

STUDIENZENTRUM GERZENSEE
 Doctoral Course 2010
 Day 1 Study Problems for Macroeconomics

Objective of problem set: The steady state or stationary state of a dynamic macroeconomic model is frequently the starting point for macroeconomic analysis, as it is a long-run situation of sorts and also provides information for construction of local dynamics. This problem set explores the steady-state of the neoclassical model of capital formation. It introduces some core economic ideas about the nature of the stationary state.

Problem #1: The Steady State. The stationary point in the basic model of capital accumulation is a triplet k^*, λ^*, c^* specified to be consistent with the three equations,

$$u_c(c^*) = \lambda^* \quad (1)$$

$$f_k(k^*) - \delta - \rho = 0 \quad (2)$$

$$f(k^*) - \delta k^* - c^* = 0 \quad (3)$$

(a) In class, it was asserted that this was a "recursive system", in that one could use one of the equations above to solve first for one member of the triplet; then another equation, along with the value of the first variable, to solve for the second; and finally solve for the third. Write down the order in which you would use the equations in this manner, the variable that would be solved for in each, and the solution under the Cobb-Douglas assumption, i.e., if $f(k) = ak^{1-\alpha}$.

	Equation #	Variable	Value with CD production function
First	(2)	k	$k^* = (a(1-\alpha)/(\rho+\delta))^{(1/\alpha)}$
Second	(3)	c	$c^* = f(k^*) - \delta k^*$
Third	(1)	λ	$\lambda = u_c(c^*)$

(b) It has been noted, by William Brock, that the steady-states of growth models can be viewed as distorted static general equilibrium models. This question pursues that interpretation.

(b-1) In the current context, imagine a static model in which there is a competitive firm with a production function $y = F(k, n)$ which minimizes costs given a wage rate w and a rental rate q . Assuming the firm does not earn economic profits, what equilibrium conditions does this produce?

ANSWER: Consider minimizing unit cost, $wn + qk$, subject to the requirement that one unit of the good is produced, $F(k, n) = 1$. Forming $L = -(wn + qk) + \psi(F(k, n) - 1)$, where ψ is a multiplier interpretable as marginal cost, we find that the FOCS are $-w + \psi F_n = 0$ and $-q + \psi F_k = 0$. However, with zero profits, $\psi = 1$, so that the conditions further simplify to $w = F_n$ and $q = F_k$.

(b-2) Suppose next that there is a household which maximize $u(c)$ subject to a budget constraint

$$c + \delta k \leq wn + (q - \tau)k + T$$

in which $n = 1$ is the amount of labor supply and k is the amount capital supply. Also in this expression, w is the wage rate earned by a unit of labor; $(q - \tau)$ is the "after-tax" rental rate earned by holding capital; δk is the cost of production of a unit of capital and T is a level of transfer payments which the household receives from the government. Finally, assume that the household has an endowment of time n and that it can freely vary the stock of capital that it will hold, subject to $k \geq 0$.

(b-2-i) Under what condition on $q - \tau - \delta$ will the household not be willing to supply any capital? Why?

ANSWER: If $(q - \tau - \delta) < 0$, then the household loses this amount for each unit of k that it supplies. It won't want to supply any capital.

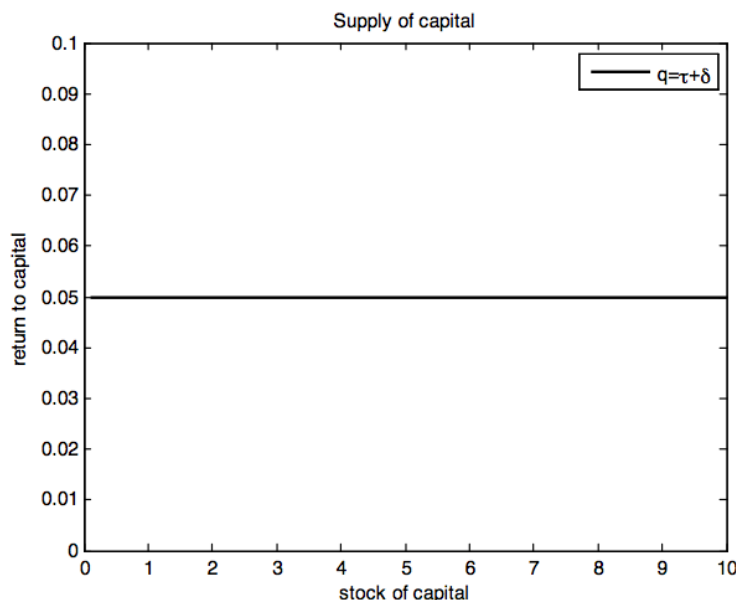
(b-2-ii) Under what condition on $q - \tau - \delta$ will the household want to supply an infinite amount of capital? Why?

ANSWER: If $(q - \tau - \delta) > 0$, then the household gains this amount for each unit of k that it supplies. It will want to supply an arbitrarily large amount to maximize its wealth.

(b-2-iii) Under what condition on $q - \tau - \delta$ will the household supply any amount of capital demanded by the firm? Why?

ANSWER: If $(q - \tau - \delta) = 0$, then there is no gain or loss to the supply of capital. The household has the same income for any value of k .

(b-2-iv) Draw a graph of the supply curve for capital in this economy.



(b-3) Assuming that the condition in (b-2-iii) is satisfied, form a Lagrangian for the household's maximization problem with λ being the multiplier on the budget constraint. What is the FOC for an interior solution?

ANSWER: The Lagrangian is $L = u(c) + \lambda[w + T - c]$. The consumption FOC is $u_c(c) - \lambda = 0$.

(c) Compare selected the SS equations from part (a) to conditions from part (b) which describe the static model's outcomes, under the assumption that the government rebates tax revenue as transfers, $T = \tau k$. Do you agree with Brock's argument that the SS can be represented as a tax-distorted static competitive equilibrium? To make the equivalence precise, what value must τ take? Why?

ANSWER:

	static	SS-dynamic
(1)	$u_c(c) = \lambda$	$u_c(c) = \lambda$
(2)	$q = f_k = \tau + \delta$	$f_k = \rho + \delta$
(3)	$c = w + qk - (\delta + \tau)k + T$ $= f(k) - \delta k$	$c = f(k) - \delta k$

These conditions are identical with $\tau = \rho$, which rationalizes Brock's claim. The interest rate plays the role of a "tax" relative to the stationary model. This is the same "wedge" that drives the distinction between the golden rule and modified golden rule.