SZG Macro 2010: Lecture 1

Introduction to dynamic macroeconomics: Analysis of saving and investment

Background

- Modern macroeconomics is concerned with understanding the factors that influence development (“economic growth”) and fluctuations (“business cycles”)
- Since each of these topics involves <u>time</u> in an essential way, we use economic theories and empirical strategies that highlight this dimension.

Background (cont'd)

- Macroeconomic modeling sometimes abstracts from heterogeneity across households, firms, countries – but incorporating such heterogeneity in a tractable manner is the subject of much work on the the frontiers of macroeconomics
- The most basic dynamic question that has concerned (macro) economists is the evolution of saving and investment, so that we start with this topic. Elements of it will be present in all four weeks of the class.

Outline of lecture

A. Basic facts about consumption and investment

B. Core concepts in intertemporal models and some basic approaches to about intertemporal consumption choice

A. Basic Facts

- Consumption, investment and output all trend up together (log scale in figure below is normalized so all have same mean, emphasizing trend and cycle variation)
- Consumption is about 2/3 of output and investment is about 1/6 of output
- Investment is proportionately more volatile than output, which is in turn more volatile than consumption

Figure 1: Trends and cycles
(data adjusted so all series have same mean as log output)

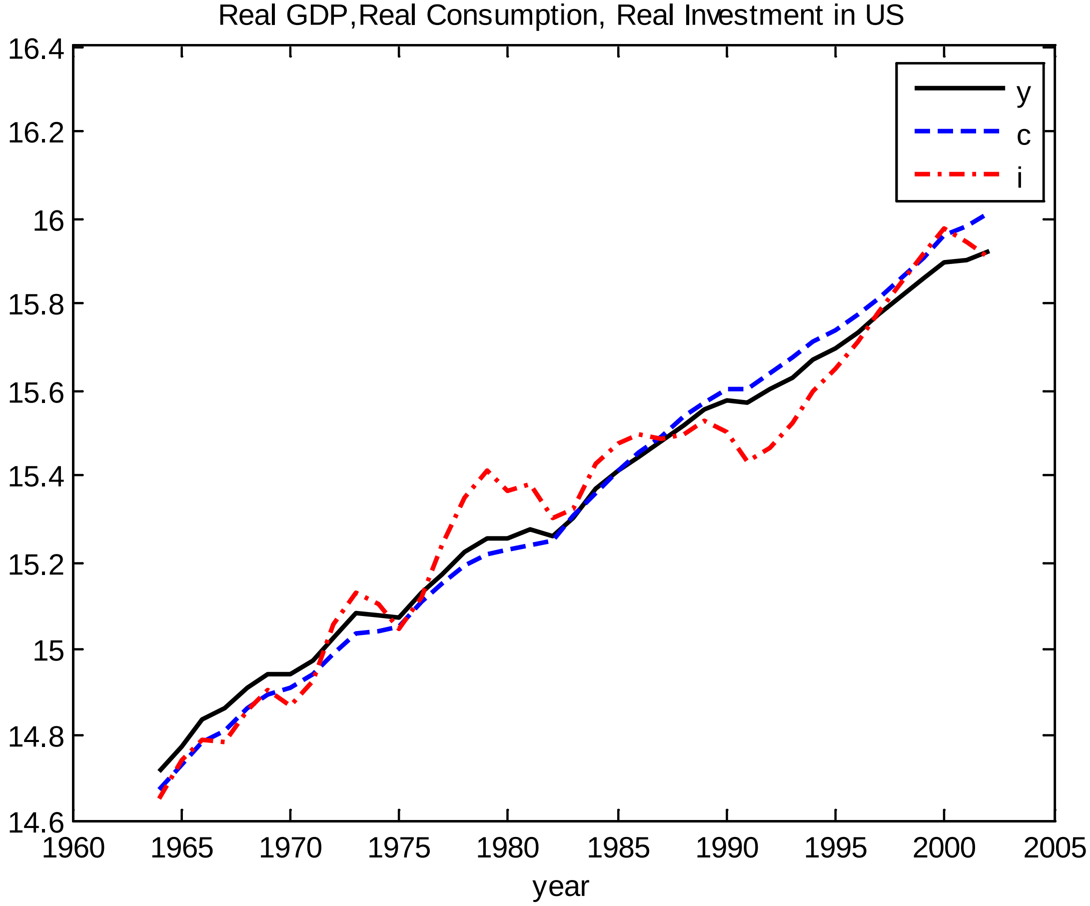
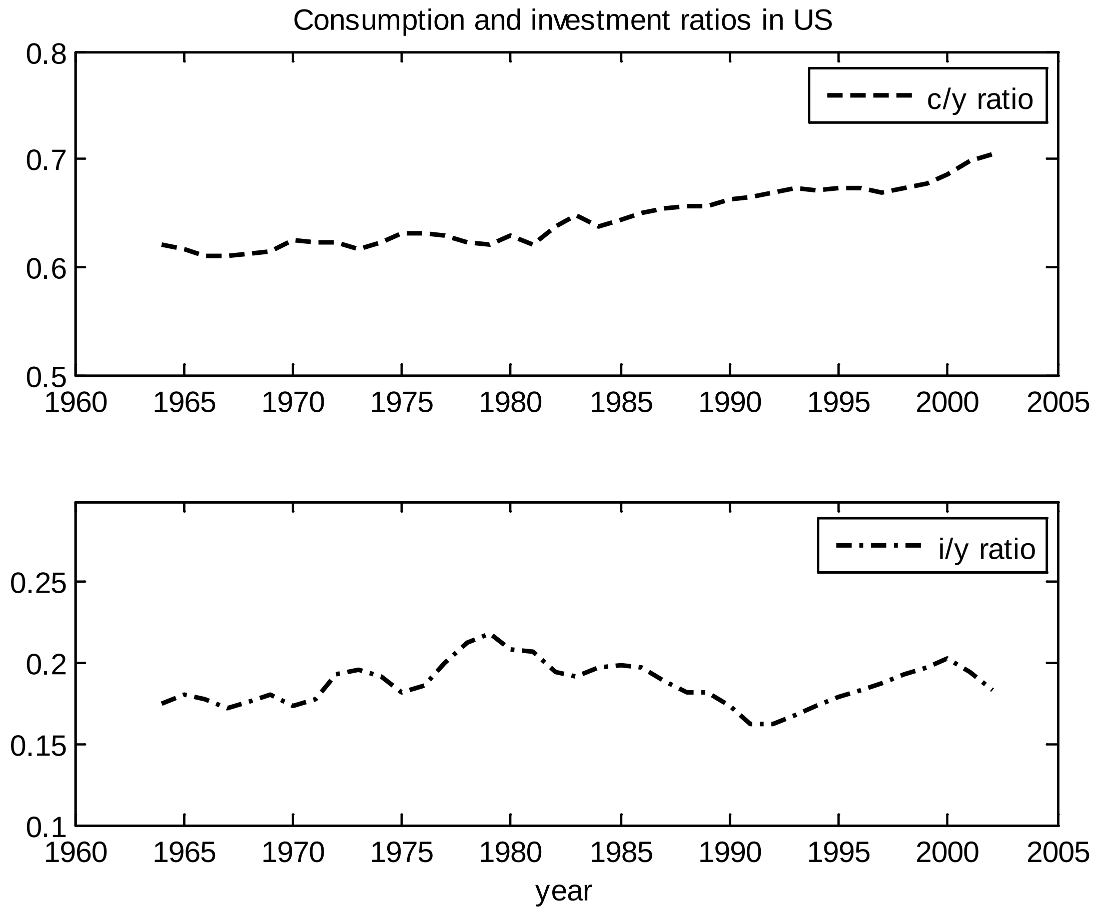


Figure 2: Great Ratios



Implications for modeling

- Evidence of common trends (rough stability of ratios) has motivated development of models with such a property (steady state growth, balanced growth)
- First modeling attack on capturing these saving and investment outcomes also employs idea that proportionate fluctuations are not related to level of development (this requires restrictions on preferences and possibilities)

B. Core ideas

- Economists have long been interested in dynamic economic problems
- Analysis of consumption and investment is hard because:
  - It involves the study of choices over time and under uncertainty
  - It involves general equilibrium (at a point in time and over time)
- Start by reviewing the elements emphasized and the abstractions made (“short cuts” taken)

B.1 Austrian capital theory: Eugen Bohm-Bawerk

- Interested in role of capital in economic systems
- Interested in how markets work to coordinate activity
- Finance minister under alternative parties, due to his practical competence
- However, to study capital formation, he advocated a very abstract approach: imagine that a single individual, Robinson Crusoe, must decide directly on how much to consume now and how much to use to produce goods later.

http://www.econlib.org/library/Enc/bios/BohmBawerk.html

Abstracting from markets

- Although he was intensely interested in the operation of markets, BB recognized that preferences and production possibilities were central determinants of saving, investment, and output even if there was a single individual
- BB suggested that one should learn about these topics by considering the problem of an isolated individual: Robinson Crusoe, the hero of the then current and now-classic story by Defoe about a man shipwrecked on an island.

BB’s preferences and production approach in a simpler atemporal setting

- Crusoe’s problem is to collect coconuts (c); work (n) is an alternative to leisure (*l*)
- Preferences: u(c,*l*)
- Time constraint: n+l=1
- Goods production: c=y=a f(n), with b(0)=0, f increasing and concave in n

Representing the abstraction in a figure: Robinson Crusoe

B2. Irving Fisher on consumption and interest

- Interested in real and nominal interactions
  - The "Fisher equation" linking nominal rate (R), real rate (r) and expected inflation $E\pi$: $R=r+E\pi$
  - Monetary effects on economic activity: "I discovered the Phillips curve"
  - Invented "distributed lags" for this purpose
  - Index number theorist
  - Investment: practice as well as theory
- However, our focus is on his real theory, as described in *Theory of Interest*.

http://www.econlib.org/library/Enc/bios/Fisher.html

Fisher’s *Theory of Interest*

- Consumption over time: used utility analysis (then applied to static demand), building in
  - Preference for current over future consumption
  - Desire for consumption smoothing
  - Willingness to substitute across time
- Equilibrium: theory of interest rate determination in sequential markets
  - First approximation: endowment economy (much progress)
  - Second approximation: production economy (little progress)

Two period model

- Preferences over time
  - specialize to $U(c_0,c_1)=u(c_0)+\beta u(c_1)$ with β reflecting preference for earlier consumption
  - [slope of indifference curve at $c_0=c_1$) is $1/\beta$]
  - "momentary" utility u is increasing and strictly concave so as to produce desire to smooth consumption and willingness to substitute across time
- Saving (z) at interest rate "r"
  - links current and future consumption: $c_0+z_0=y_0$ and $c_1=(1+r)z_0+y_1$ combine to yield $(1+r)c_0+c_1= (1+r)y_0+y_1$
  - The relative price of current consumption in terms of future consumption is (1+r)

Preference specification: time preference and slope of indifference curve

Theory of consumption over time:

(1) consumption smoothing is feasible with asset accumulation;

(2) consumption level adjusts to lifetime resources;

(3) consumption differs across time based on interest rate

(4) saving is residual (z=y-c)

Fisher's first approximation:

Theory of interest in endowment economy involves key idea that the interest rate adjusts to make c=y (or z=0)

Multiperiod extensions: The present value budget constraint in a sequence of one-period bond markets

$$a_{t+1} = (1+r_t)(a_t + z_t) \Leftrightarrow \frac{1}{(1+r_t)} a_{t+1} = (a_t + z_t)$$

$$a_{T+1} \geq 0 \Leftrightarrow a_0 + \sum_{t=0}^{T} P_t z_t \geq 0$$

with $P_t = [(1+r_0)(1+r_1)...(1+r_{t-1})]^{-1}$ an implicit present value price

Note: $\frac{1}{(1+r_t)}$ is a one period discount bond price

Multiperiod Fisherian preferences

$$\sum_{t=0}^{T} \beta^t u(c_t) \qquad \beta = \frac{1}{1+\upsilon} < 1; \qquad \upsilon > 0 \text{ is rate of pure time preference}$$

$$u(c) = \begin{cases} \frac{1}{1-\sigma}(c^{1-\sigma} - 1) \text{ with } \sigma > 0 \text{ and } \sigma \neq 1 \\ \log(c) \end{cases}$$

σ controls elasticity of intertemporal substitution

Modern version of Fisher's dynamic optimization
(via a Lagrangian, with integrated budget constraint)

$$L = \sum_{t=0}^{T} \beta^t u(c_t) \ + \Lambda[a_0 + \sum_{t=0}^{T} P_t (y_t - c_t)]$$

FOC: $\beta^t u_c(c_t) - \Lambda P_t = 0$

Ratio (mrs): $\dfrac{\beta^{t+1} u_c(c_{t+1})}{\beta^t u_c(c_t)} = \dfrac{P_{t+1}}{P_t}$

$$\Rightarrow \beta[\frac{c_{t+1}}{c_t}]^{-\sigma} = \frac{1}{1+r_t}$$

Fisher's rule

- Can be used for theory of consumption growth *and* for equilibrium theory of interest: just switch the dependent variable.

$$\log(c_{t+1}/c_t) = \frac{1}{\sigma}\log(\frac{1+r_t}{1+\upsilon}) \cong \frac{1}{\sigma}[r_t - \upsilon]$$

$$[r_t - \upsilon] \cong \log(\frac{1+r_t}{1+\upsilon}) = \sigma \log(y_{t+1}/y_t)$$

C. John Hicks

- *Value and Capital* approach: models involving time are to be analyzed just like other models – "just need to expand the commodity space"

- "Markets out of time"
- "Forward Markets"

- Effect on modern analyses with uncertainty (Arrow-Debreu)

Hicksian intertemporal preferences

- Just as with a static analysis, a general form of utility can be used when general questions are being studied

$$U(c_0, c_1, ...c_T)$$

- Special constructs like separability, time preference, constant elasticity are imposed for practical rather than conceptual reasons

Two versions of the Hicksian BC

- Concept: markets “out of time” or at “date 0” for all dated goods

- Straightforward Version:

- Convenient version: $$\sum_{t=0}^{T} \beta^t p_t c_t \leq \sum_{t=0}^{T} \beta^t p_t y_t$$

(with Fisherian preferences)

Lagrangian for demand behavior

- Objective, 1 constraint, 1 multiplier

$$L = U(\{c_t\}_{t=0}^{T}) + \Lambda[\sum_{t=0}^{T} P_t y_t - \sum_{t=0}^{T} P_t c_t] \qquad \text{OR}$$

$$L = U(\{c_t\}_{t=0}^{T}) + \Lambda[\sum_{t=0}^{T} \beta^t p_t y_t - \sum_{t=0}^{T} \beta^t p_t c_t]$$

- Multiplier is interpretable as “marginal utility of wealth”, i.e., of relaxing budget constraint by a small amount

First order conditions

General utility:

$$0 = \frac{\partial U}{\partial c_t} - \Lambda P_t$$

Restriction to $U = \sum_{t=0}^{T} \beta^t [x_t^{\sigma} \frac{1}{1-\sigma}(c_t^{\ 1-\sigma} - 1)]$

$$0 = \beta^t [x_t^{\sigma} c_t^{\ -\sigma} - \Lambda p_t]$$

Demand conditional on Λ (Frisch demand)

- Working in applied demand analysis, Ragnar Frisch asked the question: "when is it legitimate to exclude other prices from a system of demand equations?"
  - Separability is answer
  - Analysis also suggested the value of multiplier as a scale index

$$\text{Restriction to } U = \sum_{t=0}^{T} \beta^t [x_t^\sigma \frac{1}{1-\sigma}(c_t^{\,1-\sigma} - 1)]$$

$$\text{FOC: } 0 = x_t^\sigma c_t^{\,-\sigma} - \Lambda p_t$$

$$\text{Implied demand: } c_t = x_t (\Lambda p_t)^{-1/\sigma}$$

Demand just depends on own price (p), shifter (x) and Λ

Determining the level of consumption plan

$$0 = \sum_{t=0}^{T} \beta^t p_t [y_t - c_t]$$

$$= \sum_{t=0}^{T} \beta^t p_t [y_t - x_t (\Lambda p_t)^{(-1/\sigma)}]$$

$$\Rightarrow \Lambda = \left[\frac{\sum_{t=0}^{T} \beta^t p_t y_t}{\sum_{t=0}^{T} \beta^t x_t (p_t)^{(1-\frac{1}{\sigma})}} \right]^{-\sigma}$$

Hicksian equilibrium in endowment economy

- Use single agent's FOCs evaluated at endowments to determine price

General utility price implication :

$$P_t = \frac{1}{\Lambda}\frac{\partial U}{\partial c_t}(\{c_j = y_j\}_{j=0}^{T})$$

Restriction to $U = \sum_{t=0}^{T} \beta^t [x_t^{\sigma} \frac{1}{1-\sigma}(c_t^{1-\sigma} - 1)]$

Price implication: $p_t = \frac{1}{\Lambda} x_t \, y_t^{-\sigma}$

Challenges for Fisher and Hicks

- In general equilibrium (except with endowment economy), everything depends on everything else.
- Interest depends on preferences *and* production possibilities
- Can describe production economy equilibrium conditions but hard to determine outcomes

D Fisher's Heirs: Friedman and Modigliani

- Consumption smoothing implications for individuals and aggregates
  - Permanent income theory: differential response to sustained and transitory income shocks
  - Life cycle model: allocation of consumption over time given life cycle changes

Friedman's permanent income model

- Typically focused on special case in which interest rate equal time preference

$$c_t = r[a_t + \sum_{t=0}^{\infty} (\frac{1}{1+r})^j y_{t+j}] = y_t^p$$

modern description:

consumption is the annuity value of wealth

(the constant level with same present value)

Friedman's
Theory of the consumption function

- Worked out against the backdrop of Keynesian $c = a + b\ y$ and its relatives, with "b" being marginal propensity to consume.
- MPC smaller out of transitory income than out of permanent income
  - Occupations (farmers, dentists)
  - Trend versus cycle in macroeconomic data (note in figures above that c/y roughly invariant to trend, c deviations from trend less volatile than y deviations)

Modigliani’s life cycle model

- Aimed at explaining micro data (cross-section, panel) of consumption and then aggregating up
- Fact 1: life cycle variations in income
- Fact 2: life cycle variations in household demographics and activities

Life cycle income profile and simple consumption profile

Asset profile implication

(interpret as prediction about cross-section)

Modigliani style
Lifecycle model of consumption

- Note: sequence of constraints

max $$U = \sum_{t=0}^{T} \beta^t u(c_t, x_t)]$$

with $$u(c_t, x_t) = [x_t^{\sigma} \frac{1}{1-\sigma}(c_t^{\,1-\sigma} - 1)]$$

subject to $$a_t + y_t - c_t = \frac{1}{1+r_t} a_{t+1} \quad \text{for } t = 0,1,\ldots T$$

and $$a_{T+1} \geq 0 \quad \text{(enforced by credit market)}$$

Lagrangian for problem with sequence of constraints

(multipliers scaled by β^t for convenient FOCs)

$$L = \sum_{t=0}^{T} \beta^t u(c_t, x_t)]$$

$$+\sum_{t=0}^{T} (\beta^t \lambda_t)[a_t + y_t - c_t - \frac{1}{1+r_t} a_{t+1}]$$

$$+\Theta_{\mathrm{T+1}} a_{T+1}$$

First Order Conditions (interior)

$$c_t : 0 = \beta^t [x_t^\sigma c_t^{-\sigma} - \lambda_t]$$ for t=0,1,...T

$$a_{t+1} : 0 = \beta^t [-\lambda_t \frac{1}{1+r_t} + \beta \lambda_{t+1}]$$ for t=0,1,...T-1

$$a_{T+1} : 0 = -\beta^T \lambda_T \frac{1}{1+r_t} + \Theta_{T+1}$$

$$(\beta^t \lambda_{\mathrm{t}}): 0 = a_t + y_t - c_t - \frac{1}{1+r_t} a_{t+1}$$ for t=0,1,...T

Terminal wealth: the transversality condition

$$\frac{\partial L}{\partial \Theta_{T+1}} = a_{T+1} \geq 0$$

$$\Theta_{T+1} \frac{\partial L}{\partial \Theta_{T+1}} = \Theta_{T+1} a_{T+1} = 0$$

Formally: Kuhn-Tucker condition

Economics: valued wealth should not be left at the end

Either: Θ_{T+1}=0 or a_{T+1}=0 or both

Equivalently: $\beta^{\mathrm{T}} \lambda_T a_{T+1} = 0$

Solving for optimal outcomes: Step 1 is conditionally optimal consumption

$$c_t : 0 = \beta^t [u_c(c_t , x_t) - \lambda_t] \Rightarrow c_t = c(\lambda_t , x_t)$$

$$c_t : 0 = \beta^t [x_t^\sigma c_t^{-\sigma} - \lambda_t] \Rightarrow c_t = x_t (\lambda_t)^{(-1/\sigma)}$$

Solving for optimal outcomes: Step 2 is optimal shadow price dynamics

$$a_{t+1}: 0 = \beta^t[-\lambda_t \frac{1}{1+r_t} + \beta\lambda_{t+1}] \Rightarrow \lambda_{t+1} = (\frac{1}{\beta(1+r_t)})\lambda_t = \gamma_{\lambda t}\lambda_t$$

Words: shadow price falls at rate depending

on gap between interest rate and time preference rate

$$[\log(\lambda_{\mathrm{t+1}} / \lambda_t) \simeq -(r_t - \upsilon)]$$

$$\lambda_t = [\gamma_{\lambda,t-1}\gamma_{\lambda,t-2}...\gamma_{\lambda,0}]\lambda_0$$

Words: shadow price is fully determined by λ_0

Solving for optimal outcomes: Step 3 is optimal asset stock dynamics

#2: Once we know λ_0, we know $\{\lambda_t\}$ path

#1: Once we know $\{\lambda_t\}$ and $\{x_t\}$ path, we know $\{c_t\}$ path

#3: Once we know $\{c_t\}$ path, we determine asset path via

$$a_{t+1} = (1+r_t)[a_t + y_t - c_t]$$

from given a_0 for all dates.

However, an arbitrary λ_0 does not make $a_{T+1}=0$: only one does

In fact, since we have already studied this problem several times, we know that there is one and we could work it out algebraically

Summary of key points

1. General facts about consumption and investment
2. The Robinson Crusoe device
3. Consumption smoothing
4. Saving as a residual
5. PDV budget constraint in sequential markets
6. Intertemporal substitution and the interest rate
7. Interest rate determination in an endowment economy
8. Markets out of time: the Hicksian equilibrium device
9. The LC-PI model of consumption
10. Optimization over time with a sequence of constraints and the transversality condition for a household

What's next?

- The analysis so far suggest why consumption might be smoother than income (output)
- But it determines either saving (investment) or the interest rate (with saving zero) but <u>not</u> both at the same time
- Our next step is to analyze investment in more detail
- Then we will turn to the general equilibrium determination of consumption, investment, output and the interest rate.