Large Class of Macro Models

$$E_t F(Y_{t+1}, Y_t, \varepsilon_{t+1}) = 0$$

dimension: n_Y elements of column vector Y_t

n_ε elements of column vector ε_t of iid (unforecastable) shocks

n_Y elements of column vector F (same number of equations, unknowns)

n_p elements of column vector Y_t have exogenous forecast error or are predetermined

first part of solution: stationary certainty point

$$F(Y, Y, 0) = 0$$

claim: such a model has a unique, stable rational expectations solution as a first order approximation if the following three conditions are satisfied

(1) There is an equal number of stable eigenvalues as predetermined + exogenous: $n_s = n_p$

(2) A technical "rank" condition is satisfied

(3) A determinant condition is satisfied, which basically says that there are as many equations as unknow

such as solution takes the form

$$Y_t - Y = G(Y_{t-1} - Y) + H\varepsilon_t$$

where G and H are readily computed.

What would one do with such a solution?

#1 Forecasting

$$E_t(Y_{t+k} - Y) = G E_t(Y_{t+k-1} - Y)$$

$$= G^k(Y_t - Y)$$

Q: How do we expect economy to evolve?

~~#2 Stochastic simulation~~

#2 Analyze response of forecasts to shocks

$$E_t(Y_{t+k} - Y) - E_{t-1}(Y_{t+k} - Y)$$

$$= G^k\{E_t(Y_t - Y) - G(Y_{t-1} - Y)\}$$

$$= G^k H \varepsilon_t$$

q: How do we expect economy will in response to "news"?

#3 Stochastic simulation

$$Y_t = Y + G(Y_{t-1} - Y) + H\varepsilon_t$$

$$Y_{t+1} = Y + G(Y_t - Y) + H\varepsilon_{t+1}$$

$$Y_{t+2} = Y + G(Y_{t+1} - Y) + H\varepsilon_{t+2}$$

and so on

~~#4 [illegible]~~

#4 Compute moments (variance, covariance, correlation)

#5 Do econometrics

$$(Y_{t+1} - Y) = G(Y_t - Y) + H\varepsilon_t$$

is vector autoregression

If parameter vector θ enters into functions F then $G(\theta)$, $H(\theta)$ generally.

RE models impose nonlinear restrictions on G, H at each θ.

Various approaches can be used to estimate θ given criterion, construct confidence intervals and so on.

Example: Neoclassical growth model with uncertainty

$$V(k_t, a_t) = \max_{c_t} \left\{ u(c_t) + \beta E_t V(k_{t+1}, a_{t+1}) \right\}$$

$$k_{t+1} = f(k_t, a_t) + (1-\delta) k_t - c_t$$

$$a_{t+1} = a + \rho (a_{t\cancel{+1}} - a) + \varepsilon_{t+1}$$

FOCs

$$u_c(c_t) - \lambda_t = 0$$

$$\lambda_t - \beta E_t \{ V_k (k_{t+1}, a_{t+1} \} = 0$$

$$k_{t+1} = f(k_t, a_t) + (1-\delta) k_t - c_t$$

Exogenous state, variable

$$a_{t+1} = a + \rho (a_t - a) + \varepsilon_{t+1}$$

Envelope theorm

$$V_k(k_{t+1}, a_{t+1}) = \lambda_t \left[f(k_t, a_t) + 1 - \delta \right]$$

5 equations = 5 unknowns

$$Y_t = [c_t, \lambda_t, V_{kt}, \underset{\substack{\uparrow \\ \text{predetermined}}}{k_t}, \underset{\substack{\uparrow \\ \text{exogenous forecast error}}}{a_t}]$$

Linear approximation

$$0 = E_t\left\{ \underset{\uparrow\, n_Y \times n_Y}{F_{Y'}} (Y_{t+1} - Y) + \underset{\uparrow\, n_Y \times n_Y}{F_Y} (Y_t - Y) + \underbrace{F_{\varepsilon'}\, \varepsilon_{t+1}}_{=0} \right\}$$

Recall approximation of example

$$A \underset{\uparrow \text{ "add expectation"}}{E_t} (Y_{t+1} - Y) = B(Y_t - Y)$$

Condition #3 from page 1: There must be a value of z such that $|Az - B| \neq 0$.

Consider scalar case

$$a(y_{t+1} - y) = b(y_t - y)$$

$|az - b| = 0$ always only if $a = b = 0$. That is, you did not specify an equation to determine y_t.

Comment – does not occur in Dynare but must be satisfied for model to run.