STUDIENZENTRUM GERZENSEE
Doctoral Course 2010
Day 1 Study Problems for Macroeconomics

Objective of problem: We have discussed optimization over time for individual decision-makers and for a social planner. We now consider a revenue-maximizing government's optimal money creation decisions, when it is faced with a forward-looking demand for money. This revenue maximization differs in important ways from the others that we have faced, due to forward-looking constraints. In week 3 of the class, we will study recursive methods for optimal policy design in such settings, but now we begin with a simple and direct approach.

Problem #2: The Revenue from Money Creation. Suppose that a government seeks to maximize the revenue that it derives from inflation, within a model with purely flexible prices (P_t is the price level at date t). In real terms, the government's revenue at date t is

$$z_t = m_t - \frac{1}{\pi_t} m_{t-1} \tag{1}$$

where π_t is the inflation rate – defined as $\pi_t = P_t/P_{t-1}$– and m_t is the amount of real balances held by agents in period t.[1]

The demand for real money balances is given by

$$m_t = \beta f(\pi_{t+1}) \tag{2}$$

which is assumed to be positive, but declining in the inflation rate ($f(\pi) \geq 0$ for all $\pi \geq 0$ and $f_\pi < 0$). The parameter β satisfies $0 < \beta < 1$. In some of the analysis below, it is also assumed that the money demand function takes the particular functional form

$$m_t = \kappa \pi_{t+1}^{-\alpha} \tag{3}$$

with κ and α being positive parameters.

(a) What inflation rate maximizes steady-state rate revenue,

$$m(1 - \frac{1}{\pi}) \tag{4}$$

subject to the particular money demand function $m = \kappa\pi^{-\alpha}$? How does revenue-maximixing this inflation rate depend on κ and α?

[1] This revenue may be derived as follows. First, the nominal money stock in period t is M_t and the newly printed money in period t is $M_t - M_{t-1}$. The real value of this newly printed money is $(M_t - M_{t-1})/P_t$, with P_t being the price level. Hence, the real revenue is as specified in the body of the question, if $\pi_t = P_t/P_{t-1}$.

(b) Consider next a government that maximizes the present discounted value of its revenue,

$$\sum_{t=0}^{\infty} \beta^t z_t \tag{5}$$

and assume that the government can commit to a series of inflation rates at dates $t = 0, 1, 2, ...$ Form a dynamic Lagrangian for the government's revenue maximization problem, treating the money demand function as an inquality of the form

$$m_t \leq \beta f(\pi_{t+1}) \tag{6}$$

$t = 0, 1, 2, ...$ That is: assume that the government can pick a "tax base" for the inflation tax which is no larger than the real balances that individuals are willing to hold. Call the multiplier on this constraint ϕ_t.

(c) Find the first-order conditions for optimal choice of m_t, π_t, and ϕ_t for all dates $t = 0, 1, 2, ...$ Record these as follows

$$\pi_0 \quad : \tag{7}$$
$$m_0 \quad : \tag{8}$$
$$\phi_0 \quad : \tag{9}$$

for $t > 0$.

$$\pi_t \quad : \tag{10}$$
$$m_t \quad : \tag{11}$$
$$\phi_t \quad : \tag{12}$$

(d) Using the economics of the problem and the first-order conditions, explain why the government has sharply different inflation incentives at date t=0 and date $t > 0$.

(e) Work out the stationary level of the revenue from money creation. How does the optimizing goverment's revenue compare to the answer from part (a)? Why?