Lab 2 of Part 1: Stochastic Systems

Key concepts

A. **State space form of linear difference equation**
B. **Stochastic simulation: a recursive approach**
C. **Impulse response from linear stochastic system**
D. **Computing population mean and variance**
E. **Monte Carlo simulation of finite sample moments**

Concept A: State space form of Linear Difference Equations.

Consider the state-space form

$$y_t = \Pi s_t$$
$$s_t = M s_{t-1} + G e_t$$

We know from class that the difference equation $y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + e_t$ **can be represented in this form as follows**

$$y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix}$$
$$\begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} e_t$$

We also know from class that the state space form is convenient for many purposes, as will be illustrated below.

Problem 1: Write a Short MATLAB Program that sets up the system above, creating the PI,G, and M matrices for specified values of theta ($\theta_1 = 1.3$ and $\theta_2 = -.4)$.

Concept B: Stochastic simulation: a recursive approach.

A stochastic simulation in state space form is implemented recursively, as follows. We begin by setting initial conditions s_0 , which implies $y_0 = \Pi s_0$. Then, at date 1, we first draw a random variable (in this case, a standard normal variable (N(0,1)) using randn.m). We then update to get a new state.

$$s_1 = Ms_{t-1} + Ge_1$$

We next compute the y variable,

$$y_t = \Pi s_t$$

We finally place the y variable in the vector Y , starting with $Y_0 = y_0$

$$Y_1 = [Y_0 \;\; y_1]$$

Then, we can restart the process from s_1 to compute s_2, y_2 and so forth.

Problem 2: Write a short program to stochastically simulate a y series, using the MATLAB built-in function randn.m (for information type "help randn") to generate a series of normal shocks.

Problem 3: change the simulation so that the initial conditions are $y_1 = y_2 = 2$ and show how the simulation changes from the $y_1 = y_2 = 0$ simulation using the same set of shocks in both cases.

Concept C: Impulse response.

The system impulse response is given by

$$E_t y_{t+k} - E_{t-1} y_{t+k} = \Pi(E_t s_{t+k} - E_{t-1} s_{t+k}) = M^k G e_t$$

It can be computed recursively.

Problem 4: Write a short program to compute the impulse response. The first few terms of the answer are: 1.0000, 1.3000, 1.2900, 1.1570, 0.9881.

Problem 5: Change the θ parameters so that you have a set of complex roots. Graph the impulse response.

Concept D: Computing the population mean and variance

There are a number of ways of computing population second moments for a state space system. One is to use the difference equation solution,

$$s_t = \sum_{j=0}^{\infty} \phi_j e_{t-j}$$

with the weights from the impulse response analysis above. Then, one can approximate the variance by using a large finite sum.

Another way is to view the variance of the states, V_{ss} **as given by**

$$E_t(s_t s_t^T) = ME_t(s_{t-1} s_{t-1}^T)M^T + GV_{ee}G^T$$

so that it is constrained by

$$V_{ss} = MV_{ss}M^T + GV_{ee}G^T$$

Using the column-stacking operator, $vec(V_{ss})$ **, it will be the case that**

$$\begin{aligned} vec(V_{ss}) &= M \otimes M * vec(V_{ss}) + vec(GV_{ee}G^T) \\ &= (I - M \otimes M)^{-1} vec(GV_{ee}G^T) \end{aligned}$$

Problem 6: Write a program which implements these two methods.

Concept E: "Monte Carlo" analysis

This method can be used to study how finite sample means and variances are related to their population values. In such a simulation, one draws H samples of length T, computing the sample mean for each and the sample variance for each.

Problem 7: For H=1000 and T=100, conduct a Monte Carlo analysis of the mean and variance of y . Use the built-in MATLAB function hist.m to display your results.