STUDIENZENTRUM GERZENSEE Doctoral Course

Introductory Study Problem #3: A Wealth Difference Equation

Consider an agent who can accumulate wealth (a_t) according to

$$a_{t+1} = (1+r_t)[a_t + z_t]$$

where z_t is his saving and r_t is the rate of return.

(a) Assuming that the rate of return is constant over time, solve this difference equation from an initial condition a_0 given an exogenous sequence of savings, $\{z_t\}_{t=0}^T$, to determine wealth at date T + 1.

(b) Supposing that wealth must be positive at date T + 1, show that the condition

$$a_{T+1} \ge 0$$

is equivalent to an economic requirement on the present value of saving

$$a_0 + \sum_{t=0}^T (\frac{1}{1+r})^t z_t \ge 0$$

(c) Now suppose that the rate of return is not constant through time. Solve this difference equation from an initial condition a_0 given an exogenous sequence of savings, $\{z_t\}_{t=0}^T$, to determine wealth at date T + 1.

(d) Show that the requirement that wealth must be positive at date T+1, $a_{T+1} \ge 0$, is equivalent to an economic requirement on the present value of saving

$$a_0 + \sum_{t=0}^T p_t z_t \ge 0$$

and determine how the p_t relate to the r_t .