

Lecture 7:

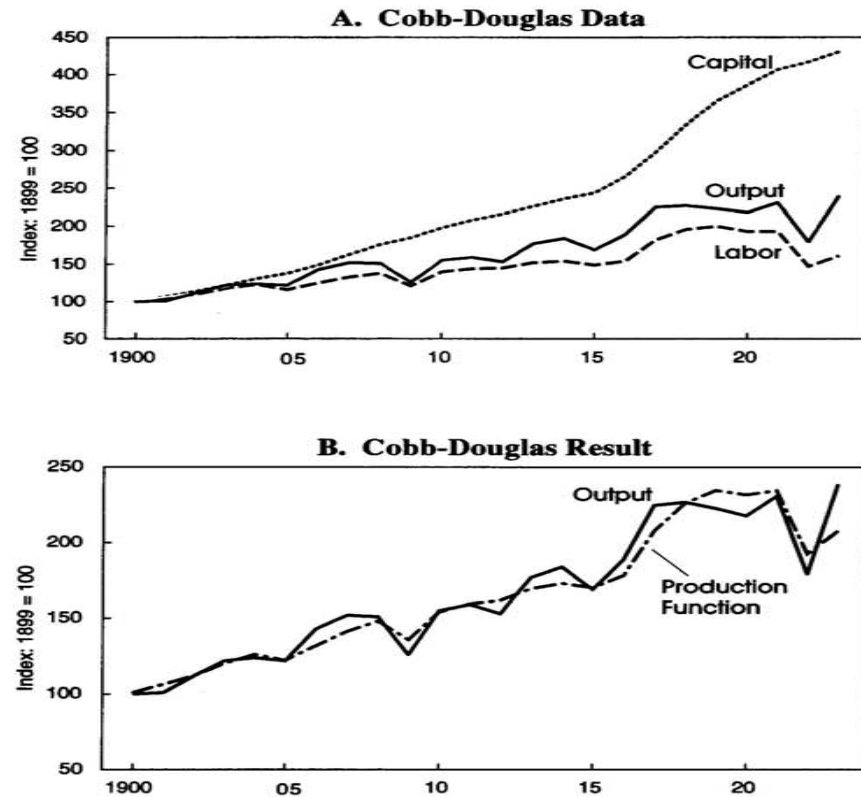
Real business cycles

1. Solow and macroeconomic accounting
2. The real business cycle view: Kydland and Prescott, Long and Plosser

1. The aggregate production function

- It is standard for us to think about economic activity at the level of the firm or industry using a production function
- But it once was not standard: it took work by economists like Paul Douglas to turn it from a theoretical device to one that was an a practical tool.
- Douglas noticed that shares paid to factors (like w_n/y) were relatively constant as levels of inputs and outputs changed.
- Working with a UC mathematician, he determined a production function that had these properties and explored its fit

Figure 1



Production Function

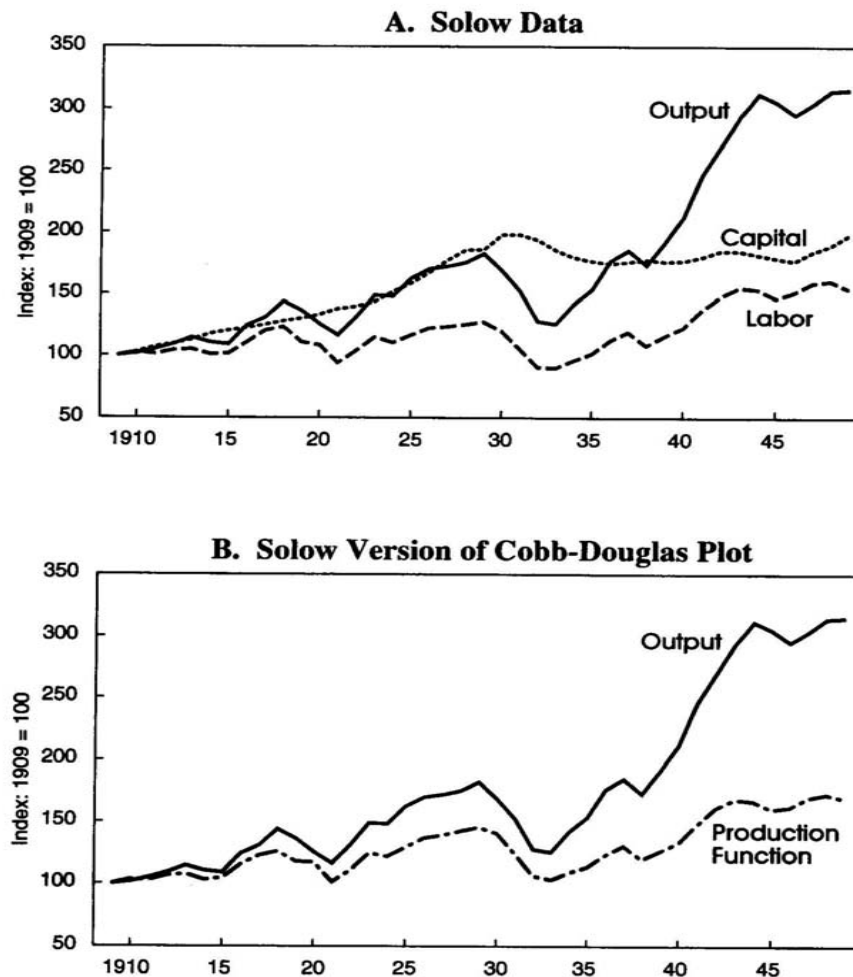
- CD production function

$$\log(Y_t) = \log(A) + \alpha \log(K_t) + (1 - \alpha) \log(N_t) + e_t$$

Output = [Production Function] + error

- “Production Function” explains most of output

Figure 2



Solow

- Similar strategy, but very different findings
- [Some elements – such as using unemployment rate to correct for utilization are very important for year to year umbers, but not for longer-term evolution]
- Figure is based on equation below
$$\log(A_t) = \log(Y_t) - \alpha \log(K_t) - (1 - \alpha) \log(N_t)$$

Residual (Productivity) = Output - [Total Factor Input]

SGZ Macro 2010, Lecture 7.

Real Business Cycles

2. The Real Business Cycle hypothesis

F. Kydland and E.C. Prescott, “Time to Build and Aggregate Fluctuations,” *Econometrica*, 1982.

J. Long and C.I. Plosser, “Real Business Cycles,” *Journal of Political Economy*, 1983.

Background

Although real factors were suggested to be an important of sources by various authors (notably Hayek and Schumpeter in the 1920s and 1930s), the Keynesian revolution in macroeconomics led to a situation in which (a) changes in aggregate demand were viewed as the major source of business cycle impulses; and (b) various private market failures –involuntary unemployment– and various market frictions—including sticky prices and wages—were viewed as essential to understanding the positive nature and the normative consequences of economic fluctuations

Background

- Circa 1975, it was standard for macroeconomists to view “equilibrium activity” as a largely constant or slowly evolving value (a trend, perhaps) and to view cyclical variations as “disequilibrium phenomena”
- Circa 1975, the main debates among macroeconomists were mainly about whether monetary or fiscal impulses (or rules) had larger effects on economic activity and which should be used to stabilize fluctuations and to eliminate them if possible.

Background:

The New Stimulae circa 1975

- Education of a generation of economists on the dynamic models of Solow and Cass that we just studied, focusing attention on economic dynamics in theory and the role of productivity in growth.
- Rise of quantitative (if static) models of general equilibrium in public finance and other areas.
- Lucas critique which suggested that existing policy models were inadequate and a new generation of models were necessary.

Basic RBC papers

- The initial set of dynamic general equilibrium models of the 1980s focused on productivity shocks as a source of economic fluctuations
- The Kydland-Prescott (henceforth KP) and Long-Plosser (henceforth LP) each described how a well-functioning market economy would respond to productivity shocks, concluding that it would display important characteristics of business cycles in the real world

Basic RBC papers

- Kydland and Prescott studied a variant of the one-sector neoclassical growth model, augmented to include variable labor supply, investment according to a “time to build” investment process and various other elements aimed at producing a modern business cycle model. Their model was driven by an aggregate productivity shock.
- Long and Plosser constructed a multisector neoclassical growth model, in which changes in sectoral productivity produced responses in other sectors and ultimately at the aggregate level in the economy.

Impact

- These papers launched a revolution in macroeconomics, which has resulted in methodology that is now used widely to study a wide range of problems (including in “New Keynesian” models that we will discuss later in the course).
- These papers stimulated much research into the influence of productivity on macroeconomic activity including work on its measurement, its origin and its consequences.

2A. Kydland-Prescott

- KP specified a version of the neoclassical growth model with several complications of preferences and the production function:
 - An investment technology designed to capture a range of phenomena described through the early section of the article, from which the paper takes its name.
 - Inventories (since inventories vary substantially over business cycles)
 - Variable labor (since labor varies over business cycles)
 - A utility function allowing for nonseparabilities in labor, designed to allow for different short-run and long-run labor supply elasticities.

Time to Build

- Let s_{jt} be the number of investment projects with j stages to go until they are completed and increment the capital stock.
- Suppose that there are J stages, so that

$$k_{t+1} - k_t = s_{1t} - \delta k_t$$

$$s_{j-1,t+1} = s_{j,t} \quad \text{for } j = 2, 3, 4, J$$

In this expression, s_{Jt} is the number of “starts”, i.e., the number of new projects initiated in period t . Other projects simply move toward completion (they cannot be cancelled)

Investment

- Investment expenditures are distributed through time on any given project, with weights suggested by data on actual investment expenditure

$$i_t = \sum_{j=1}^J \varphi_j s_{jt} + (m_{t+1} - m_t)$$

where m_t is the predetermined stock of inventories at t so $m_{t+1} - m_t$ is inventory investment and the other part is fixed investment (inventory notation differs from KP)

Production function

- Inventories are factor of production for KP, so that

$$y_t = f(a_t, n_t, k_t, m_t)$$

where we use our standard notation with “a” being productivity, “n” being labor input

- Further, the constraint on goods is then

$$c_t + i_t \leq y_t$$

Utility Function

- KP posit a utility function of the form

$$u(c_t, l_t)$$

$$l_t = 1 - \gamma n_t - (1 - \gamma) \eta \sum_{j=1}^{\infty} \eta^j n_{t-j}$$

$$= 1 - \gamma n_t - (1 - \gamma) \eta z_t$$

$$z_{t+1} = (1 - \eta) z_t + n_t$$

Utility Function (cont'd)

- This utility function imbeds the idea that supply of work is more costly if one has recently worked a lot.
- It therefore allows for a higher response of work to temporary variations in opportunities (temporary changes in wages) than permanent ones
- It is set up to allow for dynamic programming: z_t is a state variable at date t

Other features not emphasized in lecture

- Two stage decision structure, with some variables chosen before others
- Incomplete information about permanent or temporary nature of aggregate productivity situation (originally included to allow a potential source of monetary nonneutrality).

Computing Equilibrium

- KP set up a dynamic programming problem, essentially

$$v(k_t, [s_{jt}]_{j=1}^{J-1}, m_t, z_t, \varsigma_t) = \max \{ u(c_t, l_t) \\ + \beta E_t v(k_{t+1}, [s_{j,t+1}]_{j=1}^{J-1}, m_{t+1}, z_{t+1}, \varsigma_{t+1}) \}$$

$$st \quad c_t + i_t = f(a(\varsigma_t), k_t, n_t, m_t)$$

$$i_t = \sum_{j=1}^J \varphi_j s_{jt} + (m_{t+1} - m_t)$$

$$k_{t+1} - k_t = s_{1t} - \delta k_t$$

$$s_{j-1,t+1} = s_{j,t} \quad for \quad j = 2, 3, 4, J$$

$$l_t = 1 - \gamma n_t - (1 - \gamma) \eta z_t$$

$$z_{t+1} = (1 - \eta) z_t + n_t$$

Computing equilibria (cont'd)

- KP designed their economy so that it was linear in the constraints, once they substituted out for consumption from the first constraint.
- They knew that optimization models with quadratic objectives and linear constraints (like those which we studied last term) could be rapidly and easily computed, resulting in linear decision rules, so that they sought a quadratic approximation to a reduced form objective (or indirect utility function).

Computing Equilibrium(cont'd)

“To determine the equilibrium process for this model, we exploit the well-known result that, in the absence of externalities, competitive equilibria are Pareto optima. With homogenous households, the relevant Pareto optimum is the one which maximizes the utility of the representative consumer subject to the technology constraints and the information structure” KP, pg. 1354.

Computing equilibria (cont'd)

“Approximations are necessary before equilibrium decision rules are computed. Our procedure is to determine the steady state for the model with no shocks to technology. Next, quadratic approximations (of an objective) are made in the neighborhood of the steady state. Equilibrium decision rules of the resulting approximate economy are then computed. These decision rules are linear, so ... the approximated economy is generated by a system of stochastic difference equations for which covariances are easily determined.” KP 1355.

Testing the theory

- KP propose a specific test of their theory, explained on pages 1359-1360.
- They begin by choosing most of the parameters of their model to match micro observations and steady state facts, a method of parameter selection which they call “calibration”.
- They then evaluate how well their model does at reproducing certain facts about business cycles (which they refer to as “the deviations”).

Example

- “The model is consistent with the high (percentage) variability in investment and the low variability in consumption and their high correlation with output.” KP, page 1364.

KP's conclusion

- “The preference-technology environment was the simplest one that explained the quantitative comovements and serial correlation properties of output. The fit of the model was surprisingly good in light of the model's simplicity.”
- “A crucial component of the model that contributed to persistence of output movements was the time-to-build requirement.”

KP's conclusions (cont'd)

- KP suggest that models of this class could be used to study design of policy rules, in a manner consistent with the Lucas critique, but that these analyses will require further developments of methods, as models with general policy rules are typically not models in which competitive equilibria are pareto optima.

2B. Long-Plosser

- Considered how two key economic ideas could lead to business cycle phenomena.
 1. The idea that consumers generally desire variety of goods and so will spread increments to wealth across many different commodities at many different dates.
 2. That producers generally require many different produced inputs to efficiently produce final goods.

LP's focus on particular aspects of business cycles

- Business cycles involve strong positive serial correlation of aggregates: *persistence*
- Over the course of business cycles expansions and contractions, many different sectors rise or fall together: *comovement*

LP's approach

- Describe general model in which ideas can be explored.
- Analyze in detail a particular analytical solution, obtained by making specific restrictions on preferences and technology (essentially loglinear preferences and Cobb-Douglas technology).
- We will discuss this model using notation which is compatible with the preceding presentation of KP and the later presentation of other models.

LP model elements

- Preferences
 - Depend on a J element vector of consumer goods, with c_t being the date t vector and c_{jt} the quantity of the jth good (jth element of vector).
 - Restricted so demand for each good increases with wealth (positive wealth elasticities)
 - Depend on the quantity of leisure (do not introduce dynamic elements as in KP)

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

LP model elements

- Sectoral production $y_{i,t+1}$ depends on
 - labor allocated to the sector n_{it}
 - the amount of intermediate inputs from other sectors m_{ijt} , with i being the sector of use and j being the sector of origin
 - a sector-specific shock $a_{i,t+1}$

$$y_{j,t+1} = f_j(a_{j,t+1}, n_{jt}, [m_{ijt}]_{j=1}^J)$$

LP elements

(indicated by notation on prior page)

- f_j indicates that production function in sectors need not be same
- Dating of outputs ($t+1$) and inputs (t) indicates that length of period is the amount of time that production requires, which is assumed constant across sectors

LP elements

- Write production function compactly as

$$y_{t+1} = F(a_{t+1}, n_t, m_t)$$

where

- y is the output vector (J by 1)
- a is the productivity vector (J by 1)
- n is the labor input vector (J by 1)
- m is the intermediate input matrix (J by J)

LP model elements

- There are constraints

$$c_{jt} + \sum_{i=1}^J m_{ijt} = y_{jt}$$

$$1 = l_t + \sum_{i=1}^J n_{it}$$

Equilibrium computation: Solving Robinson Crusoe's problem

$$v(y_t, \varsigma_t) = \max \{u(c_t, l_t) + \beta E_t v(y_{t+1}, \varsigma_{t+1})\}$$

s.t.

$$c_t + (\phi m_t)' = y_t$$

$$y_{t+1} = F(a_{t+1}, n_t, m_t)$$

$$l_t + \phi n_t = 1$$

with ϕ being a J element row vector of 1s

so that this sums elements of labor vector and

$(\phi m_t)'$ is the column vector of total input use.

Example

- Preferences

$$u(c_t, l_t) = \theta_0 \log(l_t) + \sum_{j=1}^J \theta_j \log(c_{jt})$$

- Production

$$\log(y_{i,t+1}) = \log(a_{i,t+1}) + \alpha_i^n \log(n_{it}) + \sum_{j=1}^J \alpha_{ij}^x \log(m_{ijt})$$

LP solution

- Remarkable (positively) in that they are able to actually generate an exact loglinear difference system as the solution for the “policy functions”. Generalizes solution we explored in earlier “computational dp lecture” to many goods
- Remarkable (negatively) that labor in the economy as a whole and to each sector is constant in this solution, reflecting a complicated general equilibrium pattern of offsetting wealth and substitution effects.

LP solution

- Takes the form

$$\log(y_{t+1}) = \kappa + \alpha^x \log(y_t) + \log(a_{t+1})$$

$$\text{where } \alpha^x = [\alpha_{ij}^x]$$

- Depends centrally on “input output” information from matrix of cost-shares α^x
- Is invariant to nature of driving process (analogous to Sandmo’s offsetting effects of return level and uncertainty)

Calibration

- Choice of sectors (degree of disaggregation)
- Input-output information

Analysis

- Response of system to transitory shocks: how does output shock in sector i respond to events in sector j through time? (depends on input-output structure)
- How much persistence – measured by serial correlation – do mechanisms produce? (some)
- How much comovement – measured by correlation—do mechanisms produce? (some)
- How does system respond to permanent productivity shocks, e.g., in a simulation? (looks more like macro data)

LP conclusions

- Real shocks and real mechanisms may be important for business cycle (enduring)
- Sectoral structure and sectoral shocks are important (largely unexplored, but tantalizing research problem)
- “Technical progress is possible in obtaining explicit solutions to models with alternative specifications of preferences and technology”: 20 years later, we don’t have many additional ones
- “Empirical research”: active with highly aggregated models as in KP but relatively little with models of sectoral interactions and/or shocks

Questions raised by initial RBC studies

- How should we best measure productivity change and to what extent is it exogenous to business cycle?
- How important are various mechanisms stressed in individual studies? (e.g., time-to-build)
- How should we select parameters for and evaluate the performance (fit) of simple dynamic models?
- How should we solve models with suboptimal equilibria?