frac{r^{(2)}\lambda^{(2)}\left(B^{(2)}(\boldsymbol{\lambda},\kappa,R) - B^{(2)}(\boldsymbol{\lambda},n,R)\right)}{\lambda^{(1)}B^{(1)}(\boldsymbol{\lambda},n,R)}. \end{aligned} \tag{9}$$

Now note that the system occupancy distribution seen by an admitted type 1 connection is given by

$$\pi_o^{(1)}(n) = \begin{cases} \frac{\pi_\lambda(n)}{\sum_{i=0}^{\kappa-1}\pi_\lambda(i)} & \text{if } 0 \leq n \leq \kappa-1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Also for an admitted type 2 connection

$$\pi_o^{(2)}(n) = \begin{cases} \frac{\pi_\lambda(n)}{\sum_{i=0}^{R-1}\pi_\lambda(i)} & \text{if } 0 \leq n \leq R-1 \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

The *average* implied cost $c^{(m)}$ of admitting a connection of type m can be obtained by averaging $\sigma_R(n)$ over the system occupancy distribution; i.e.,

$$c^{(m)} = \sum_{n=0}^{\kappa-1} \pi_o^{(m)}(n)\sigma_R(n), \qquad m = 1, 2. \tag{12}$$

Theorem 3.1. *(Average Implied Cost in an Isolated Cell): For* $m = 1, 2$

$$c^{(m)} = \left(1 - B^{(m)}(\boldsymbol{\lambda}, \kappa, R)\right)^{-1} \sum_{k=1,2} r^{(k)}\lambda^{(k)} \frac{d}{d\lambda^{(m)}} B^{(k)}(\boldsymbol{\lambda}, \kappa, R). \tag{13}$$

Proof. The proof of the theorem follows directly from computing formula (12) using expressions (8,9,10,11). □

In view of Theorem 3.1, the average implied cost in an isolated cell can be exactly and directly computed. However, it will be important in the subsequent discussion of general topologies to consider an insightful form of (13). Therefore, assume each cell receives a fictitious flow of connection request of each type m with rate $\hat{\lambda}^{(m)}$ and reward per connection $\hat{r}^{(m)}$ such that $\hat{\lambda}^{(m)} = \hat{r}^{(m)} = 0$. Thus, the long-term revenue rate, $W(R)$, as given by (2) remains unchanged. Now consider the derivative of $W(R)$ with respect to $\hat{\lambda}^{(m)}$. Namely:

$$\frac{d}{d\hat{\lambda}^{(m)}} W(R) = \frac{d}{d\hat{\lambda}^{(m)}} \left(\sum_{k=1,2} r^{(k)}\lambda^{(k)}(1 - B^{(k)}(\boldsymbol{\lambda}, \kappa, R)) \right), \tag{14}$$

where now $\boldsymbol{\lambda} = (\lambda^{(1)}, \lambda^{(2)}, \hat{\lambda}^{(1)}, \hat{\lambda}^{(2)})$. Note that connection requests arrive independently and therefore $\frac{d\lambda^{(k)}}{d\hat{\lambda}^{(m)}} = 0$ for $k, m = 1, 2$. Thus, equality (14) can be written as

$$\frac{d}{d\hat{\lambda}^{(m)}} W(R) = -\sum_{k=1,2} r^{(k)}\lambda^{(k)} \frac{d}{d\hat{\lambda}^{(m)}} B^{(k)}(\boldsymbol{\lambda}, \kappa, R)). \tag{15}$$

Now by combining (14) and (15), the average implied cost in an isolated cell can be written in the equivalent form

$$c^{(m)} = -\left(1 - B^{(m)}(\boldsymbol{\lambda}, \kappa, R)\right)^{-1} \frac{d}{d\hat{\lambda}^{(m)}} W(R). \tag{16}$$

3.2. General Topologies

An extension of Theorem 3.1 to general topologies can be pursued under the RLA. Namely, each blocking probability $B_j^{(m)}$ is approximated by the quantity $\hat{B}_j^{(m)}$ as given in expressions (5,6,7). In this case, the long-term revenue rate (2) can be approximated by

$$\hat{W}(\mathbf{R}) \doteq \sum_{j \in N} \sum_{m=1,2} r^{(m)} \lambda_j^{(m)} (1 - \hat{B}_j^{(m)}), \tag{17}$$

by replacing $B_j^{(m)}$ with $\hat{B}_j^{(m)}$. The definition of fictitious flows can be extended to the present context so that for any cell i, the fictitious flow of type m is such that $\hat{\lambda}_i^{(m)} = \hat{r}_i^{(m)} = 0$, $w_{ii} = 1$, and $w_{ij} = 0$ for $i \neq j$. Expression (17) can be used as a proxy to $W(\mathbf{R})$ and, by mimicking (16), the average implied cost of type m connections at cell j is defined as

$$c_j^{(m)} \doteq -(1 - \hat{b}_j^{(m)})^{-1} \frac{d}{d\hat{\lambda}_j^{(m)}} \hat{W}(\mathbf{R}),$$

where $b^{(m)}$ is given by (6).

Theorem 3.2 and Theorem 3.3 below are generalization of [15, Theorem 2.2 and Theorem 2.3] to those cases where the parameters w_{ij}'s are not restricted to values taken from the set $\{0, 1\}$. The theorems here amount to a nontrivial extension of the analysis of [15]. Proofs of the theorems are provided in the appendix.

Theorem 3.2. *: For $m = 1, 2$ and $j \in N$*

$$\begin{aligned} c_j^{(m)} &= (1 - b_j^{(m)})^{-1} \sum_{k=1,2} \frac{\partial B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j)}{\partial \rho_j^{(m)}} \times \\ & \sum_{i \in N} \rho_{ij}^{(k)} \left(r^{(k)} - (w_{ij} - 1) c_j^{(k)} - \sum_{l \in N-j} w_{il} c_l^{(k)} \right), \end{aligned} \tag{18}$$

where $b_j^{(m)}$ is defined by (6), $\rho_j^{(m)}$ is defined by (7), and

$$\rho_{ij}^{(k)} = (1 - b_j^{(k)})^{-1} w_{ij} \lambda_i^{(k)} \prod_{l \in N} (1 - b_l^{(k)})^{w_{il}}. \tag{19}$$

In Theorem 3.2, $\rho_{ij}^{(k)}$ represents the arrival rate of units interference of type k from cell i to cell j, after being thinned at other cells in the network. The total arrival rate of interference to cell j in (7) can be verified to satisfy $\rho_j^{(k)} = \sum_{i \in N} \rho_{ij}^{(k)}$. Note that the relation (18) is linear in the implied costs; hence $\{c_j^{(k)} : k = 1, 2,\ j \in N\}$ can be computed via matrix inversion methods.

Now for each cell j, let $\Delta_j^- \hat{W}(\mathbf{R})$ denote the *left derivative* of $\hat{W}(\mathbf{R})$ in the jth entry. That is, $\Delta_j^- \hat{W}(\mathbf{R})$ is the amount by which $\hat{W}(\mathbf{R})$ increases when the R_j is decreased by 1

$$\Delta_j^- \hat{W}(\mathbf{R}) = \hat{W}(\mathbf{R}) - \hat{W}(\mathbf{R} - \mathbf{e}_j), \tag{20}$$

where $\mathbf{e}_j = (e_j(i) : i \in N)$ is a binary vector that has a value 1 only at the jth entry. Also let $\Delta_j^+ H$ denote the *right derivative*

$$\Delta_j^+ \hat{W}(\mathbf{R}) = \hat{W}(\mathbf{R} + \mathbf{e}_j) - \hat{W}(\mathbf{R}). \tag{21}$$

The following theorem identifies the sensitivity of $\hat{W}(\mathbf{R})$ to individual reservation parameter values in terms of the average implied costs. The theorem forms the basis of the adaptive admission control algorithms studied in the next section. First, consider the following matrices:

$$\left[\frac{d}{d\rho_j^{(m)}} B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j)\right]_{2\times 2} \qquad j \in N, \tag{22}$$

and assume they are invertible.

Theorem 3.3. *: For $j \in N$,*

$$\begin{aligned} \Delta_j^{\pm} \hat{W}(\mathbf{R}) \ =& - \sum_{k=1,2} \Delta_j^{\pm} B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j) \sum_{i \in N} \rho_{ij}^{(k)} \times \\ & \left(r^{(k)} - (w_{ij} - 1) c_j^{(k)} - \sum_{l \in N - j} w_{il} c_l^{(k)} \right), \end{aligned} \tag{23}$$

where

$$\Delta_j^{-} B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j) = B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j) - B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j - 1) \tag{24}$$

and

$$\Delta_j^{+} B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j) = B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j + 1) - B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j). \tag{25}$$

4. Revenue Maximization via Adaptive Reservation

A guiding principle for revenue maximization involves updating the reservation parameters to improve $\hat{W}(\mathbf{R})$ based on the increments/decrements $\Delta_j^{\pm} \hat{W}(\mathbf{R})$. A straightforward implementation of this principle may have two pitfalls:

1. Centralized computation of $\Delta_j^{\pm} \hat{W}(\mathbf{R})$ may not scale gracefully to large networks. Thus, a localized procedure for computing these quantities is required.

2. Unimodality of $\hat{W}(\mathbf{R})$ cannot be guaranteed in light of the generality of considered topologies; hence classical techniques based on steepest ascent may lead to local maxima of $\hat{W}(\mathbf{R})$.

In this section, the first issue is addressed by exploiting expressions (18) and (23) which are of particular interest from the standpoint of distributed implementation. The second issue will be addressed by resorting to algorithms based on simulated annealing; a generic method for global optimization widely used in problems which involve discrete state spaces [19].

First note that obtaining the quantities $b_j^{(m)}$'s and $c_j^{(m)}$'s requires solving systems of fixed point equations (6) and (18), respectively. Thus, given a vector $\mathbf{x} = (x_j^{(m)} : m = 1, 2;\ j \in N)$, define the following linear mapping using formula (6)

$$f^{(m)} : \mathbb{R}^{2|N|} \to \mathbb{R}^{|N|},\ f^{(m)} = (f_1^{(m)}, f_2^{(m)}, \cdots, f_{|N|}^{(m)}), \tag{26}$$

where

$$f_j^{(m)}(\mathbf{x}) \doteq B^{(m)}\left(\boldsymbol{\rho}_j(\mathbf{x}), \kappa_j, R_j\right), \quad j \in N. \tag{27}$$

Here $\boldsymbol{\rho}_j(\mathbf{x}) = (\rho_j^{(1)}(\mathbf{x}), \rho_j^{(2)}(\mathbf{x}))$ is defined as in (7) so that

$$\rho_j^{(m)}(\mathbf{x}) \quad \doteq (1 - x_j^{(m)})^{-1} \sum_{i \in N} w_{ij} \lambda_i^{(m)} \prod_{l \in N} (1 - x_l^{(m)})^{w_{il}}. \tag{28}$$

Define also the mapping

$$g^{(m)} : \mathbb{R}^{2|N|} \to \mathbb{R}^{|N|},\ g^{(m)} = (g_1^{(m)}, g_2^{(m)}, \cdots, g_{|N|}^{(m)}), \tag{29}$$

based on formula (18). Namely, for a given vector $\mathbf{y} = (y_j^{(m)} : m = 1, 2;\ j \in N)$,

$$g_j^{(m)}(\mathbf{y}) \doteq (1 - b_j^{(m)})^{-1} \sum_{k=1,2} \frac{\partial B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j)}{\partial \rho_j^{(m)}} \times \\ \sum_{i \in N} \rho_{ij}^{(k)} \left(r^{(k)} - (w_{ij} - 1) y_j^{(k)} - \sum_{l \in N - j} w_{il} y_l^{(k)} \right). \tag{30}$$

Note that $\{b_j^{(m)} : m = 1, 2;\ j \in N\}$ and $\{c_j^{(m)} : m = 1, 2;\ j \in N\}$ are fixed points of the mappings $(f^{(1)}, f^{(2)})$, and $(g^{(1)}, g^{(2)})$, respectively.

An important observation is that values of w_{il} typically decrease as cell l is further located from cell i. Thus, it is reasonable to assume $w_{il} = 0$ for cells beyond a certain distance from cell i. We measure such distance by the number of consecutive neighbouring cells and we refer to it as the neighbourhood distance. We use this observation to propose a procedure for computing $b_j^{(m)}$'s and $c_j^{(m)}$'s using localized message passing between cells that are within a neighbourhood distance of each other.

Assume that each cell l periodically measures the average arrival rate $\lambda_l^{(m)}$ for each type m. Assume also that each cell l starts with arbitrary initial values of $b_l^{(m)}$ and $c_l^{(m)}$ for $m = 1, 2$. The values are updated as follows:

1. Each cell l broadcasts the quantities $b_l^{(m)}$ and $c_l^{(m)}$ to cells within its neighbourhood distance. Thus, each cell i can compute the quantity $\lambda_i^{(m)} \prod_{l \in N} (1 - b_l^{(m)})^{w_{il}}$ for each type m.
2. Each cell i broadcasts the quantity $\lambda_i^{(m)} \prod_{l \in N} (1 - b_l^{(m)})^{w_{il}}$ to its neighbouring cells. Thus each cell j can compute $\rho_{ij}^{(m)}$ and $\rho_j^{(m)}$, where $\rho_{ij}^{(m)}$ is given by (19) and $\rho_j^{(m)} = \sum_{i \in N} \rho_{ij}^{(m)}$.
3. Each cell j iterates the mappings (26), (27), (28) (29), (30) and broadcasts the new values to its neighbouring cells.

The procedure is repeated for sufficient time till the iterations converge and each cell has all the quantities required to compute (23). Each step of the procedure should be allowed sufficient time so that the network comes to general consensus. It is important to report here that there is no solid result that proves convergence of this iterative approach for any topology. However, the numerical example we provide in Section (5) for a 7-cell lattice topology shows convergence.

Distributed Simulated Annealing:

On a longer time scale, let each cell j update its reservation parameter R_j at rate γ_j. Namely, for a given R_j define the set of neighbouring states $\{R_j - 1, R_j + 1 : 0 \leq R_j \pm 1 \leq \kappa_j\}$. When the internal clock ticks, the cell chooses a neighbouring state R'_j according to a certain probability distribution. The cell then computes the corresponding value $\Delta_j^{\pm}\hat{W}(\mathbf{R})$ based on the conventions in (20) and (21); that is:

$$\begin{array}{llll} \text{case (1):} & \text{If } R'_j = R_j - 1 & \text{compute} & \Delta_j^{-}\hat{W}(\mathbf{R}) \\ \text{case (2):} & \text{If } R'_j = R_j + 1 & \text{compute} & \Delta_j^{+}\hat{W}(\mathbf{R}). \end{array}$$

In case (1), the cell adopts $R'_j = R_j - 1$ as a new reservation parameter if $\Delta_j^{-}\hat{W}(\mathbf{R}) < 0$. If $\Delta_j^{-}\hat{W}(\mathbf{R}) \geq 0$ the cell adopts the new reservation parameter with probability $\exp\left(\frac{-\Delta_j^{-}\hat{W}(\mathbf{R})}{s_{j_t}}\right)$, where s_{j_t} represents a time decreasing temperature schedule at the cell such that s_{j_t} goes to 0 as time $t \to \infty$. This way a local maxima of the function $\hat{W}(.)$ can be avoided. In case (2), the cell adopts $R'_j = R_j + 1$ if $\Delta_j^{+}\hat{W}(\mathbf{R}) > 0$ or otherwise with probability $\exp\left(\frac{\Delta_j^{+}\hat{W}(\mathbf{R})}{s_{j_t}}\right)$. A pseudo-code for the distributed algorithm is given as follows:

Distributed Simulated Annealing Algorithm

1. Initialize $R_{j_{start}}$
2. $R_j \leftarrow R_{j_{start}}$
3. With rate γ_j

   3.1 Choose a neighbouring state $R'_j \in \{R_j - 1, R_j + 1\}$ with probability p_j

   3.2 If $R'_j = R_j - 1$

   3.2.1 If $\min\left\{1, \exp\left(\frac{-\Delta_j^- W(\mathbf{R})}{s_{j_t}}\right)\right\} > \text{random}[0,1)$

   $R_j \leftarrow R'_j$

   3.3 If $R'_j = R_j + 1$

   3.3.1 If $\min\left\{1, \exp\left(\frac{\Delta_j^+ W(\mathbf{R})}{s_{j_t}}\right)\right\} > \text{random}[0,1)$

   $R_j \leftarrow R'_j$

   3.4 $t \leftarrow t + 1$.

The algorithm can be considered as a distributed version of the simulated annealing algorithm. It can adapt to traffic fluctuations due to the time-of-day use of the network. In particular, when traffic rate changes at a certain cell, the iterative procedure converges to new values of $b_j^{(m)}$'s and $c_j^{(m)}$'s. Consequently, the cells update their reservation parameters according to the new set of values.

Choosing a simulated annealing approach seems natural in light of the generality of network topologies considered for this work and the lack of knowledge of the shape of revenue function under such generalization. Convergence properties of simulated annealing have been studied for example in [19]. Still, for some topologies, the shape of the revenue function can be closely characterized and thus, more efficient algorithms can be implemented. For example, in the special case of the isolated cell, it has been shown in [13] that, under certain traffic assumptions, the revenue function is unimodal and thus, logarithmic search techniques such as Fibonacci search [20] can be used.

5. Numerical Example

In the following, we give a numerical example for updating reservation parameters based on formula (23) via the distributed simulated annealing algorithm and show its adaptivity to fluctuations in traffic rates. Consider for this purpose a CDMA wideband system of 7-cell lattice topology with a graph representation shown in Figure 2. Assume that an established connection at a given cell generates interference at that cell and every other cell that it shares a boundary with. Thus, interference between cells that share no boundary is neglected. The system has the following parameters: channel bandwidth = 1.25 MHz, uplink data rate = 64.4 kbps, path loss exponent = 4.0, and the fraction of time a user is expected to be active = 40%. We compute w_{ij} and κ_i using a Chernoff bound approach as detailed in [18] with mobile terminal locations taken to be random and uniformly distributed in the cells. Namely, we obtain:

$$\begin{cases} w_{ii} = 15.0 & \\ w_{ij} = 1.0 & \text{if } i \text{ and } j \text{ are neighbors,} \end{cases}$$

and $\kappa_i = 54.0$ for all i.

Now assume that the license holder opens cell 1 for type 2 traffic; i.e., $\lambda_i^{(2)} = 0$ for $i = 2, \cdots, 7$. Let $r^{(1)}$ and $r^{(2)}$ be 1.0 and 0.75, respectively. Assume also that each cell $i = 1, 2, \cdots, 7$ updates its reservation parameter using a Poisson clock with rate $\gamma_i = 1.0$. Once the clock ticks, the cell chooses a neighboring state $R_i' = R_i \pm 1$ with probability 0.5. The cell then computes $\Delta_i^+ \hat{W}(\mathbf{R})$, or $\Delta_i^- \hat{W}(\mathbf{R})$, and updates its reservation parameter if necessary. To obtain values of $\Delta_i^\pm \hat{W}(\mathbf{R})$, we first solve offline equations (6) and (18) iteratively via repeated substitution. In this experiment, we set $s_j^t = 0$, i.e., we do not use a temperature schedule at the cells as it is not the purpose of the example to verify local maxima avoidance techniques.

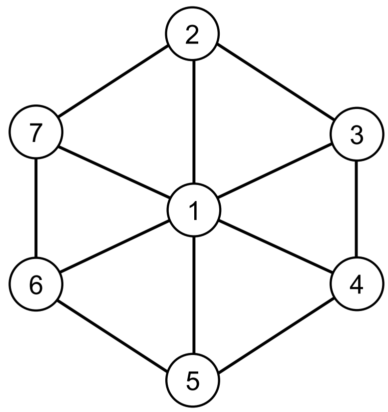


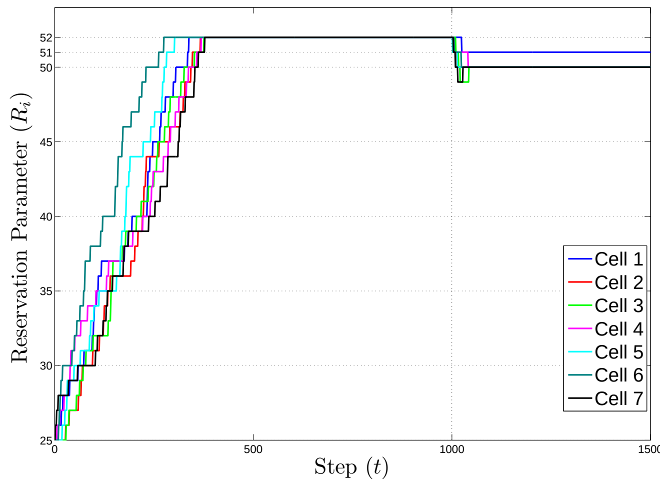
Figure 2: Network graph of a 7-cell hexagonal lattice topology used for the numerical example of section 5.

Figure 3(a) shows the trajectories of updating the reservation parameters at the different cells. For the first 1000 steps, $\lambda_i^{(1)}$ for $i = 1, \cdots, 7$ are taken to be 1.0 and $\lambda_1^{(2)}$ is 5.0. The algorithm starts with each cell having a reservation parameter $R_i = 25$. As each cell updates its parameter according to its own Poisson clock rate, the cells converge to the value $R_i^* = 52$ for all i.

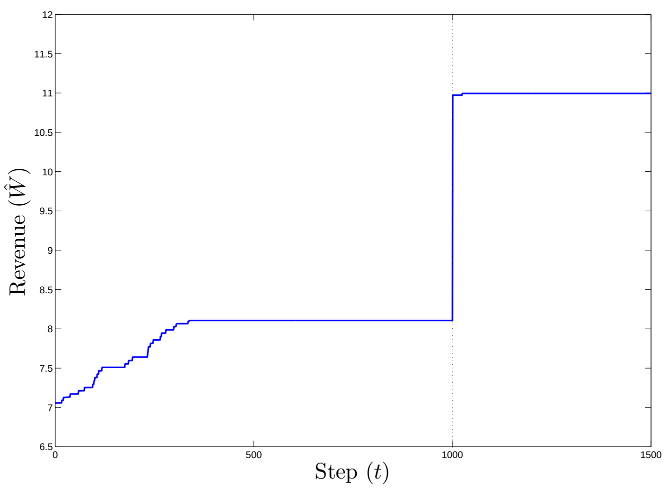
In the second part of the experiment, we change the traffic rates so that for the rest of the experiment $\lambda_i^{(1)} = 1.5$ for all i and $\lambda_1^{(2)} = 4.0$. The result shows that all the cells adapt their reservation parameters and quickly converge to the values $R_1^* = 51$ and $R_i^* = 50$ for $i = 2, \cdots, 7$.

Figure 3(b) shows the rate of revenue from the network at the different time steps of implementing the algorithm. For the first 1000 steps and while $\lambda_1^{(2)} = 1.0$, the revenue rate increases gracefully to the value 8.11. When traffic rates change, the algorithm adapts and the revenue improves and converges to the new value 10.99.

Verifying optimality of algorithm performance requires an exhaustive search over all the possible reservation parameters. This task is computationally intense. However, the previous results show that the achieved polices are symmetric and have at most two distinct reservation parameter values;



(a)



(b)

Figure 3: (a) Trajectories for updating the reservation parameters in a 7-cell lattice topology. In the first 1000 steps, the reservation parameters converge to the values $R_i^* = 52$ for $i = 1, \cdots 7$. When traffic rates change, the algorithm adapts and the reservation parameters converge fast to the new values $R_1^* = 51$ and $R_i^* = 50$ for $i = 2, \cdots 7$. (b) Rate of revenue at different time steps of the implementation. In the first 1000 steps, the revenue rate improves to the value 8.11. After the change in traffic rates, the algorithm adapts the reservation parameters and the rate improves to the new value 10.99.

one for cell 1 and one for cells $2-7$. In this context, we limit our search to a sub-domain that covers all reservation parameter vectors $\mathbf{R}$ which have at most two distinct entry values. For each set of parameters, we compute revenue rate under the RLA. The maximum rate is found to be 8.11, achieved at the same set of parameters obtained by the algorithm. In the same manner, the results of the second part of the experiment have been verified.

6. Conclusion

In this paper, we have considered an admission control problem for wireless cellular networks under the objective of revenue maximization. The problem involved service measures which favor primary users by reserving part of the capacity of each cell for their exclusive use. We have developed an analytical framework that captures the average implied cost of establishing a connection in the network. An important value of this work lies in the fact that it develops concepts from wireline to wireless telephony and proves their usefulness for computational tractability. We have used simplified reasoning in explaining these concepts and making them appeal to readers who are more interested in the practical side of the problem. In this respect, and starting with an isolated cell, we have exactly and explicitly characterized the average implied cost of establishing a connection by using notions from dynamic programming. An extension of this result to general topologies has been pursued under the reduced load approximation and led to guiding principles for updating reservation parameters at the cells. Given the complexity of implementing a reservation-based policy in a centralized fashion, we have suggested a distributed online algorithm that can be employed at the base stations and can adapt to fluctuations in traffic rates.

Appendix A. Proofs of Theorems 3.2 and 3.3

For convenience of the proofs, we consider an extended network model by augmenting each (original) cell $i \in N$ by cell i'. Fictitious call requests at cell i are now assumed to arrive at cell i', instead of cell i, and such that

$$w_{i'j} = \begin{cases} 1 & \text{if } j = i, \\ 0 & \text{else.} \end{cases}$$

Note that the specification on the right hand side involves cell $i \in N$ rather than i'. Here we refer to the arrival rate of fictitious call requests by $\lambda_{i'}^{(m)}$, instead of $\hat{\lambda}_i^{(m)}$, and the reward rate by $r_{i'}^{(m)}$, instead of $\hat{r}_i^{(m)}$, where as before $\lambda_{i'}^{(m)} = r_{i'}^{(m)} = 0$ for all i. Hence, the extended model is analytically equivalent to the original model. We denote the set of cells in this new model by

$$D = \{i, i' : i \in N\}.$$

For the purposes of reducing notational burden, we will consider more than one form of revenue function (17) by emphasizing or suppressing its dependence on the parameters $\mathbf{R}$ and $\boldsymbol{\lambda}$. Now consider the following form of the revenue function (17) with emphasis on its dependence on the demand vector $\boldsymbol{\lambda}$:

$$\hat{W}(\boldsymbol{\lambda}) = \sum_{m=1,2} \sum_{i \in N} r^{(m)} \xi_i^{(m)}(\boldsymbol{\lambda}), \tag{A.1}$$

where we define

$$\xi_i^{(m)}(\boldsymbol{\lambda}) = \lambda_i^{(m)} \prod_{j \in N} (1 - b_j^{(m)}(\boldsymbol{\lambda}))^{w_{ij}} \tag{A.2}$$

and

$$b_j^{(m)}(\boldsymbol{\lambda}) = B^{(m)}(\boldsymbol{\rho}_j(\boldsymbol{\lambda}), \kappa_j, R_j) \tag{A.3}$$

where

$$\boldsymbol{\rho}_j(\boldsymbol{\lambda}) = (\rho_j^{(1)}(\boldsymbol{\lambda}), \rho_j^{(2)}(\boldsymbol{\lambda}))$$

and

$$\rho_j^{(m)}(\boldsymbol{\lambda}) = (1 - b_j^{(m)}(\boldsymbol{\lambda}))^{-1} \sum_{i \in D} w_{ij} \lambda_i^{(k)} \prod_{l \in N} (1 - b_l^{(m)}(\boldsymbol{\lambda}))^{w_{il}}, \quad m = 1, 2. \tag{A.4}$$

Here (A.3) and (A.4) are functional representations of formulas (6) and (7), respectively.

Lemma 1. *For $j \in N$ and $m = 1, 2$*

$$\frac{d}{d\lambda_{j'}^{(m)}}\hat{W}(\boldsymbol{\lambda}) \;\; = \;\; -\sum_{k=1,2}\sum_{l\in N}(1-b_l^{(k)}(\boldsymbol{\lambda}))^{-1}\left(\sum_{i\in D} w_{il} r_i^{(k)} \xi_i^{(k)}(\boldsymbol{\lambda})\right)\frac{d}{d\lambda_{j'}^{(m)}} b_l^{(k)}(\boldsymbol{\lambda}).$$

Proof. Given a vector $\mathbf{b} = (b_l^{(k)} : k = 1,2; l \in N) \in [0,1]^{2N}$, define the following equivalent form to (A.1)

$$\hat{W}(\boldsymbol{\lambda}, \mathbf{b}) = \sum_{k=1,2}\sum_{i\in D} r_i^{(k)} \lambda_i^{(k)} \prod_{l\in N}(1-b_l^{(k)})^{w_{il}}. \tag{A.5}$$

Thus, we can write

$$\frac{d}{d\lambda_{j'}^{(m)}}\hat{W}(\boldsymbol{\lambda}) = \left[\frac{d}{d\lambda_{j'}^{(m)}} + \sum_{k=1,2}\sum_{l\in N}\frac{d}{d\lambda_{j'}^{(m)}} b_l^{(k)} \frac{\partial}{\partial b_l^{(k)}}\right]\hat{W}(\boldsymbol{\lambda}, \mathbf{b})\Bigg|_{\mathbf{b}=\mathbf{b}(\boldsymbol{\lambda})}$$

where $\mathbf{b}(\boldsymbol{\lambda}) = (b_l^{(k)}(\boldsymbol{\lambda}) : k = 1,2; l \in N)$. The claim of the lemma can be then verified by observing that $\frac{d}{d\lambda_{j'}^{(m)}}\hat{W}(\lambda; \mathbf{b}) = 0$ and

$$\begin{aligned}\frac{\partial}{\partial b_l^{(k)}}\hat{W}(\boldsymbol{\lambda}, \mathbf{b}) &= -\sum_{i\in D} r_i^{(k)} \lambda_i^{(k)} w_{il}(1-b_l^{(k)})^{w_{il}-1}\prod_{s\in N-l}(1-b_s^{(k)})^{w_{is}} \\ &= -(1-b_l^{(k)})^{-1}\sum_{i\in D} w_{il} r_i^{(k)} \lambda_i^{(k)} \prod_{s\in N}(1-b_s^{(k)})^{w_{is}}.\end{aligned}$$

□

Lemma 2. *Let δ_{jl} be 1 if $j = l$ and 0 otherwise. Then, for $l, k \in N$ and*

$k, m \in \{1,2\}$:

$$\frac{d}{d\lambda_{j'}^{(m)}} b_l^{(k)}(\boldsymbol{\lambda}) = \delta_{jl} \frac{\partial}{\partial \rho_l^{(m)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l)$$
$$- \sum_{u=1,2} \frac{\partial}{\partial \rho_l^{(u)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l)(1 - b_l^{(u)}(\boldsymbol{\lambda}))^{-2} \left(\sum_{i \in D} w_{il}(w_{il} - 1)\xi_i^{(u)}(\boldsymbol{\lambda}) \right) \frac{d}{d\lambda_{j'}^{(m)}} b_l^{(u)}(\boldsymbol{\lambda})$$
$$- \sum_{u=1,2} \frac{\partial}{\partial \rho_l^{(u)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l)(1 - b_l^{(u)}(\boldsymbol{\lambda}))^{-1} \times$$
$$\sum_{s \in N - l} (1 - b_s^{(u)}(\boldsymbol{\lambda}))^{-1} \left(\sum_{i \in D} w_{il} w_{is} \xi_i^{(u)}(\boldsymbol{\lambda}) \right) \frac{d}{d\lambda_{j'}^{(m)}} b_s^{(u)}(\boldsymbol{\lambda}).$$

Proof. Given $\mathbf{b} = (b_l^{(k)} : k = 1, 2; l \in N)$, we define the following form of (A.3)

$$b_l^{(k)}(\boldsymbol{\lambda}, \mathbf{b}) = B^{(k)} \left(\boldsymbol{\rho}_l(\boldsymbol{\lambda}, \mathbf{b}), \kappa_l, R_l \right) \tag{A.6}$$

so that

$$\frac{d}{d\lambda_{j'}^{(m)}} b_l^{(k)}(\boldsymbol{\lambda}) = \left[\frac{d}{d\lambda_{j'}^{(m)}} + \sum_{u=1,2} \sum_{s \in N} \frac{d}{d\lambda_{j'}^{(m)}} b_s^{(u)}(\boldsymbol{\lambda}) \frac{\partial}{\partial b_s^{(u)}} \right] b_l^{(k)}(\boldsymbol{\lambda}; \mathbf{b}) \Bigg|_{\mathbf{b} = \mathbf{b}(\boldsymbol{\lambda})} . \tag{A.7}$$

Note that

$$\frac{d}{d\lambda_{j'}^{(m)}} b_l^{(k)}(\boldsymbol{\lambda}, \mathbf{b}) = \delta_{jl} \frac{\partial}{\partial \rho_l^{(m)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l). \tag{A.8}$$

By the same token and for $s \neq l$

$$\begin{aligned}
&\frac{\partial}{\partial b_s^{(u)}} b_l^{(k)}(\boldsymbol{\lambda}, \mathbf{b}) \\
&\quad = \; -\frac{\partial}{\partial \rho_l^{(u)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l)(1 - b_l^{(u)})^{-1} \times \\
&\qquad \sum_{i \in D} w_{il} \lambda_i^{(u)} w_{is} (1 - b_s^{(u)})^{w_{is}-1} \prod_{j \in N-s} (1 - b_j^{(u)})^{w_{ij}} \\
&\quad = \; -\frac{\partial}{\partial \rho_l^{(u)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l)(1 - b_l^{(u)})^{-1}(1 - b_s^{(u)})^{-1} \times \\
&\qquad \sum_{i \in D} w_{il} w_{is} \lambda_i^{(u)} \prod_{j \in N} (1 - b_j^{(u)})^{w_{ij}}. \qquad \text{(A.9)}
\end{aligned}$$

While for $s = l$

$$\begin{aligned}
&\frac{\partial}{\partial b_l^{(u)}} b_l^{(k)}(\boldsymbol{\lambda}, \mathbf{b}) \\
&= -\frac{\partial}{\partial \rho_l^{(u)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l) \sum_{i \in D} w_{il} \lambda_i^{(u)} (w_{il} - 1) \times \\
&\quad (1 - b_l^{(u)})^{w_{il}-2} \prod_{j \in N-l} (1 - b_j^{(u)})^{w_{ij}} \\
&= -\frac{\partial}{\partial \rho_l^{(u)}} B^{(k)}(\boldsymbol{\rho}_l, \kappa_l, R_l)(1 - b_l^{(u)})^{-2} \sum_{i \in D} w_{il}(w_{il} - 1) \lambda_i^{(u)} \prod_{j \in N} (1 - b_j^{(u)})^{w_{ij}}. \\
&\qquad \text{(A.10)}
\end{aligned}$$

The desired result is obtained by substituting (A.8)-(A.10) in (A.7). □

Next, we express Lemmas 1 and 2 in a matrix form that will be useful in the sequel. Namely, define the following matrices

$$
\begin{aligned}
w &= \begin{bmatrix} [w_{ij}]_{D\times N} & 0 \\ 0 & [w_{ij}]_{D\times N} \end{bmatrix}_{2D\times 2N} \\
\frac{db}{d\lambda} &= \begin{bmatrix} \left[\mathcal{E}_{jl}^{(1,1)}\right]_{N\times N} & \left[\mathcal{E}_{jl}^{(1,2)}\right]_{N\times N} \\ \left[\mathcal{E}_{jl}^{(2,1)}\right]_{N\times N} & \left[\mathcal{E}_{jl}^{(2,2)}\right]_{N\times N} \end{bmatrix}_{2N\times 2N}, \qquad \textit{where} \\
& \mathcal{E}_{jl}^{(k,m)} = \frac{d}{d\lambda_{l'}^{(m)}} b_j^{(k)}(\boldsymbol{\lambda}) \\
\eta &= \begin{bmatrix} \left[\eta_{jl}^{(1,1)}\right]_{N\times N} & \left[\eta_{jl}^{(1,2)}\right]_{N\times N} \\ \left[\eta_{jl}^{(2,1)}\right]_{N\times N} & \left[\eta_{jl}^{(2,2)}\right]_{N\times N} \end{bmatrix}_{2N\times 2N}, \qquad \textit{where} \\
& \eta_{jl}^{(k,m)} = \delta_{jl} \frac{\partial}{\partial \rho_l^{(m)}} B^{(k)}(\boldsymbol{\rho}_j, \kappa_i, R_i). \\
\Lambda &= \begin{bmatrix} \left[\Lambda_{jl}^{(1)}\right]_{N\times N} & 0 \\ 0 & \left[\Lambda_{jl}^{(2)}\right]_{N\times N} \end{bmatrix}_{2N\times 2N}, \qquad \textit{where} \\
& \Lambda_{jl}^{(k)} = \begin{cases} \sum_{i\in D} w_{il}(w_{il}-1)\xi_i^{(k)}(\boldsymbol{\lambda}) & \text{if } j = l \\ \sum_{i\in D} w_{ij} w_{il} \xi_i^{(k)}(\boldsymbol{\lambda}) & \text{otherwise.} \end{cases}
\end{aligned}
$$

We define also the following diagonal matrices

$$
\begin{aligned}
\beta &= \begin{bmatrix} \left[diag[1-b_i^{(1)}(\boldsymbol{\lambda})]\right]_N & 0 \\ 0 & \left[diag[1-b_i^{(2)}(\boldsymbol{\lambda})]\right]_N \end{bmatrix}_{2N\times 2N} \\
\xi &= \begin{bmatrix} \left[diag[\xi_i^{(1)}(\boldsymbol{\lambda})]\right]_D & 0 \\ 0 & \left[diag[\xi_i^{(2)}(\boldsymbol{\lambda})]\right]_D \end{bmatrix}_{2D\times 2D}
\end{aligned}
$$

and the row vectors

$$\begin{aligned}
r &= [row[r_i^{(1)}]_D \quad row[r_i^{(2)}]_D]_{2D}, \\
\frac{d\hat{W}}{d\lambda} &= \left[row[\frac{d}{d\lambda_{j'}^{(1)}}\hat{W}(\boldsymbol{\lambda})]_N \quad row[\frac{d}{d\lambda_{j'}^{(2)}}\hat{W}(\boldsymbol{\lambda})]_N \right]_{2N}.
\end{aligned}$$

Lemmas 1 and 2 can be expressed respectively in terms of these matrices as

$$\begin{aligned}
\frac{d\hat{W}}{d\lambda} &= -r\xi w\beta^{-1}\frac{db}{d\lambda} \\
\frac{db}{d\lambda} &= \eta(I - \beta^{-1}\Lambda\beta^{-1}\frac{db}{d\lambda}).
\end{aligned} \tag{A.11}$$

In particular, the last equality can be written as

$$\frac{db}{d\lambda} = (I + \eta\beta^{-1}\Lambda\beta^{-1})^{-1}\eta. \tag{A.12}$$

Proof of Theorem 3.2

Appeal to equalities (A.11)-(A.12) to obtain the matrix equation

$$\begin{aligned}
\frac{d\hat{W}}{d\lambda} &= -r\xi w\beta^{-1}(I + \eta\beta^{-1}\Lambda\beta^{-1})^{-1}\eta \\
&= -r\xi w\beta^{-1}(I - (I + \eta\beta^{-1}\Lambda\beta^{-1})^{-1}\eta\beta^{-1}\Lambda\beta^{-1})\eta \\
&= -r\xi w\beta^{-1}(I - \frac{db}{d\lambda}\beta^{-1}\Lambda\beta^{-1})\eta \\
&= -\left(r\xi w\beta^{-1} + \frac{d\hat{W}}{d\lambda}\beta^{-1}\Lambda\beta^{-1} \right)\eta.
\end{aligned} \tag{A.13}$$

The second equality uses the Matrix Inversion Lemma and can be verified directly, the third equality uses expression (A.12), and the last equality follows from (A.11). The jth component of this vector equality is:

$$\frac{d}{d\lambda_{j'}^{(m)}}\hat{W}(\boldsymbol{\lambda}) = -\sum_{k=1,2}\frac{\partial}{\partial\rho_j^{(m)}}B^{(k)}(\boldsymbol{\rho}_j,\kappa_j,R_j)\sum_{i\in D}r_i^{(k)}w_{ij}\xi_i^{(k)}(\boldsymbol{\lambda})(1-b_j^{(k)}(\boldsymbol{\lambda}))^{-1}$$
$$+\left(\sum_{i\in D}w_{ij}(w_{ij}-1)\xi_i^{(k)}(\boldsymbol{\lambda})\right)(1-b_j^{(k)}(\boldsymbol{\lambda}))^{-2}\frac{d}{d\lambda_{j'}^{(k)}}\hat{W}(\boldsymbol{\lambda})$$
$$+\sum_{l\in N-j}\left(\sum_{i\in D}w_{ij}w_{il}\xi_i^{(k)}(\boldsymbol{\lambda})\right)(1-b_j^{(k)}(\boldsymbol{\lambda}))^{-1}(1-b_l^{(k)}(\boldsymbol{\lambda}))^{-1}\frac{d}{d\lambda_{l'}^{(k)}}\hat{W}(\boldsymbol{\lambda}).$$

Using expression (17) in the theorem, the equality can be written as

$$\frac{d}{d\lambda_{j'}^{(m)}}\hat{W}(\boldsymbol{\lambda}) = -\sum_{k=1,2}\frac{\partial}{\partial\rho_j^{(m)}}B^{(k)}(\boldsymbol{\rho}_j,\kappa_j,R_j)\times$$
$$\sum_{i\in D}\rho_{ij}^{(k)}\left(r_i^{(k)}+(w_{ij}-1)(1-b_j^{(k)}(\boldsymbol{\lambda}))^{-1}\frac{d}{d\lambda_{j'}^{(k)}}\hat{W}(\boldsymbol{\lambda})\right.$$
$$\left.+\sum_{l\in N-j}w_{il}(1-b_l^{(k)}(\boldsymbol{\lambda}))^{-1}\frac{d}{d\lambda_{l'}^{(k)}}\hat{W}(\boldsymbol{\lambda})\right).$$

The proof is completed by multiplying both side of the equality with $(1-b_j^{(m)}(\boldsymbol{\lambda}))^{-1}$ and letting

$$c_j^{(m)} = -(1-b_j^{(m)}(\boldsymbol{\lambda}))^{-1}\frac{d}{d\lambda_{j'}^{(m)}}\hat{W}(\boldsymbol{\lambda}) \quad \textit{for all } j. \tag{A.14}$$

□

Proof of Theorem 3.3

We will prove the formula for $\Delta_j^-\hat{W}(\mathbf{R})$. The proof for $\Delta_j^+\hat{W}(\mathbf{R})$ can be obtained by following the same procedure in this proof. Here we will use an informal argument similar to the one used in the proof of [15, Theorem 2.3]. Namely, given that the matrices in (22) are invertible, there exists a solution

θ_{uj} for $u = 1, 2$ which solves the system of equations

$$\Delta_j^- B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j) + \sum_{u=1,2} \theta_{uj} \frac{d}{d\rho_j^{(u)}} B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j) = 0, \quad k = 1, 2, \quad \text{(A.15)}$$

where

$$\Delta_j^- B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j)$$

is given by (24).

Now consider the perturbation where R_j is reduced by one unit. Assume this reduction is compensated for by a change in $\lambda_{j'}^{(k)}$ for $k = 1, 2$ so that $b_j^{(k)}$ stays constant. Thus, given that $r_{j'}^{(k)} = 0$, the change of revenue due to reducing R_j has been compensated for. Namely, we can write

$$\Delta_j^- \hat{W}(\mathbf{R}) + \sum_{m=1,2} \theta_{mj} \frac{d\hat{W}(\mathbf{R})}{d\lambda_{j'}^{(m)}} = 0.$$

Using(A.14), we obtain

$$\Delta_j^- \hat{W}(\mathbf{R}) = \sum_{m=1,2} \theta_{mj} (1 - b_j^{(m)}) c_j^{(m)}.$$

Now by substituting for $c_j^{(m)}$ from (18), we obtain

$$\begin{aligned} \Delta_j^- \hat{W}(\mathbf{R}) \quad = \quad & \sum_{m=1,2} \theta_{mj} \sum_{k=1,2} \frac{\partial B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j)}{\partial \rho_j^{(m)}} \times \\ & \sum_{i \in N} \rho_{ij}^{(k)} \left(r^{(k)} - (w_{ij} - 1) c_j^{(k)} - \sum_{l \in N - j} w_{il} c_l^{(k)} \right). \end{aligned}$$

The expression can be rearranged in the following form

$$
\begin{aligned}
\Delta_j^- \hat{W}(\mathbf{R}) \quad = \quad & \sum_{k=1,2} \left(\sum_{m=1,2} \theta_{mj} \frac{\partial B^{(k)}(\boldsymbol{\rho}_j, \kappa_j, R_j)}{\partial \rho_j^{(m)}} \right) \times \\
& \sum_{i \in N} \rho_{ij}^{(k)} \left(r^{(k)} - (w_{ij} - 1) c_j^{(k)} - \sum_{l \in N-j} w_{il} c_l^{(k)} \right).
\end{aligned}
$$

The result in the theorem now follows from (A.15). □

References

[1] Federal Communications Commission, In the Matter of Unlicensed Operation in the TV Broadcast Bands, Second Report and Order and Memorandum Opinion and Order, FCC 08-260, November, 2008.

[2] M. BH Weiss, Secondary Use of Spectrum: A Survey of the Issues, *Info,* vol. 8, no. 2, pp. 74–82, 2006.

[3] M. Cave, C. Doyle, and W. Webb, *Essentials of Modern Spectrum Management,* Cambridge University Presss, 2007.

[4] P. J. Weiser and D. Hatfield, Spectrum Policy Reform and the Next Frontier of Property Rights, *George Mason Law review,* vol. 15, no. 3, pp. 549–609, 2008.

[5] P. B. Key, Optimal Control and Trunk Reservation in Loss Networks, *Probability in the Engineering and Informational Sciences,* vol. 4, pp. 203–242, 1990.

[6] S. Jordan and P. Varaiya, Control of Multiple Service, Multiple Resource Communication Networks, *IEEE Transactions on Communications*, vol. 42, no. 11 pp. 2979–2988, 1994.

[7] C. P. Paschalidis and Y. Lui, Pricing in Multiservice Loss Networks: Static Pricing, Asymptotic Optimality, and Demand Substitution Effects, *IEEE/ACM Transactions on Networking,* vol. 10, no. 3, pp. 171–184, June, 2002.

[8] N. Gans and S. Savin, Pricing and Capacity Rationing for Rentals with Uncertain Durations, *Management Sciences,* vol. 53, no. 3, pp. 390–407, 2007.

[9] D. P. Bertsekas, *Dynamic Programming and Optimal Control,* Athena Scientific, Belmont, 1984.

[10] M. Ahmed, Call Admission Control in Wireless Networks: A Comprehensive Survey, *IEEE Communications Surveys,* vol. 7, no. 1, pp. 50–69, 2005.

[11] S. A. Lippman, Applying a New Device in the Optimization of Exponential Qeueing Systems, *Operations Research,* vol. 23, no. 4, pp. 687–710, 1975.

[12] R. Ramjee, R. Nagarajan, and D. F. Towsley, On Optimal Call Admission Control in Cellular Networks, *Infocom,* pp. 43–50, 1996.

[13] H. Mutlu, M. Alanyali, and D. Starobinski, Spot Pricing of Secondary Spectrum Access in Wireless Cellular Networks, *IEEE/ACM Transactions on Networking,* vol. 17, no. 6, pp. 1794–1804, 2009.

[14] F. P. Kelly, Loss networks, *Annals of Applied Probability,* vol. 1, pp. 319–378, 1991.

[15] F. P. Kelly, Routing and Capacity Allocation in Networks with Trunk Reservation, *Math. of Oper. Res.,* vol. 15, pp. 771–793, 1990.

[16] F. P. Kelly, Loss Networks, *Annals of Applied Probability* vol. 1, no. 3,319–378, 1991.

[17] J. S. Evans and D. Everitt, On the Teletraffic Capacity of CDMA Cellular Networks, *IEEE Trans. on Vehicular Tech.*, vol. 48, no. 1, pp. 153–165, 1999.

[18] J. S. Evans and D. Everitt, Effective Bandwidth-Based Admission Control for Multiservice CDMA Cellular Networks, *IEEE Trans. on Vehicular Tech.*, vol. 48, no. 1, pp. 36–46, 1999.

[19] B. Hajek, A Tutorial Survey of Theory and Applications of Simulated Annealing, *IEEE Conference on Decision and Control CDC*, 1985.

[20] D. Wilde, *Optimum seeking methods,* Prentice-Hall, 1964.