Asset pricing in endowment and production economies: general ideas and loglinear benchmarks

Overview

A. Review and consolidation

B. Asset pricing in endowment economies

   1. Mehra-Prescott

   2. Campbell loglinear benchmark

C. Asset pricing in production economies

   1. General theory

   2. Jermann loglinear benchmark

A. Review and consolidation

- Last lecture's complete markets model implies

$$p(s_t) = \sum_{s_{t+1}} Q_1(s_{t+1} \mid s_t)[p(s_{t+1}) + d(s_{t+1})] = \sum_{j=0}^{\infty} \sum_{s_{t+j}} Q_j(s_{t+j} \mid s_t) d(s_{t+j})$$

$$= \sum_{s_{t+1}} \pi(s_{t+1} \mid s_t) \beta m(s_{t+1} \mid s_t)[p(s_{t+1}) + d(s_{t+1})]$$

$$= \sum_{j=0}^{\infty} \sum_{s_{t+j}} \pi(s_{t+j} \mid s_t) \beta^j m(s_{t+j}, s_t) d(s_{t+j})$$

$$= E_t\{\beta m(s_{t+1} \mid s_t)[p(s_{t+1}) + d(s_{t+1})]\} = E_t\{\sum_{j=0}^{\infty} \sum_{s_{t+j}} \beta^j m(s_{t+j}, s_t) d(s_{t+j})\}$$

where the latter expression renormalizes contingent claims prices

What is “m”?

- Normalized contingent claims price
- Stochastic discount factor
- Asset pricing kernel
- What could it be in complete markets?
  - Social planner’s endowment multliplier
  - Any asset holders marginal utility

$$m(s_{t+j}, s_t) = \frac{\bar{\theta}(s_{t+j})}{\bar{\theta}(s_t)} = \frac{u'(c^i_{t+j})}{u'(c^i_t)}$$

Why combine macro and finance?

- Finance suggests diversifiable risk should not be priced
- Macroeconomics explains common factors across households, firms and industries
- Macroeconomics is thus important for understanding aggregate risks
- Financial markets are arguably important for macroeconomic developments

Holy Grail: Find “m”

- Practical (Markets): identify profits opportunities in asset markets (deviations from predicted asset prices)
- Practical (Markets): Create new assets
- Practical (Policy): Extract information about agent beliefs from asset prices
- Academic: Learn about asset price determination
- Academic: Additional discipline on modern macroeconomic models

Mechanics

- Evaluate pricing formulae and create practical model. That is, solve

$$p(s_t) = E_t\{\beta m(s_{t+1} \mid s_t)[p(s_{t+1}) + d(s_{t+1})]\}$$

$$= E_t\{\sum_{j=0}^{\infty} \sum_{s_{t+j}} \beta^j m(s_{t+j}, s_t) d(s_{t+j})\}$$

Digression on risk neutrality

- Notion of risk neutral probability measure as in LS (and in finance).
- Alternative risk neutral equation (units of prices and cash flows denominated in marginal utility)

$$p(s_t) = E_t\{\beta m(s_{t+1} \mid s_t)[p(s_{t+1}) + d(s_{t+1})]\} = E_t\{\sum_{j=0}^{\infty} \sum_{s_{t+j}} \beta^j m(s_{t+j}, s_t) d(s_{t+j})\}$$

$$p(s_t) = \overline{E}_t\{\beta[p(s_{t+1}) + d(s_{t+1})]\} = \overline{E}_t\{\sum_{j=0}^{\infty} \sum_{s_{t+j}} \beta^j d(s_{t+j})\}$$

where $\overline{\pi}(s_{t+1} \mid s_t) = \pi(s_{t+1} \mid s_t) m(s_{t+1} \mid s_t)$

$$\theta(s_t) p(s_t) = E_t\{\beta\theta(s_{t+1t})[p(s_{t+1}) + d(s_{t+1})]\} = E_t\{\sum_{j=0}^{\infty} \sum_{s_{t+j}} \beta^j \theta(s_{t+jt}) d(s_{t+j})\}$$

B. Endowment Economies

1. Lucas, Mehra and Prescott
2. Discount bonds, coupon bonds, and strips
3. Mechanics of discount bond yields
4. The stochastic discount factor and bond pricing
5. A simple model of the term structure
6. Stripping other assets

1. Basic Endowment economy

- Lucas tree is complex asset. Ownership of tree today is claim to fruit tomorrow (d’) and future ownership (p’).
- Mehra-Prescott studied returns on this sort of tree, interpreting it as the stock market and contrasting the returns on stocks to those on bonds.
- Assumption of power utility (constant relative risk aversion)
- Markov chain on consumption growth rate.

Endowment

- Markov chain on growth rate

$$\text{states:} \quad \gamma_1 < \gamma_2 < \ldots \gamma_J$$

$$prob(g_{t+1} = \gamma_j \mid g_t = \gamma_h) = \pi_{hj}$$

- Asset pricing condition

$$\text{Stock:} \;\; p(g) = \beta E\{(\frac{c'}{c})^{-\sigma}[p' + d']\}$$

$$\text{Bond:} \;\; b(g) = \beta E\{(\frac{c'}{c})^{-\sigma}\}$$

Asset returns

- Conceptually (conjectured solution)

$$\text{Stock: } p_t = f(g_t; g_{t-1})c_t; \quad \mathrm{d}_t = \mathrm{c}_t$$

$$1 = \beta E\{(\frac{c'}{c})^{-\sigma}[\frac{p'+d'}{p}]\}$$

$$= \beta E\{(g')^{-\sigma}[g' f(g'; g) + g']\} \mid g$$

$$R_s(g'; g) = g' f(g'; g) + g'$$

$$\text{Bond: } 1 = \beta E\{(g')^{-\sigma}(R_b(g))\} \mid g$$

Computationally

- For each state h (Markov-Bellman sense)

$$1 = \beta \sum_{j=1}^{J} \pi_{jh} (g_j)^{-\sigma} [R_s(\gamma_j ; \gamma_h)]$$

$$1 = \beta \sum_{j=1}^{J} \pi_{jh} (g_j)^{-\sigma} [R_b(\gamma_h)]$$

MP finding

- For consumption process that matched aspects of US data (mean, standard deviation, serial correlation), there was a critical asset pricing anomaly: the spread in expected return between stocks and bonds was small for “reasonable” levels of risk aversion (less than 10) relative to observed data: .5%, say rather than 6% per annum.

B2. Stripping coupon bonds

- Pure discount bond of maturity n: face value payment of "f" dollars n periods from now
- Coupon bond
  - Regular <u>coupon</u> payments of "c" dollars (say, every year, for simplicity) and one terminal payment <u>face value</u> payment of "f" dollars
- Strip market
  - Discount bonds created by selling coupons and face values separately
  - http://www.riskglossary.com/link/treasury_strips.htm

B3. Mechanics of discount bond yields

Discrete compounding: yield to maturity

p_{nt} : n period discount bond pric

i_{nt}: n period discount bond yield

$$p_{nt} = [(1 + i_{nt})]^{-n}$$

Continous compounding: yield to maturity

$$p_{nt} = \exp(-n * i_{nt})$$

B3. Stochastic discount factor and asset pricing

- (Gross) holding period yield on any asset

$$h_{t+1} = \frac{P_{t+1} + d_{t+1}}{P_t}$$

- Stochastic discount factor m

$$\begin{aligned} p_t &= E_t[m_{t+1}(p_{t+1} + d_{t+1})] \\ &= E_t[m_{t+1}d_{t+1}] + E_t[m_{t+2}m_{t+1}(p_{t+2} + d_{t+2})] \end{aligned}$$

Bond pricing with an SDF

- Discount bond notation: bonds of maturity n at t are bonds of maturity n-1 at t+1

$$
\begin{aligned}
p_{nt} &= E_t[m_{t+1} p_{n-1,t+1}] \\
&= E_t[m_{t+n} m_{t+n-1} ... m_{t+2} m_{t+1}] \\
&= E_t \exp([\sum_{j=1}^{n} \log(m_{t+j})])
\end{aligned}
$$

$$
i_{nt} = -\frac{1}{n} \log(p_{nt})
$$

- Expressions like these embolden Campbell and Jermann to study loglinear (lognormal) asset pricing

Key properties of lognormal random variable

- Suppose that y is normal with mean μ and variance q, then

$$E(\exp(y)) = \exp(\mu + \frac{1}{2}q)$$

State space system

$$\log(m_{t+n}) = \kappa + \pi s_{t+n}$$

$$s_t = Ms_{t-1} + Ge_t$$

$$s_{t+n} = M^n s_t + \{Ge_{t+n} + MGe_{t+n-1} + \ldots M^{n-1}Ge_{t+1}\}$$

$$\sum_{j=1}^{n}\log(m_{t+j})$$

$$= n\kappa + \pi[s_{t+n} + s_{t+n-1} + s_{t+n-2} + \ldots s_{t+1}]$$

$$= n\kappa + \pi[Ge_{t+n} + (I+M)s_{t+n-1} + s_{t+n-2} + \ldots s_{t+1}]$$

$$= n\kappa + \pi(\mathrm{M}+\ldots\mathrm{M}^{\mathrm{n}})s_t \quad \text{forecastable (known at t)}$$

$$+ \pi[Ge_{t+n} + (I+M)Ge_{t+n-1} + (I+M+M^2)Ge_{t+n-2}$$

$$(\mathrm{I}+\mathrm{M}+\ldots\mathrm{M}^{\mathrm{j-1}})Ge_{t+n-j} + \ldots(\mathrm{I}+\mathrm{M}+\ldots\mathrm{M}^{\mathrm{n-1}})Ge_{t+1}]$$

$$\mathrm{q_n} = q_{n-1} + \pi(\mathrm{I}+\mathrm{M}+\ldots\mathrm{M}^{\mathrm{n-1}})GE(ee')G'(\mathrm{I}+\mathrm{M}+\ldots\mathrm{M}^{\mathrm{n-1}})'\pi'$$

Why useful?

- Simple formula for yield.
- Time-varying rate, but no change in risk premium

$$p_{nt} = E_t \exp(\sum_{j=1}^{n} \log(m_{t+j,t+j-1}))$$

$$i_{nt} = -\frac{1}{n}\log(p_{nt}) = -\frac{1}{n}[n\kappa + \pi(\mathrm{M}+...+\mathrm{M}^{\mathrm{n}})s_t + \frac{1}{2}q_n]$$

- More generally, can price single period cash flow period instruments in simple manner

Term structure

$$p_{nt} = \exp(n\kappa + \pi(\mathrm{I{+}M{+}...M^n})s_t + \frac{1}{2}q_n)$$

$$i_{nt} = -\frac{1}{n}\log(p_{nt}) = -(\kappa + \frac{q_n}{2n}) - \frac{1}{n}\pi(\mathrm{I{+}M{+}...M^n})s_t$$

$$i_{1t} = -(\kappa + \frac{q_1}{2}) - \pi s_t$$

$$E_t i_{1,t+j} = -(\kappa + \frac{q_1}{2}) - \pi M^j s_t$$

$$i_{nt} = K_n + \frac{1}{n}E_t[\sum_{j=0}^{n-1} i_{1,t+j}]$$

5. Stripping other assets

- Strip off and sell just the dividends at each date

$$v_t[d_{t+n}] = E_t[m_{t+1} * v_{t+1}[d_{t+n}]]$$

C. Production economies

- Equations derived in Jermann (also IHW-part D)
- Interpretation:
  - "θ" is RC's marginal value of having a little more of the consumption good. It depends of k_t,c_{t-1},a_t
  - The social planner's Lagrange multiplier on consumption goods in lecture 9, where these were endowments.

$$c_t : \theta_t = u_1(c_t, c_{t-1}) + \beta E_t v_{c,t+1}$$

$$z_t : \theta_t = \lambda_t h'(z_t)$$

$$k_{t+1} : \lambda_t = \beta E_t v_{k,t+1}$$

$$p_t : c_t + i_t = a_t f(k_t, n)$$

$$\lambda_t : k_{t+1} = h(z_t)k_t + (1-\delta)k_t$$

$$ET : v_{c,t} = u_2(c_t, c_{t-1})$$

$$ET : v_{kt} = h(z_t) + (1-\delta)$$

$$Y = [k_t, c_{t-1}, z_t, \theta_t, \lambda_t, v_{ct}, v_{kt}]$$

What is "m"?

- According the analysis above,

$$\theta(s_t)\,p(s_t) = \beta E_t\{\theta(s_{t+1})[\,p(s_{t+1}) + d(s_{t+1})]\}$$

- For arbitrary p,d
- Note that the DP decision rules are

$$\theta(s_t)$$

$$s_{t+1} = \begin{bmatrix} k_{t+1} \\ c_t \\ a_{t+1} \end{bmatrix} = \begin{bmatrix} k_t[h(z(s_t)) + (1-\delta)] \\ c(s_t) \\ f(a_t, e_t) \end{bmatrix}$$

Jermann loglinear/lognormal approximation

- Approximate second block of equations with loglinear RE model with normal shocks
- Treat the first equation, the pricing equation, as exactly as possible
- What does “as exactly as possible” mean?

- Loglinearly approximate pricing kernel (θ) and payouts (d)
- Strip payouts t+j periods ahead.
- Use exact lognormal approximation to price strips
- Add the strips back up to get multiperiod payouts as a non-loglinear function of the state.

See you next October!

- We’ll study asset pricing in production economies as an example of use of higher order approximation methods.
- We’ll see how well Jermann’s approximation holds up.

6. Consumption and the SDF

$$m_{t+1} = \frac{u_c(c_{t+1})}{u_c(c_t)} = (\frac{c_{t+1}}{c_t})^{-\sigma}$$

$$\beta = \exp(-\upsilon)$$

$$\log(m_{t+1}) = -\sigma[\log(c_{t+1}) - \log(c_t)]$$

7. Valuing a stock

- A stock is just a portfolio of stripped dividends.
- Value the strips via loglinear asset pricing using a joint process for m and d.
- Add up the values

8. Equity premium puzzle once again

- Suppose that log consumption is a random walk with normal innovations
- Suppose we have constant elasticity marginal utility
- What is the return on a pure consumption discount bond?

Bond return

$$1 = R_b \beta E_t[m_{t+1}]$$

$$E_t[\beta(\frac{c_{t+1}}{c_t})^{-\sigma}] = \beta \exp(-\sigma\gamma + \frac{1}{2}\sigma^2 E(e^2))$$

Fisher's rule under uncertainty

$$\log(R_b) = \upsilon + \sigma\gamma - \frac{1}{2}\sigma^2 E(e^2)$$

Suppose stock price is proportional to consumption

$$R_{t+1} = \frac{(p_{t+1} + d_{t+1})}{p_t} = (\frac{c_{t+1}}{c_t})k$$

$$E(\log(R_{s,t+1})) = \log(k) + \gamma + \frac{1}{2}E(e^2)$$

Determining "k"

$$1 = E_t[m_{t+1} R_{s,t+1}]$$

$$1 = E_t[\beta(\frac{c_{t+1}}{c_t})^{1-\sigma} k] = \beta k \exp((1-\sigma)\gamma + \frac{1}{2}(1-\sigma)^2 E(e^2))$$

$$\log(k) = \upsilon - (1-\sigma)\gamma - \frac{1}{2}(1-\sigma)^2 E(e^2) =$$

$$= [\upsilon - \sigma\gamma - \frac{1}{2}\sigma^2 E(e^2)] - \gamma - \frac{1}{2} E(e^2) + \sigma E(e^2)$$

MP puzzle

$$ER_s = R_b + \sigma E(e^2) + \frac{1}{2}E(e^2)$$