SZG Macro 2010 Lecture 4: Empirical dynamic models

Modern perspectives

- Consumption is forward-looking because people prefer smooth consumption profiles and can manage their saving to this end.
- Investment is forward-looking because firm's demand for new capital goods depends on a present-value of profits from such investments.
- Labor supply is forward-looking, as individuals decide how to substitute leisure intertemporally, toward those periods with high wages.

Three models

- The modern variant of the "life-cycle/ permanent-income" model of consumption as developed by Hall.
- The modern variant of investment with "capital adjustment costs" as developed by Lucas, Prescott and Hayashi.
- The life-cyle labor supply analysis of Heckman and MaCurdy

General empirical problem

- Theory says that behavior depends on present-discounted value (pdv)
- Evaluating the theory requires modeling this pdv, which can be complicated.
- Modern modern approaches circumvent this pdv modeling problem by clever, but different eliminations of it.

Approaches

- Consumption (Hall): work off efficiency condition ("euler equation") and use general property of expectation forecasting errors, which is that errors should be unrelated to available information
- Investment (LHP): work off efficiency condition and relate unobserved pdv to observable variable, the ratio of firm value to replacement cost of capital (Tobin's Q)
- Labor (HM): use lambda-constant approach to estimating labor supply elasticity

A. Consumption

• "Permanent income" model relates consumption to "annuity value of future income"

$$c_{t} = k * y_{pt}$$

$$[\sum_{j=0}^{\infty} b^{j}]y_{pt} = [\sum_{j=0}^{\infty} b^{j}E_{t}y_{t+j}]$$

Variants in prior work

 Friedman: treat permanent income as average of past income (only rational in some settings)

$$y_{pt} = \theta y_{p,t-1} + (1 - \theta) y_t = \sum_{j=0}^{\infty} \theta^j y_{t-j}$$

• Sargent: treat permanent income as outcome of forecasting with specific model

$$y_t = \pi s_t \qquad s_t = M s_{t-1} + G e_t$$

$$y_{t} = \sum_{j=0}^{\infty} b^{j} E_{t} y_{t+j} = \pi \sum_{j=0}^{\infty} b^{j} E_{t} s_{t+j} = \pi [I - bM]^{-1} s_{t}$$

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Hall's work on consumption

• Consider dynamic model of optimal consumption over time

$$\max E_t [\sum_{j=0}^{\infty} \beta^j u(c_{t+j})]$$

s.t.
$$p(\varsigma_t)a_{t+1} = a_t + y(\varsigma_t) - c_t$$

where $p_t = 1/R_t$

 Note that income and interest rate depend on a set of state variables, but this dependence is not made explicit.

Euler equation

 Utility consequences of a little more wealth and consumption tomorrow, at expense of consumption today, must be zero at an optimum

$$-u_{c}(c_{t})p(\varsigma_{t}) + \beta E_{t}[u_{c}(c_{t+1})] = 0$$

or

$$-u_{c}(c_{t}) + \beta R(\varsigma_{t}) E_{t}[u_{c}(c_{t+1})] = 0$$

Hall's observation

- Realized marginal utility is expected marginal utility plus an error term
- Under RE, this error term should be uncorrelated with available information (prior work on "efficient markets" in finance had exploited this observation as well)
- With utility that was quadratic (or approximately so) then marginal utility would be linear (or approximately so)
- With interest rate just offestting time preference, current and expected future consumption should then be equal under RE PIH model

Steps

$$u_c(c_t) = \beta R(\zeta_t) E_t[u_c(c_{t+1})]$$
$$R\beta = 1 \Longrightarrow \quad u_c(c_t) = E_t[u_c(c_{t+1})]$$
$$\Longrightarrow \quad u_c(c_{t+1}) = u_c(c_t) + \xi_{t+1}$$

$$u_{c} = \phi - \gamma c_{t} \Longrightarrow c_{t+1} = c_{t} - \frac{1}{\gamma} \xi_{t+1}$$

Test

- Linear regression, testing various x's
 - Additional lags of consumption
 - Additional variables
 - Past income
 - Past wealth

$$c_t = c_{t-1} + \theta x_{t-1} + e_t$$

 $PIH \Rightarrow \theta = 0$

Initial findings

- Changes in consumption were suprisingly, largely unpredictable: Hall found consumption was "random walk"
- Sometimes written in log form: growth rate of consumption unpredictable
- Tests on micro data by Hall and Mishkin
 - Surprisingly hard to predict individual consumption changes also

Extensions

• Time varying interest rate (under loglinearity)

$$\log(c_{t+1}) - \log(c_t) = \frac{1}{\sigma} [\log(R_t / \beta)] + e_{t+1}$$

- Result is small intertemporal substitution elasticity $1/\sigma$ is small.
- Estimated using an instrumental variables approach (more details in semester 2)
- Nonlinear estimation (Hansen-Singleton) applied to Euler equations for multiple assets

Challenges

• Campbell-Mankiw studied effect of a measure of expected income growth

$$\log(c_{t+1}) - \log(c_t) = \frac{1}{\sigma} [\log(R_t / \beta)] + \kappa [E_t \log(y_{t+1} / y_t)] + e_{t+1}$$

 Estimate "big" (κ about .4) and "significant" coefficient. Much dispute about interpretation, but rejection of basic model

B. Investment

- Consider a firm seeking to maximize its present value, subject to an accumulation equation that penalizes large movements in capital (h is positive, increasing, and strictly concave).
- Firm faces time-varying productivity of capital and time-varying investment good price (p). Constant discount factor for simplicity only

$$\max\sum_{j=0}^{\infty} b^{j} E_{t}(a_{t}k_{t}-p_{t}i_{t})$$

s.t.

$$k_{t+1} - k_t = h(\frac{i_t}{k_t})k_t$$

Key features

- Homogeneity of firm's problem: a kind of dynamic constant-returns to scale.
 - Motivated by applied work showing growth rates of firms do not depend importantly on level.
- Implications for value of firm: v_t=w_tk_t with w_t not depending on capital stock k_t.

Dynamic program

$$v(k_t, \varsigma_t) = \max_{k_{t+1}, i_t} [a(\varsigma_t)k_t - p(\varsigma_t)i_t] + bE_t v(k_{t+1}, \varsigma_{t+1})$$

s.t.
$$k_{t+1} - k_t = h(\frac{\dot{k}_t}{k_t})k_t$$

Lagrangian

$$L = [a(\varsigma_t)k_t - p(\varsigma_t)i_t] + bE_t v(k_{t+1}, \varsigma_{t+1})$$

$$+\lambda_t \left[h(\frac{i_t}{k_t})k_t + k_t - k_{t+1}\right]$$

FOCs and ET

$$i_t:-p(\varsigma_t)+\lambda_t h_z(z_t)=0$$
 with $z_t=\frac{i_t}{k_t}$

$$k_{t+1} : -\lambda_t + bE_t v_k(k_{t+1}, \varsigma_{t+1})$$

$$ET: v_k(k_t, \varsigma_t) = a(\varsigma_t) + \lambda_t [1 + h(z_t) - z_t h_z(z_t)]$$

So z=(i/k)

- Can be "explained" entirely by p/λ
 - Multiplier is "marginal value" of another unity of capital tomorrow, p is current cost of terms of investment
- Depends on form of h(z), particularly its extent of decreasing returns to z.
- Problem: multiplier is unobservable

Market value

• Suppose firm pays out all profits as dividends. Then, its "ex dividend" market value is

$$v_t - [a_t k_t - p_t i_t] = bE_t v_{t+1}$$
 Ex dividend value
= $bk_{t+1}E_t v_{k,t+1}$ Homogeneity
= $k_{t+1}\lambda_t$ Efficient Investment

Why was it ok to assume that value was proportional to k? If future value is proptional to k' then current value will be proportional to k.

$$\frac{v(k_t, \zeta_t)}{k_t} = \max_{\frac{k_{t+1}}{k_t}, \frac{i_t}{k_t}} [a(\zeta_t) - p(\zeta_t) \frac{i_t}{k_t}] + bE_t \frac{v(k_{t+1}, \zeta_{t+1})}{k_{t+1}} \frac{k_{t+1}}{k_t}$$

s.t.
$$\frac{k_{t+1}}{k_t} = [h(\frac{i_t}{k_t}) + 1]$$

Tobin's Q

 James Tobin hypothesized that investment would be related to

 $Q = \frac{\text{market value of firm}}{\text{replacement cost of capital}}$

This model (and others like it) delivers Tobin's view since

$$Q = \frac{v_{t} - \pi_{t}}{p_{t}k_{t+1}} = \frac{\lambda_{t}k_{t+1}}{p_{t}k_{t+1}} = \frac{\lambda_{t}}{p_{t}}$$

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Theory and tests

- Theory predicts i/k depends on Q and only Q (perfect fit)
- Tests find some association between i/k and Q, but far from perfect fit
 - Mismeasurement of capital stock?
 - Firm value depends on other factors (eg patents)
 - Model wrong on other dimensions (homogeneity, instant adjustment of i, ...)

C. Labor supply

- How will an individual respond to a purely temporary change in the wage rate?
- Such a change should have a large intertemporal substitution effect (as it applies to a single period) and a small wealth effect (as it applies to a single period)
- Example: college students working summer jobs in US

Problems

- How isolate a purely temporary change?
- How make sure that there is no wealth effect of observed wage change?
- Object of interest is not to be found in crosssection data
- Object may be found in experiments (Denver income maintenance experiments, Fehr-Goette bicycle delivery persons)
- Some type of panel data seems essential without experiments.

Life cycle labor supply

$$\max : \sum_{t=0}^{T} \beta^{t} u(c_{t}, l_{t})$$

s.t.
$$\sum_{t=0}^{T} P_{t}[w_{t}n_{t} + \pi_{t} - c_{t}] \ge 0$$
$$n_{t} + l_{t} = 1$$

specific utility:
$$u(c,l) = b(c) - \chi \frac{1}{1+\gamma} (1-l)^{1+\gamma}$$

Efficient labor

General:
$$\beta^{t} u_{l}(c_{t}, l_{t}) = \Lambda P_{t} w_{t}$$

Specific: $\beta^{t} \chi n_{t}^{\gamma} = \Lambda P_{t} w_{t}$

Implied labor supply:

$$\log(\mathbf{n}_t) = \frac{1}{\gamma} [\log(w_t) + \log(\Lambda) + \log(P_t / \beta^t) - \log(\chi)]$$

Problems with estimating (1/ γ): the desired labor supply elasticity

- Wage is correlated with Λ (wealth effect can contaminate substitution effect)
- Intertemporal substitution in response to discount factors as well as wages
- Possible individual differences in preferences (χ parameter)
- Measurement error in wages (bias down, in general)

Empirical approach

- Difference over time eliminates Λ
- Differences from group means eliminates interest rate effects
- Instrumental variables to offset measurement error.
- All in all: predictable growth in wages over life-cycle

Finding: low elasticity

Possible issues

 Incorrect momentary utility function (interaction of consumption and labor may be important)

• Human capital may be an omitted factor, with its inclusion leading to a different measure of "wages relative to normal".

Recent articles related to these two problems

- Reverse problem: Bullard and Feigenbaum, "A Leisurely Reading of the Life Cycle Consumption Data" in JME explores effect of omitted leisure on consumption paths
- Human capital: Iwai and Kean (IER 2004) show that larger labor supply elasticities arise when human capital is accounted for as a determinant of the life-cycle labor profile.