*The Monetary Dynamics of Hyperinflation**

I. General Monetary Characteristics of Hyperinflations

Hyperinflations provide a unique opportunity to study monetary phenomena. The astronomical increases in prices and money dwarf the changes in real income and other real factors. Even a substantial fall in real income, which generally has not occurred in hyperinflations, would be small compared with the typical rise in prices. Relations between monetary factors can be studied, therefore, in what almost amounts to isolation from the real sector of the economy.

This study deals with the relation between changes in the quantity of money and the price level during hyperinflations. One characteristic of such periods is that the ratio of an index of prices to an index of the quantity of money (P/M) tends to rise. Row 6 of Table 1 gives one measure of its rise for seven hyperinflations. (These seven are the only ones for which monthly indexes of prices are available.) Another way to illustrate this characteristic is by the decline in the reciprocal of this ratio, which represents an index of the real value of the quantity of money —real cash balances (M/P). Row 15 in Table 1 gives the minimum value reached by this index. Figures 1–7 also illustrate its tendency to decline. In ordinary inflations real cash balances, instead of declining, often tend to rise. The term "*hyper*inflation" must be properly defined. I shall define hyperinflations as beginning in the month the rise in prices exceeds 50 per cent[1] and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year. The definition does not rule out a rise in prices at a rate below 50 per cent per month for the intervening months, and many of these months have rates below

* I owe a great debt to Milton Friedman for his helpful suggestions at every stage of the work. I also benefited from discussions with Jacob Marschak on certain theoretical points. The following people read the manuscript in semifinal form and offered useful suggestions: Gary Becker, Earl J. Hamilton, H. Gregg Lewis, Marc Nerlove, and my wife.

1. The definition is purely arbitrary but serves the purposes of this study satisfactorily. Few ordinary inflations produce such a high rate even momentarily. In Figs. 1–7 rates of change are given as rates per month, compounded continuously. A rate of 41 per cent per month, compounded continuously, equals a rate of 50 per cent per month, compounded monthly.

TABLE 1

MONETARY CHARACTERISTICS OF SEVEN HYPERINFLATIONS*

	Austria	Germany	Greece	Hungary	Hungary	Poland	Russia
1. Approximate beginning month of hyperinflation	Oct., 1921	Aug., 1922	Nov., 1943	Mar., 1923	Aug., 1945	Jan., 1923	Dec., 1921
2. Approximate final month of hyperinflation	Aug., 1922	Nov., 1923	Nov., 1944	Feb., 1924	July, 1946	Jan., 1924	Jan., 1924
3. Approximate number of months of hyperinflation	11	16	13	10	12	11	26
4. Ratio of prices at end of final month to prices at first of beginning month	69.9	1.02×10^{10}	4.70×10^{8}	44.0	3.81×10^{27}	699.0	1.24×10^{5}
5. Ratio of quantity of hand-to-hand currency at end of final month to quantity at first of beginning month	19.3	7.32×10^{9}	3.62×10^{6}	17.0	1.19×10^{25}†	395.0	3.38×10^{4}
6. Ratio of (4) to (5)	3.62	1.40	130.0	2.59	320.0	1.77	3.67
7. Average rate of rise in prices (percentage per month)‡	47.1	322.0	365.0	46.0	19,800	81.4	57.0
8. Average rate of rise in quantity of hand-to-hand currency (percentage per month)§	30.9	314.0	220.0	32.7	12,200†	72.2	49.3
9. Ratio of (7) to (8)	1.52	1.03	1.66	1.41	1.62	1.13	1.16
10. Month of maximum rise in prices	Aug., 1922	Oct., 1923	Nov., 1944	July, 1923	July, 1946	Oct., 1923	Jan., 1924
11. Maximum monthly rise in prices (percentage per month)	134.0	32.4×10^{3}‖	85.5×10^{6}#	98.0	41.9×10^{15}	275.0	213.0
12. Change in quantity of hand-to-hand currency in month of maximum change in prices (percentage per month)	72.0	1.30×10^{3}**	73.9×10^{3}#	46.0	1.03×10^{15}	106.0	87.0
13. Ratio of (11) to (12)	1.86	24.9	1,160	2.13	40.7	2.59	2.45
14. Month in which real value of hand-to-hand currency was at a minimum	Aug., 1922	Oct., 1923	Nov., 1944	Feb., 1924	July, 1946	Nov., 1923	Jan., 1924
15. Minimum end-of-month ratio of real value of hand-to-hand currency to value at first of beginning month	0.35	0.030††	0.0069‡‡	0.39	0.0031†	0.34	0.27

* All rates and ratios have three significant figures except those in row 15, which have two. For sources see Appendix B (pp. 96–117).

† Includes bank deposits.

‡ The value of x that sets $(1 + [x/100])^t$ equal to the rise in the index of prices (row 4), where t is the number of months of hyperinflation (row 3).

§ The value of x that sets $(1 + [x/100])^t$ equal to the rise in the quantity of hand-to-hand currency (row 5), where t is the number of months of hyperinflation (row 3).

‖ October 2 to October 30, 1923, at a percentage rate per 30 days.

\# October 31 to November 10, 1944, at a percentage rate per 30 days.

** September 29 to October 31, 1923, at a percentage rate per 30 days.

†† October 23, 1923.

‡‡ November 10, 1944.

that figure. (The three average rates of increase below 50 per cent per month shown in row 7 of Table 1 reflect low rates in some of the middle months.)

Although real cash balances fall over the whole period of hyperinflation, they do not fall in every month but fluctuate drastically, as Figures 1–7 show. Furthermore, their behavior differs greatly among the seven hyperinflations. The ratios in rows 6 and 15 have an extremely wide range. Only when we bypass short but violent oscillations in the balances by striking an average, as in row 9, do the seven hyperinflations reveal a close similarity. The similarity of the ratios in row 9 suggests that these hyperinflations reflect the same economic process. To confirm this, we need a theory that accounts for the erratic behavior of real cash balances from month to month. This study proposes and tests such a theory.

The theory developed in the following pages involves an extension of the Cambridge cash-balances equation. That equation asserts that real cash balances remain proportional to real income (X) *under given conditions*. ($M/P = kX$; k = a constant.) Numerous writers have discussed what these given conditions are. Indeed, almost any discussion of monetary theory carries implications about the variables that determine the level of real cash balances. In the most general case the balances are a function, not necessarily linear, of real income and many other variables.

The following section discusses the most important of these variables. Because one of them—the rate of change in prices—fluctuates during hyperinflations with such extreme amplitude relative to the others, I advance the hypothesis that variations in real cash balances mainly depend on variations in the expected rate of change in prices. Section III elaborates this hypothesis and relates it to observable data on money and prices. It is supported by the statistical analysis presented in Section IV. The hypothesis, with an additional assumption, implies a dynamic process in which current price movements reflect past and current changes in the quantity of money. Sections V and VI explore certain implications of the model that describes this process. Section VII analyzes the revenue collected from the tax on cash balances, which is the counterpart of the rise in prices. A final section summarizes the theory of hyperinflation that emerges from this study.

II. THE DEMAND FOR REAL CASH BALANCES

Because money balances serve as a reserve of ready purchasing power for contingencies, the *nominal* amount of money that individuals want to hold at any moment depends primarily on the value of money, or the absolute price level. Their desired *real* cash balances depend in turn on

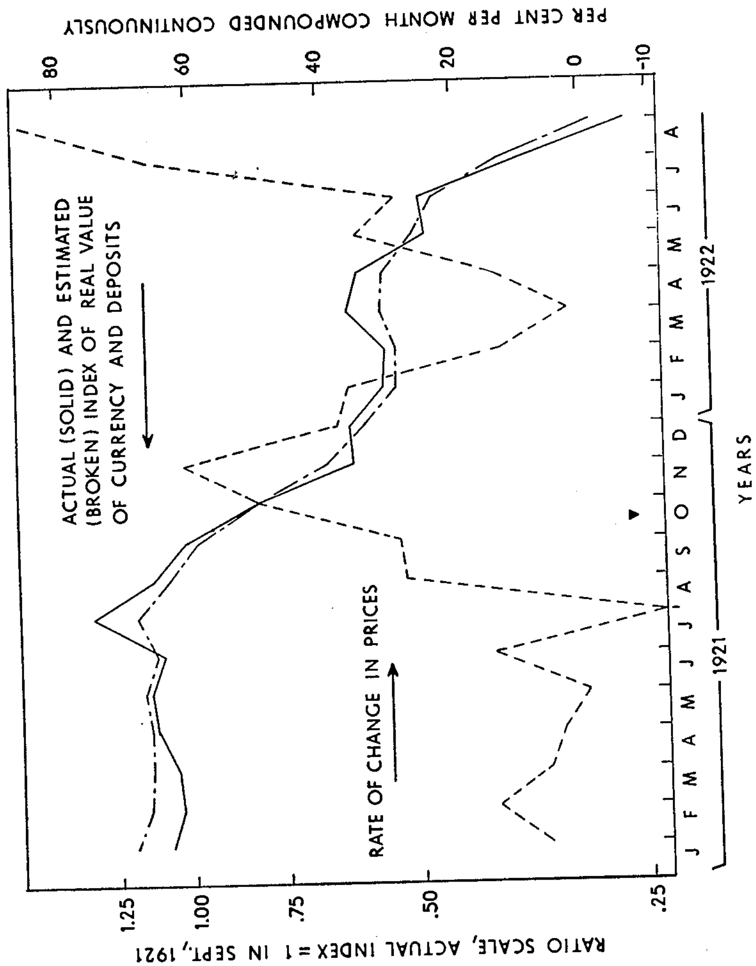

FIG. 1.—Austria—end-of-month rate of change in prices and index of real value of hand-to-hand currency and bank deposits, January, 1921, to August, 1922. (▼ Indicates beginning month of hyperinflation.)

numerous variables. The main variables that affect an individual's desired real cash balances are (1) his wealth in real terms; (2) his current real income; and (3) the expected returns from each form in which wealth can be held, including money.

If an individual's real wealth increases, he will usually desire to hold part of the increase in the form of money, because money is readily accepted in payment for goods and services or debts—it is an asset with a high liquidity.

If his current real income increases, an individual will want to substitute cash balances for part of his illiquid assets, for now he can more readily afford to forego the premium received for holding his assets in an illiquid form, and he may need larger balances to provide conveniently for his expenditures in the periods between income payments.

If the rate of interest on an asset increases, an individual is inclined to substitute this asset for some of his other assets, including his cash balances. His desired real cash balances will decrease. In addition, an increase in the rate of interest reflects a fall in the price of the asset and a decline in the wealth of holders of the asset; this decline in wealth reduces desired real cash balances.

Thus desired real cash balances change in the same direction as real wealth and current real income and in the direction opposite to changes in the return on assets other than money.

A specification of the amount of real cash balances that individuals want to hold for all values of the variables listed above defines a demand function for real cash balances. Other variables usually have only minor effects on desired real cash balances and can be omitted from the demand function. In general, this demand function and the other demand-and-supply functions that characterize the economic system simultaneously determine the equilibrium amount of real cash balances.

A simplified theory of this determination is that the amount of goods and services demanded and supplied and their relative prices are determined independently of the monetary sector of the economy. In one version of this theory—the quantity theory of money—the absolute level of prices is independently determined as the ratio of the quantity of √money supplied to a given level of desired real cash balances. Individuals cannot change the nominal amount of money in circulation, but, according to the quantity theory of money, they can influence the real value of their cash balances by attempting to reduce or increase their balances. In this attempt they bid the prices of goods and services up or down, respectively, and thereby alter the real value of cash balances.

During hyperinflation the amount of real cash balances changes

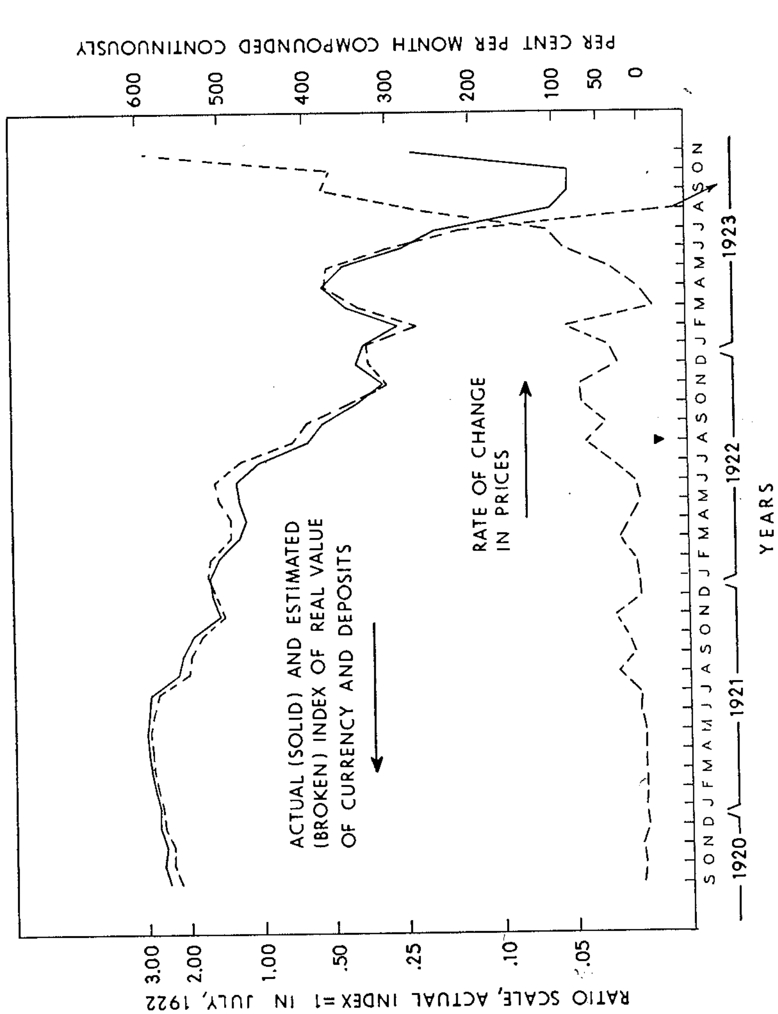

FIG. 2.—Germany—mid-month rate of change in prices and index of real value of hand-to-hand currency and bank deposits, September, 1920, to November, 1923. (▼ Indicates beginning month of hyperinflation.)

drastically (see Table 1). At first sight these changes may appear to reflect changes in individuals' preferences for real cash balances—that is, shifts in the demand function for the balances. But these changes in real cash balances may reflect instead changes in the variables that affect the desired level of the balances. Two of the main variables affecting their desired level, wealth in real terms and real income, seem to be relatively stable during hyperinflation, at least compared with the large fluctuations in an index of real cash balances. Thus to account for these fluctuations as a movement along the demand function for the balances instead of a shift in the function, we must look for large changes in the remaining variables listed above: the expected returns on various forms of holding wealth. Changes in the return on an asset affect real cash balances only if there is a change in the difference between the expected return on the asset and that on money. If this difference rises, individuals will substitute the asset for part of their cash balances. I turn, therefore, to a more detailed consideration of the difference in return on money and on various alternatives to holding money—the cost of holding cash balances.

There is a cost of holding cash balances with respect to each of the alternative forms of holding reserves, and in a wide sense anything that can be exchanged for money is an alternative to holding reserves in the form of cash balances. For practical purposes, these alternatives can be grouped into three main classes: (1) fixed-return assets (bonds); (2) variable-return assets (equities and titles to producers' goods); and (3) non-perishable consumers' goods. The cost of holding cash balances with respect to any of these alternatives is the difference between the money return on a cash balance and the money return on an alternative that is equivalent in value to the cash balance. The money return on a cash balance may be zero, as it typically is for hand-to-hand currency; negative, as it is for demand deposits when there are service charges; or positive, as it is for deposits on which interest is paid. The money return on bonds includes interest and on equities includes dividends, as well as any gains or losses due to a change in the money value of the assets. Variations in the cost of holding cash balances when the alternative is to hold consumers' goods can be determined solely by the change in the real value of a given nominal cash balance—the rate of depreciation in the real value of money. The variation in the real value of goods because of their physical depreciation is fairly constant and can be ignored.

The only cost of holding cash balances that seems to fluctuate widely enough to account for the drastic changes in real cash balances during hyperinflation is the rate of depreciation in the value of money or, equivalently, the rate of change in prices. This observation suggests the hy-

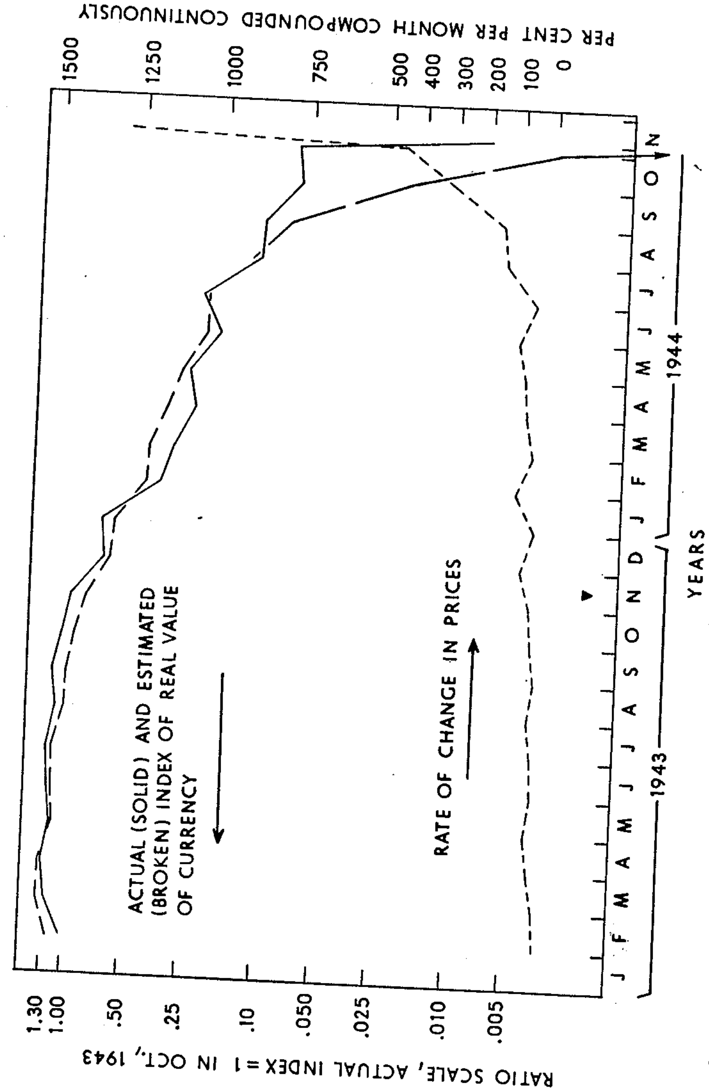

FIG. 3.—Greece—end-of-month rate of change in prices and index of real value of hand-to-hand currency, January, 1943, to October, 1944, and including November 10, 1944. (▼ Indicates beginning month of hyperinflation.)

pothesis that changes in real cash balances in hyperinflation result from variations in the expected rate of change in prices.

To be valid, this hypothesis requires that the effects of the other variables discussed above be negligible during hyperinflation. For the most part the statistical tests in Section IV uphold the hypothesis that variations in the expected rate of change in prices account for changes in desired real cash balances. For the periods in which the data do not conform to the hypothesis, what evidence there is (see Sec. IV) suggests that taking account of changes in real income would not remedy the limitations of the hypothesis. Another explanation of why the hypothesis fails to hold for these periods is offered instead as a plausible possibility.

In order to test the hypothesis statistically, the two variables, desired real cash balances and the expected rate of change in prices, must be related to observable phenomena. The assumption made about the former is that desired real cash balances are equal to actual real cash balances at all times. This means that any discrepancy that may exist between the two is erased almost immediately by movements in the price level.[2] The assumption made about the expected rate of change in prices is that it depends on the actual rate of change in a way to be explained in the next section.

With the above two assumptions, the hypothesis asserts that time series for the price level and the quantity of money are related by some

2. This assumption can be formulated as follows: Let M^d/P and M/P represent desired and actual real cash balances. Then write

$$\frac{d \log \frac{M}{P}}{dt} = \pi\left(\log \frac{M^d}{P} - \log \frac{M}{P}\right), \tag{1}$$

where π is a positive constant. This says that, when desired and actual real cash balances differ, the percentage change in the latter is proportional to the logarithm of their ratio. Prices rise and diminish the actual balances when the latter exceed desired balances. Prices fall and increase the actual balances when the latter fall short of desired balances. If we write the equation as

$$\log \frac{M^d}{P} = \log \frac{M}{P} + \frac{1}{\pi} \frac{d \log \frac{M}{P}}{dt},$$

the assumption in the text is equivalent to asserting that π is so large that

$$\frac{1}{\pi} \frac{d \log \frac{M}{P}}{dt}$$

is always almost zero.

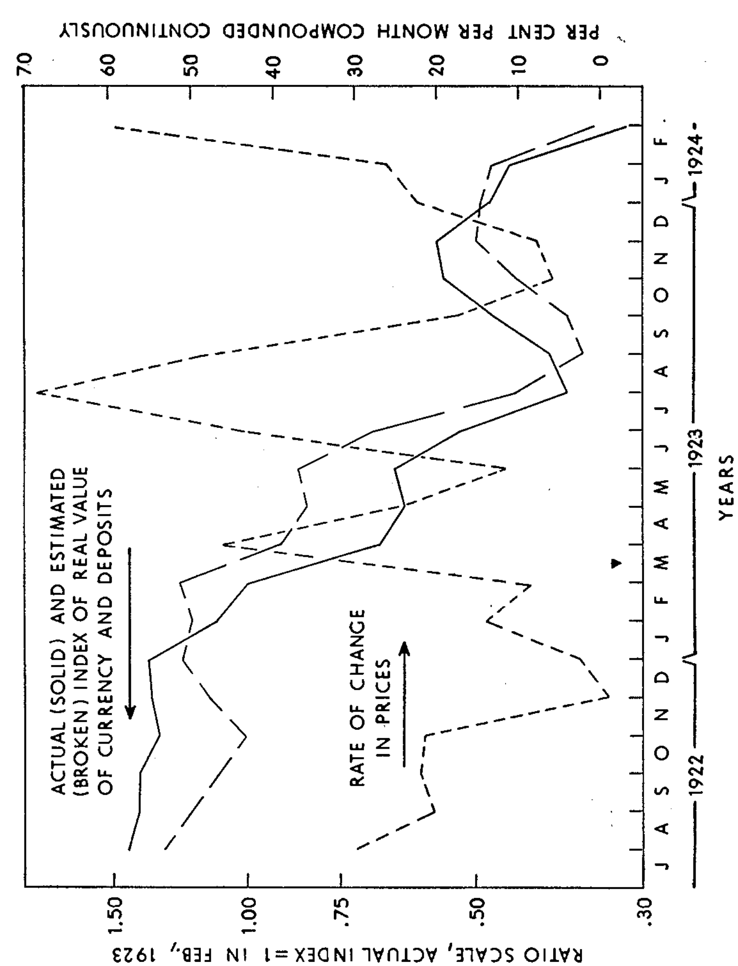

FIG. 4.—Hungary—end-of-month rate of change in prices and index of real value of hand-to-hand currency and bank deposits, July, 1922, to February, 1924. (▼ Indicates beginning month of hyperinflation.)

equation that determines real cash balances. An equation of the following form is able to account for most of the changes in real cash balances in seven hyperinflations:

$$\log_e \frac{M}{P} = -\alpha E - \gamma . \tag{2}$$

Equation (2) shows the demand for real cash balances for different levels of the expected rate of change in prices. M is an end-of-month index of the quantity of money in circulation, and P is an end-of-month index of the price level. α (which is necessarily positive) and γ are constants. E represents the expected rate of change in prices and is assumed to be a function of the actual rate of change, denoted by C. C stands for $(d \log P)/dt$ and is approximated by the difference between the logarithms of successive values of the index of prices. This difference represents the rate of change in prices per month, compounded continuously, if the logarithms have the base e.[3]

E, being the expected level of C, has the same units of measurement as C, namely, a pure number divided by the number of months. M/P is an index and therefore a pure number. Consequently, the unit of α is "months."

An implication of the above relation is that variations in the expected rate of change in prices have the same effect on real cash balances in percentage terms regardless of the absolute amount of the balances. This follows from the fact that equation (2) is a linear relation between the expected rate of change in prices and the logarithm of real cash balances. This implication seems proper for an equation that is supposed to provide an accurate approximation to the true demand function.

If we write equation (2) in the equivalent form,

$$\frac{M}{P} = e^{-\alpha E - \gamma} , \tag{3}$$

the elasticity of demand for real cash balances with respect to the expected change in prices, implied by the above relation, is

$$\frac{d \frac{M}{P}}{dE} \cdot \frac{E}{M/P} = -\alpha E , \tag{4}$$

where αE is a pure number. The elasticity is proportional to the expected rate of change in prices. It is positive when E is negative, and negative when E is positive. The elasticity is zero when E is zero.

3. If we view the change in prices from P_{i-t} to P_i in t months as a continuous rate of change at a rate of C per month, $P_i = P_{i-t}e^{Ct}$. When t is one month, $P_i = P_{i-1}e^{C}$; hence $C = \log_e P_i - \log_e P_{i-1}$.

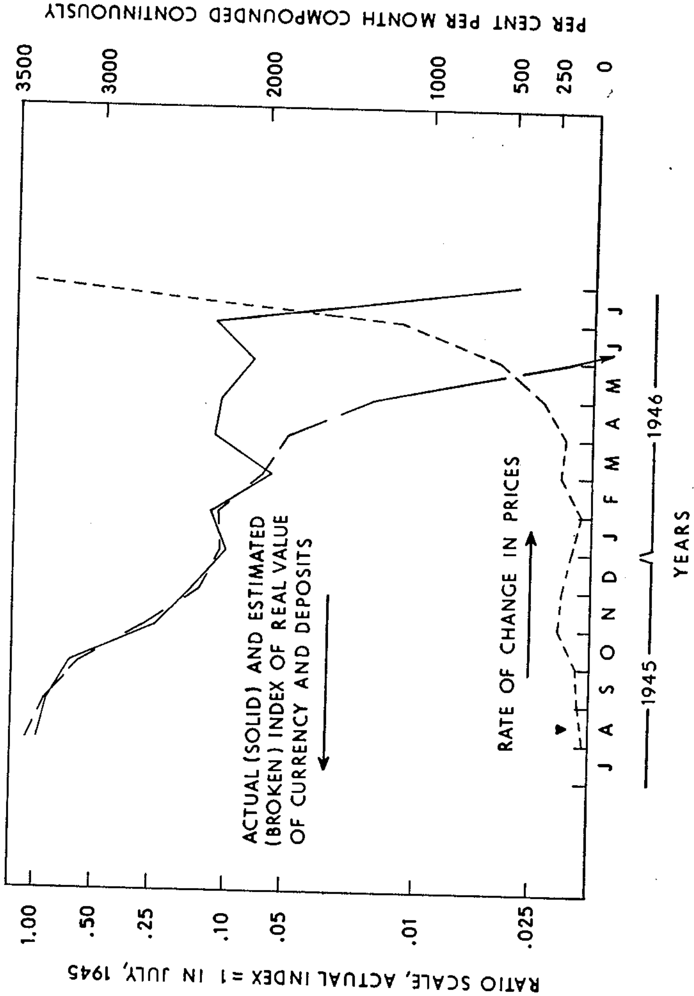

FIG. 5.—Hungary—end-of-month rate of change in prices and index of real value of hand-to-hand currency and bank deposits, July, 1945, to July, 1946. (▼ Indicates beginning month of hyperinflation.)

III. The Expected Rate of Change in Prices

The time series for the seven hyperinflations, displayed in Figures 1–7, indicate that, if desired and actual real cash balances are always equal, the actual rate of change in prices at any moment does not account for the amount of the balances at the same moment. In many months when the rates of change in prices were very low, sometimes even zero or negative, real cash balances were still much lower than they were in previous months when the rates were higher. The expected rate of change in prices seems to depend in some way on what the actual rates of change were in the past. One way is implied by the following assumption, which underlies the statistical analysis described in the next section. *The expected rate of change in prices is revised per period of time in proportion to the difference between the actual rate of change in prices and the rate of change that was expected.*

This assumption is expressed by

$$\left(\frac{dE}{dt}\right)_t = \beta\,(C_t - E_t)\,, \qquad \beta \geq 0\,, \quad (5)$$

where C_t represents $(d\log P)/dt$ at time t, and E_t is the expected level of C_t. β is a constant,[4] which can be described as a "coefficient of expectation," since its magnitude determines the rapidity with which expected rates of change in prices adjust to actual rates. The smaller is β, the slower is the adjustment.

The solution of (5) indicates what the assumption implies about the expected rate of change in prices. Equation (5) is a linear first-order differential equation in E and t with the solution,[5]

$$E_t = H\,e^{-\beta t} + e^{-\beta t}\int_{-T}^{t} \beta C_x e^{\beta x}\,dx\,, \quad (6)$$

where H is the constant of integration and $-T$ is an arbitrary lower limit of the integral. If prices had been almost constant before time $-T$, it is reasonable to assume that E was zero at time $-T$; hence

$$E_{-T} = H\,e^{\beta T} = 0\,, \qquad \text{and} \qquad H = 0\,. \quad (7)$$

4. Since C and E have the units "per month" and dE/dt, the units "per month per month," the units of β are "per month." Equation (5) is mathematically equivalent to

$$E_t = \beta\left(\log P_t - \int_{-\infty}^{t} E_x dx\right) + \text{a const.}\,,$$

where the integral term represents the expected level of prices at time t.

5. See any textbook on differential equations.

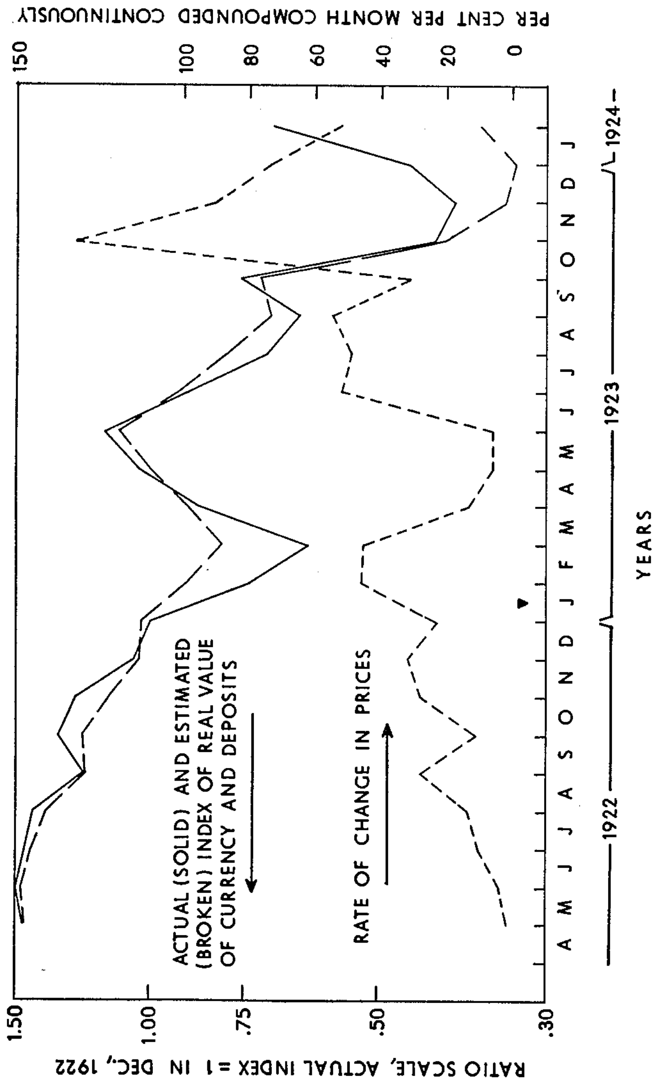

FIG. 6.—Poland—end-of-month rate of change in prices and index of real value of hand-to-hand currency and bank deposits, April, 1922, to January, 1924. (▼ Indicates beginning month of hyperinflation.)

E_t can then be written as

$$E_t = \frac{\int_{-T}^{t} C_x e^{\beta x} dx}{\frac{e^{\beta t}}{\beta}}. \tag{8}$$

In this form the expected rate of change in prices is a weighted average of past rates of change with weights given by the exponential function, $e^{\beta x}$. The denominator of the expression represents the sum of the weights, because

$$\int_{-T}^{t} e^{\beta x} dx = \frac{e^{\beta t}}{\beta} (1 - e^{-\beta [T+t]}) ,$$

and because $-T$ is chosen so that $e^{-\beta(T+t)}$ is sufficiently small to neglect (see [10], p. 41, below).

Since at best there are only monthly observations of prices during most of the hyperinflations, the expected rate of change in prices is approximated by a weighted average of a series of terms, each representing the rate of change in prices for a whole month. That is, if we approximate C_x for $t - 1 < x \leq t$ by C_t,

$$\int_{t-1}^{t} C_x e^{\beta x} dx = C_t \int_{t-1}^{t} e^{\beta x} dx = \frac{C_t e^{\beta t}}{\beta} (1 - e^{-\beta}) .$$

Equation (8) is then replaced by a series of terms, each representing a monthly period, as follows:[6]

$$E_t = \frac{(1 - e^{-\beta}) \sum_{x=-T}^{t} C_x e^{\beta x}}{e^{\beta t}} , \qquad t \geq 0. \tag{9}$$

6. A convenient procedure to follow in computing E for a series of months is to calculate the accumulated products of $C_x e^{\beta x}$ for all the months $x \geq -T$, noting the sum of the products for the months $x \geq 0$. Then E_t is simply the quotient of these accumulated products for $x = t$ divided by $e^{\beta t}/(1 - e^{-\beta})$. However, when β is large, say above .7, it is more convenient to compute each E_t separately by the following form of formula (9),

$$E_t = (1 - e^{-\beta}) \sum_{i=0}^{T} C_{t-i} e^{-\beta i} . \tag{9a}$$

In this procedure the weighting pattern, $(1 - e^{-\beta}) e^{-\beta i}$, is the same for each E_t and need only be calculated once for each value of β.

We can set the sum of the series of weights at a predetermined level by extending the

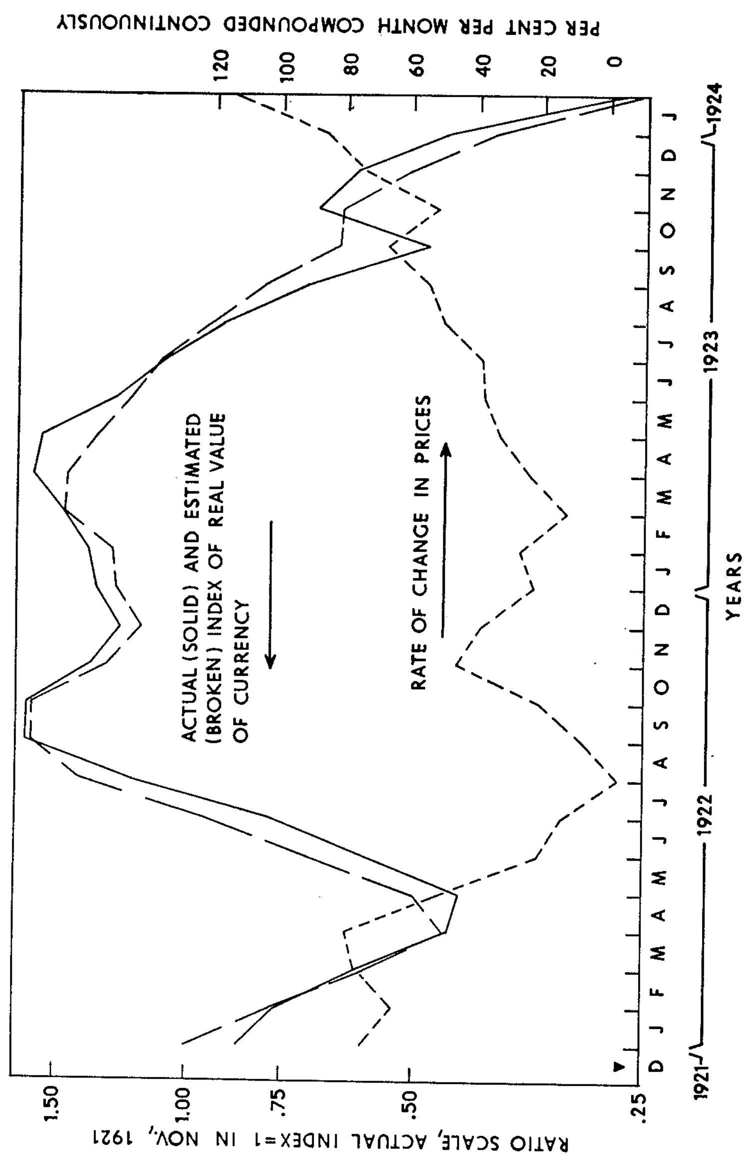

FIG. 7.—Russia—end-of-month rate of change in prices and index of real value of hand-to-hand currency, December, 1921, to January, 1924. (▼ Indicates beginning month of hyperinflation.)

Table 2 illustrates the weighting patterns by the number of months that it takes the weights in equation (9*a*) to fall by specified percentage amounts for different values of β. The average length of each weighting pattern, shown in the last column, is defined as follows:

$$\frac{-\int_{-\infty}^{0} x\, e^{\beta x} d x}{\frac{e^{\beta t}}{\beta}} = +\frac{1}{\beta}.$$

It serves as a measure of the average length of time by which expectations of price changes lag behind the actual changes.

TABLE 2

CHARACTERISTICS OF EXPONENTIAL WEIGHTS FOR DIFFERENT VALUES OF β

β (Per Month)	Value of Weight When $t=0$ (Rounded to Hundredths)	Approximate Number of Months for Weights to Fall by			Average Length of Weighting Patterns ($1/\beta$) (Months)
		50 Per Cent	75 Per Cent	90 Per Cent	
.01........	0.01	70	139	230	100.0
.05........	0.05	14	28	46	20.0
.10........	0.10	7	14	23	10.0
.15........	0.14	5	9	15	6.7
.20........	0.18	3	7	12	5.0
.25........	0.22	3	7	9	4.0
.30........	0.26	2	5	8	3.3
.35........	0.30	2	4	7	2.9
.40........	0.33	2	3	6	2.5
.50........	0.39	1	3	5	2.0
.75........	0.53	1	2	3	1.3
1.00........	0.63	1	1	2	1.0
5.00........	0.99	0	0	0	0.5
10.00........	1.00	0	0	0	0.1

IV. Statistical Analysis of Data from Seven Hyperinflations

Equation (2) and the approximation to the expected rate of change in prices given by equation (9) imply the following equation. The random variable ϵ_t is inserted to account for deviations of the left-hand side from zero.

series. In this study T was set such that

$$(1 - e^{-\beta})\, e^{\beta(-T-t)} < .00005 \tag{10}$$

for $t = 0$ (the first month used in the regressions); using the same T, the inequality is sure to hold for any $t \geq 0$. We are then sure that the series of weights for $t \geq 0$ adds up to $1 \pm .00005$ for $\beta \geq .01$.

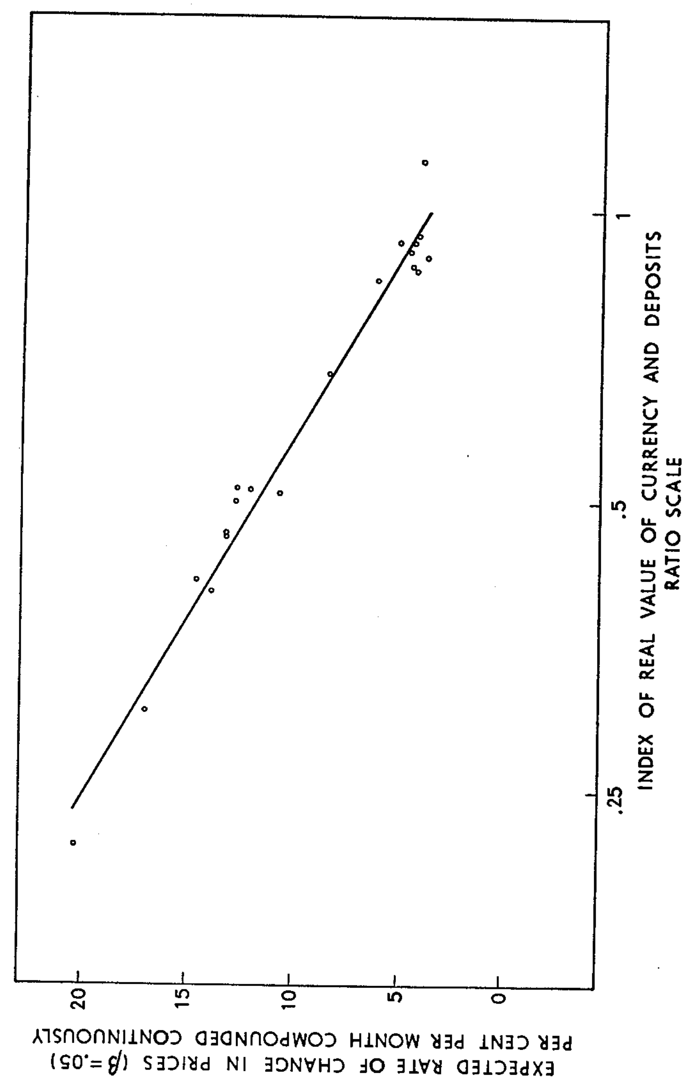

FIG. 8.—Austria—scatter diagram of end-of-month expected rates of change in prices and indexes of real value of hand-to-hand currency and bank deposits, and regression line (α = 8.55), January, 1921, to August, 1922.

$$\log_e \left(\frac{M}{P}\right)_t + \alpha \frac{(1-e^{-\beta})}{e^{\beta t}} \sum_{x=-T}^{t} C_x e^{\beta x} + \gamma = \epsilon_t . \qquad (11)$$

Table 3 contains estimates of the parameters α and β and the correlation coefficients derived by fitting equation (11) to the data from seven hyperinflations.[7] The method of fitting was by least squares. Figures 8–14 show the scatter diagrams of the regressions. Time series of real cash balances estimated from the regression functions have been plotted in

TABLE 3

LEAST-SQUARES ESTIMATES OF α AND β AND CORRELATION COEFFICIENTS FOR SEVEN HYPERINFLATIONS*

Country	Time Period (End of Month)	Degrees of Freedom	Estimated Value of α (Months)	Estimated Value of β ($\pm$.05) (Per Month)	Confidence Intervals† α (Months)	Confidence Intervals† β (Per Month)	Total Corr. Coef.
Austria	Jan., 1921–Aug., 1922	17	8.55	.05	4.43–31.0	.01 –.15	.989
Germany	Sept., 1920–July, 1923	32	5.46	.20	5.05– 6.13	.15 –.25	.992
Greece‡	Jan., 1943–Aug., 1944	17	4.09	.15	2.83–32.5§	.01§–.30	.980
Hungary	July, 1922–Feb., 1924	17	8.70	.10	6.36–42.2§	.01§–.20	.926
Hungary‡	July, 1945–Feb., 1946	5	3.63	.15	2.55– 4.73	.10 –.30	.998
Poland	Apr., 1922–Nov., 1923	17	2.30	.30	1.74– 3.94	.10 –.60	.972
Russia‡	Dec., 1921–Jan., 1924	23	3.06	.35	2.66– 3.76	.25 –.45	.971

* The estimates were computed by maximizing the total correlation coefficient for given values of β rather than by solving the normal equations. The value of β was estimated within an interval of $\pm$.05. The correct value of the correlation coefficient for each sample is therefore slightly greater than the values given in the table. The method of computing the estimates is discussed in Appendix A, and the data used are given in Appendix B.

† Confidence coefficient for intervals of α and β is .90. The confidence intervals for β are the extreme limits of a .05 interval. That is, the lower limits could be as much as .05 higher, and the upper limits could be as much as .05 lower.

‡ Greece has no adjustment to include deposits because the required data are not available. An adjustment to include deposits is not necessary for Russia. The figures for the quantity of money in Hungary following World War II include deposits for each month. See Appendix B.

§ End of confidence interval lies beyond the figure given. The correct figure was not computed because of the unreliability of the estimates of the expected change in prices for the earlier months when β is very small.

Figures 1–7. For all hyperinflations except the Russian, the regressions include observations before the beginning month shown in Table 1 in order to raise the number of degrees of freedom. But observations were not used from earlier periods in which real cash balances were subject to erratic movements. These movements were mostly increases in the balances in periods of rising prices before the beginning of hyperinflation and are inconsistent with the behavior implied by the demand function formulated above. They are discussed more fully below.

For the periods of hyperinflation covered the results indicate that an exponentially weighted average of past rates of change in prices adequate-

7. The method of deriving the estimates is discussed in Appendix A (pp. 92–96).

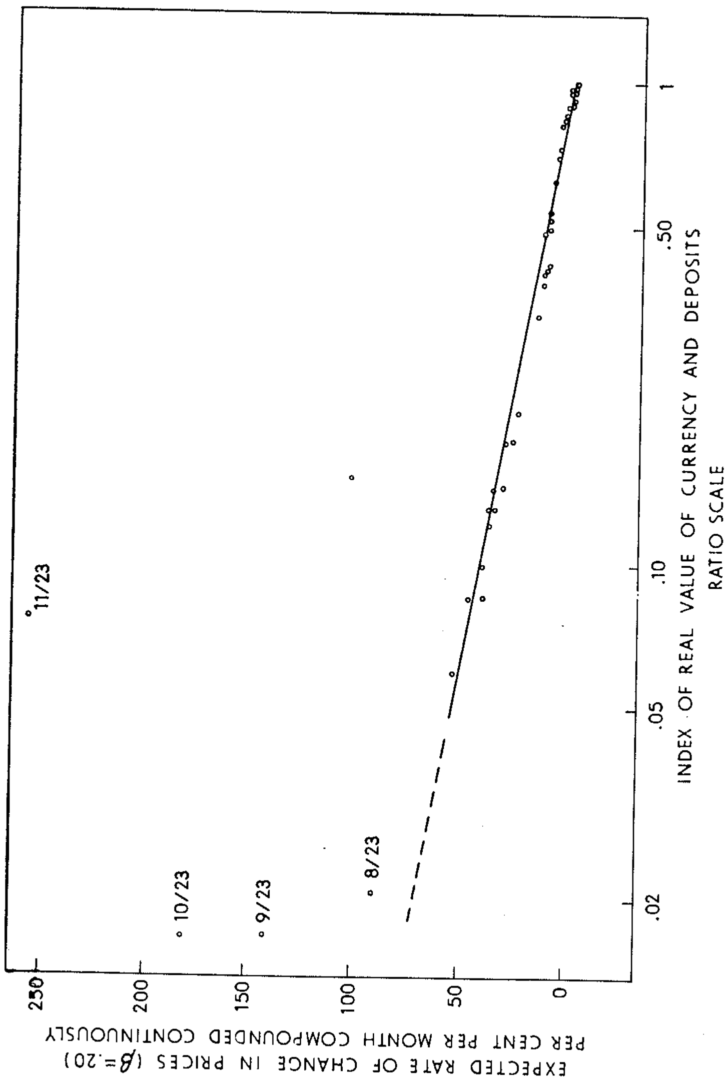

FIG. 9.—Germany—scatter diagram of mid-month expected rates of change in prices and indexes of real value of hand-to-hand currency and bank deposits, and regression line (α = 5.46), September, 1920, to November, 1923. (Points excluded from calculation of regression line are dated.)

ly accounts for movements in real cash balances. Furthermore, the confidence intervals for the estimates of β clearly indicate that expected rates of change in prices are not equal to current actual rates. The value of β that produces this equality is 10.0 (see the second column of Table 2). The highest value of β that does not differ from any of the estimates at the .10 level of significance is only .60.[8]

TABLE 4

ESTIMATES OF α AND β FOR SEVEN HYPERINFLATIONS TAKEN TOGETHER AND TEST OF SIGNIFICANCE BETWEEN ESTIMATES FOR THE HYPERINFLATIONS TAKEN TOGETHER AND SEPARATELY*

Estimated Value of		Total Corr. Coef.	Likelihood-Ratio Test of Significance between Estimates of α and β for the Hyperinflations Taken Together and Separately	
α (Months)	β ($\pm$.05) (Per Month)		Likelihood Ratio	.005 Level of Significance for Likelihood Ratio
4.68	.20	.894	72.53	28.3

* See Appendix A for statistical methods.

Table 4 gives estimates of α and β derived by fitting equation (11) to the data for all the hyperinflations taken together. These estimates differ significantly from those computed for each hyperinflation separately.[9]

8. The confidence intervals must be accepted with some caution. Besides relying on asymptotic properties of the likelihood ratio, the method of calculating these intervals assumes the independent normal distribution of the residuals from the least-squares fit. The confidence intervals can at best serve to indicate the approximate amount of random variability in the estimates of the two parameters.

9. Table 4 gives the results of fitting equation (11) to all the data under the assumption that the value of α is the same and the value of β is the same in all the hyperinflations. The value of γ varies among the hyperinflations. That the level of the likelihood ratio is significant for this fit means that the correlation coefficient of the fit is significantly lower than the correlation coefficient based on estimates derived for each hyperinflation separately.

Two other fits of equation (11) were made to all the data. In the first fit, all the hyperinflations have the same value of β, and the values of α and γ vary among the hyperinflations. In the second fit, the value of α is the same in all the hyperinflations, while the values of β and γ vary. The correlation coefficient in these two fits is smaller significantly at the .005 level than the coefficient based on the estimates derived for each hyperinflation separately.

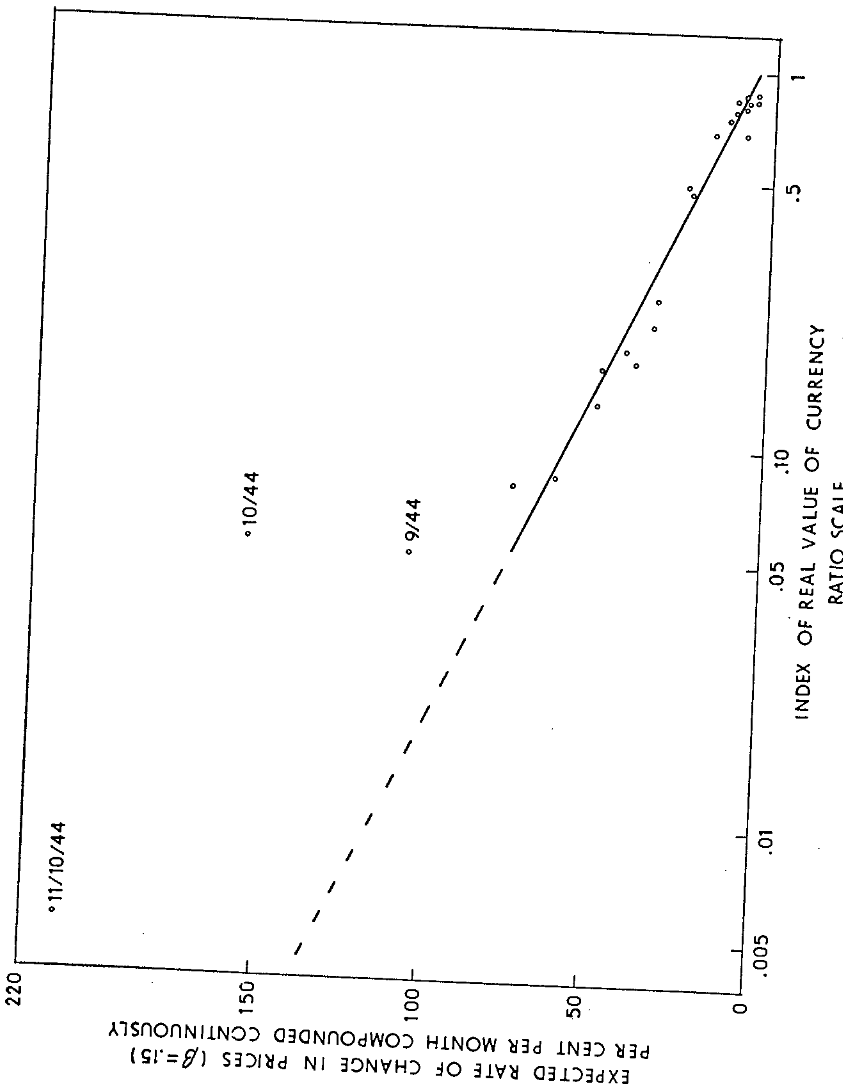

FIG. 10.—Greece—scatter diagram of end-of-month expected rates of change in prices and indexes of real value of hand-to-hand currency, and regression line ($\alpha = 4.09$), January, 1943, to October, 1944, and including November 10, 1944. (Points excluded from calculation of the regression line are dated.)

Consequently, the differences in the values of the parameters for each hyperinflation cannot be ascribed to random variability in the estimates.

However, we should not overstress these differences. The similarity in the results for the different countries is striking in some important respects. Later sections explore the economic significance of the estimated values of the parameters and point out differences and similarities in the seven hyperinflations. This section deals with the general question of the accuracy of the statistical results. They require certain reservations, not only because of unreliable data but even more because of other difficulties of an economic and statistical nature. These difficulties are taken up below under the following headings: (1) reliability of the data; (2) economic variables ignored in the regression function; (3) observations that do not fit the regressions; and (4) increases in the coefficient of expectation.

1. RELIABILITY OF THE DATA

The data for the statistical analysis are indexes of prices and the quantity of hand-to-hand currency adjusted for all the countries except Greece and Russia to include the quantity of bank deposits. As indicated in Appendix B, which describes the data, they have limited coverage. Hand-to-hand currencies are usually issued by one governmental agency, and official publications report the quantity in circulation with unquestioned accuracy. However, illegal and counterfeit currencies, which are excluded from the money figures because of the unavailability of monthly data, circulated freely in at least one hyperinflation—the German—and were issued to some extent in most of the others. The quantity of bank deposits is based on figures that lack complete coverage. As is shown below, however, an adjustment of the money figures to include deposits has only a small effect on the estimates and cannot be a source of much error. Little can be said about the price indexes except that most of them are averages of prices of economically important commodities. The indexes are far from comprehensive in scope, and their accuracy can be checked only by independent sources. When these exist, they agree on the whole with the index used.

If, as these remarks imply, much of the data could be subject to large errors, why are the correlation coefficients in Table 3 so high? Poor data tend to increase the residual errors of a least-squares fit. The high correlation coefficients suggest that the bulk of the figures are not subject to large random errors. One factor that enhances the reliability of the data is their extreme rates of change. The differences in the rates at which the prices of various commodities rise in hyperinflation are, while no doubt

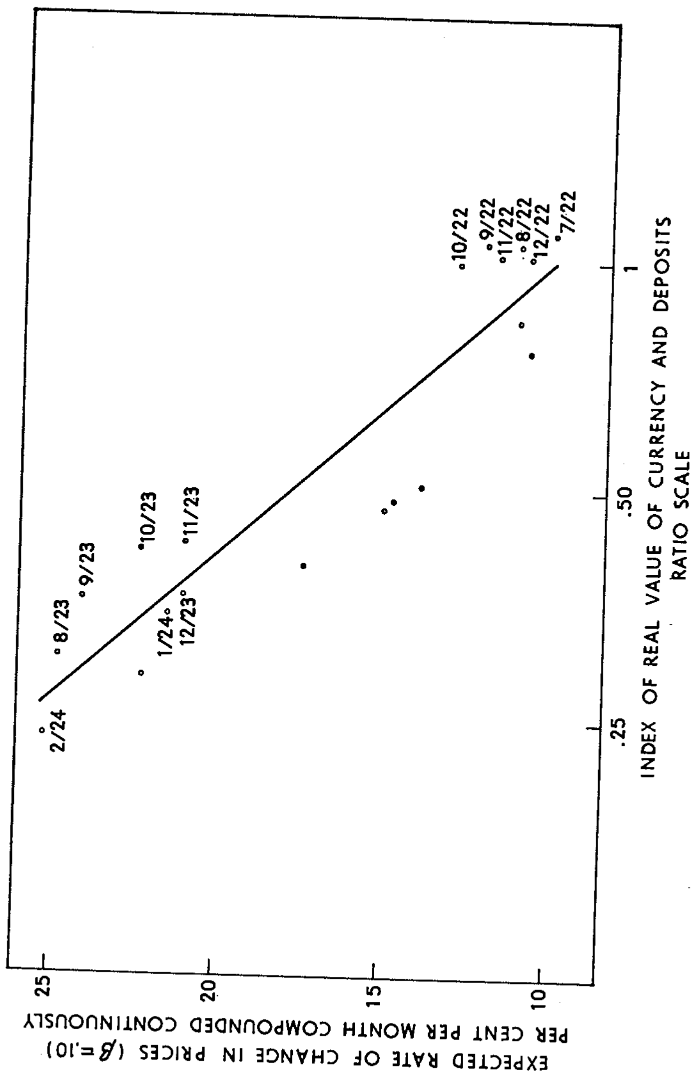

FIG. 11.—Hungary—scatter diagram of end-of-month expected rates of change in prices and indexes of real value of hand-to-hand currency and bank deposits, and regression line (α = 8.70), July, 1922, to February, 1924. (Points before January, 1923, and after July, 1923, are dated.)

large in absolute terms, probably small relative to the rates themselves. Therefore, a price index restricted in scope reflects fairly accurately the rate of rise of an index of all prices. The money figures also, while incomplete, are adequate for approximating large changes.

Although apparently free from large random errors, the data could be consistently biased in one direction. The exclusion of illegal and counterfeit currencies tends to lower the estimates of α.[10] Also limiting a price index to wholesale or retail commodities exclusively, as is necessary for want of comprehensive data, tends to make the index too high or too low, respectively. Note, however, that, if the price index used is a constant multiple of a "correct" index, α and β are estimated without bias. To produce bias in the estimates of β, an index must misrepresent the rates of change in prices. Even then, estimates of β will not be biased if there is a linear relation between the rates used and the "correct" rates. It is not unlikely that such a relation holds approximately. In hyperinflation all prices rise so rapidly that almost any group of them registers more or less accurately the time pattern of the movements in all. Consequently, bias resulting from a consistent error in the price index would appear mainly in estimates of α. But they exhibit no signs of this bias. There is no relation between the size of the estimates of α in Table 3 and the kind of index used. The estimates of α for the Austrian and first Hungarian hyperinflations are the two largest, and they are based on indexes of the cost of living and wholesale prices, respectively. The Polish and Russian estimates, which are the two smallest, depend on indexes of wholesale and retail prices, respectively. The bias is apparently not large enough to determine the rank of the estimates. Therefore, we should be able to rely at least on their general magnitude.

The data for the estimates in Table 3 include indexes of hand-to-hand currency and monthly interpolations of an annual index of the quantity of bank deposits held by individuals and enterprises. As one of the notes to Table 3 points out, the data for Greece and Russia do not include deposits; for Hungary after World War II monthly figures on deposits are available, and no interpolations are necessary. For the other four hyperinflations Table 5 gives estimates of α and β computed from data that exclude deposits. A comparison of Tables 3 and 5 indicates that including an estimate of deposits improves the correlation coefficient twice (Germany and Hungary after World War I) and diminishes it twice (Austria

10. Figures for the quantity of money that understate the correct amount by an increasing percentage over time also make the index of real cash balances progressively too low by an increasing percentage and bias the estimate of α downward.

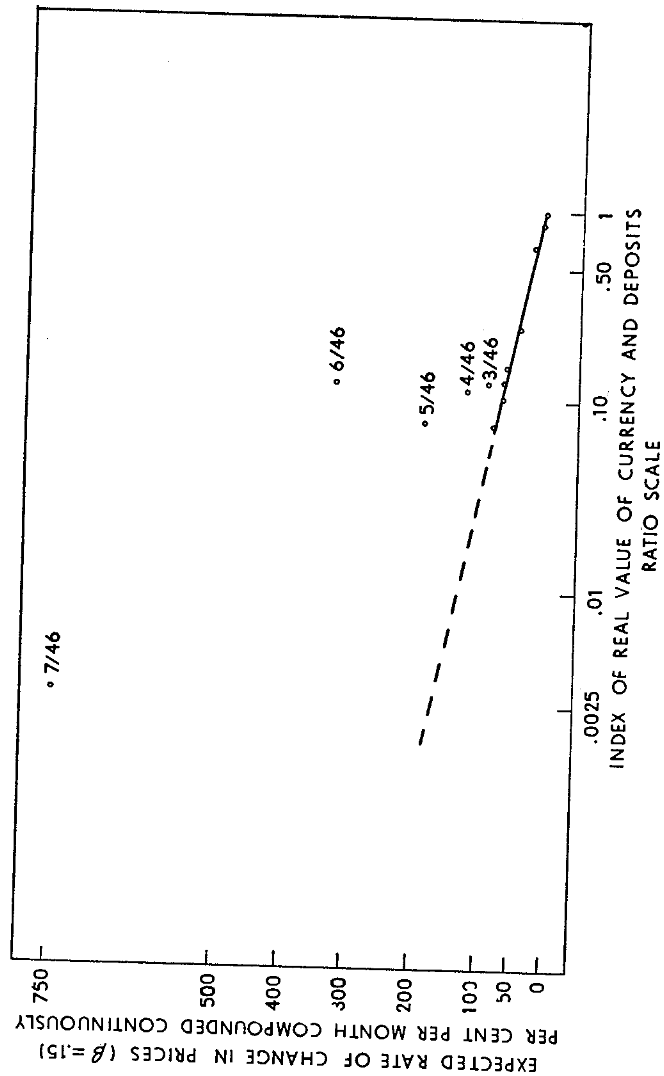

FIG. 12.—Hungary—scatter diagram of end-of-month expected rates of change in prices and indexes of real value of hand-to-hand currency and bank deposits, and regression line (α = 3.63), July, 1945, to July, 1946. (Points excluded from calculation of the regression line are dated.)

and Poland).[11] The net effects on the estimates of including deposits are unimportant. It is likely, therefore, that the use of deposit figures in the other hyperinflations would not make an appreciable difference. Moreover, the ratio of deposits to currency is not likely to change rapidly, so that the use of monthly deposit figures, if they were available, would probably not alter these results.

TABLE 5

ESTIMATED VALUES OF α AND β EXCLUDING BANK DEPOSITS FROM THE DATA*

Country	Estimated Value of α (Months)	Estimated Value of β ($\pm$.05) (Per Month)	Total Correlation Coefficient
Austria	8.78	.05	.996
Germany	5.04	.25	.987
Hungary (after World War I)	6.16	.10	.886
Poland	2.84	.25	.978

* The monthly periods and the degrees of freedom are the same as in Table 3. The remarks in n. * of Table 3 apply to these estimates.

2. ECONOMIC VARIABLES IGNORED IN THE REGRESSION FUNCTION

The figures for the quantity of money exclude all money in circulation during the hyperinflations that did not have a fixed rate of exchange with the depreciating currency. One important type of excluded money is foreign bank notes. In addition, in Russia a stable-valued currency first issued by the government at the end of 1922, the chervonets, circulated along with the depreciating rubles. The Hungarian government issued a new currency in 1946, the tax pengö, which was supposed to keep a constant purchasing power. Its issue was partly successful in that it never depreciated in value at the tremendous rates reached by the regular pengö.

These issues are not counted as part of the quantity of money, because this study concerns the cash balances of rapidly depreciating currencies only. However, the sudden introduction of these other issues opened up new alternatives to holding the depreciating currency in cash balances. The fact that a stable-valued money does not pay interest means little

11. Including deposits should improve the results, because the trend of deposits and that of hand-to-hand currency may differ from each other by a great deal. It is to be expected that using monthly interpolations of an annual index of deposits might increase the residual errors about the regression lines. Therefore, the fact that the variance of the residual errors was increased by the adjustment in two cases does not indicate that the estimates of the parameters were not improved.

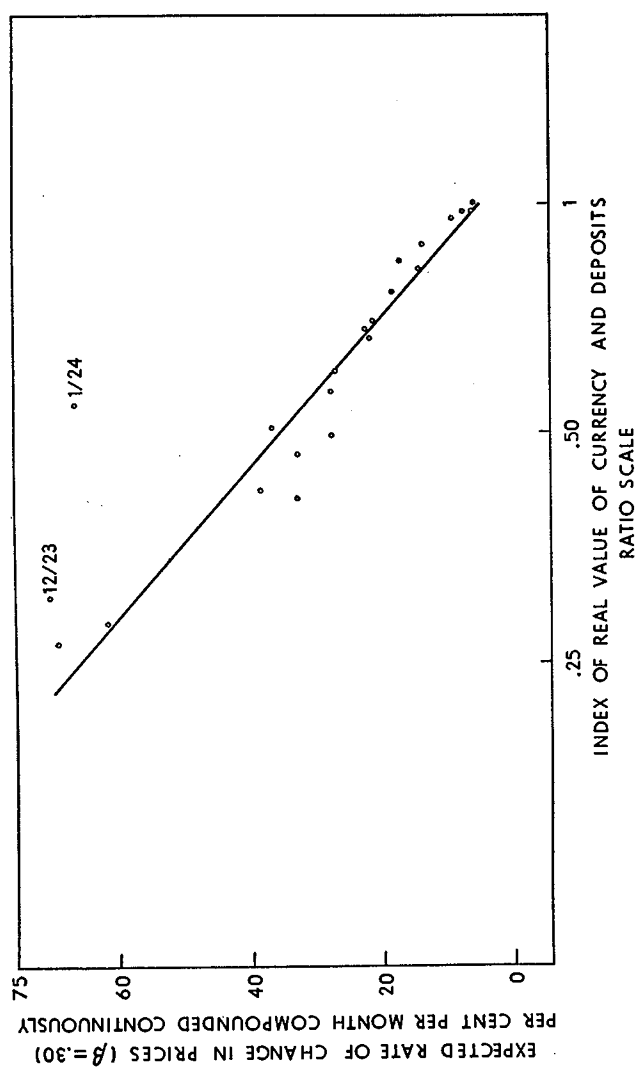

FIG. 13.—Poland—scatter diagram of end-of-month expected rates of change in prices and indexes of real value of hand-to-hand currency and bank deposits, and regression line (α = 2.30), April, 1922, to January, 1924. (Points excluded from calculation of regression line are dated.)

in time of hyperinflation. The main concern is to prevent balances of future purchasing power from completely vanishing. Therefore, the effect of issuing a stable-valued currency along with a depreciating one is to hasten the liquidation of balances composed of the latter. It is difficult, however, to find evidence of this effect. As for foreign bank notes, the quantity in circulation during the hyperinflations is not known. It can only be assumed that they could always be held and that their quantity in circulation increased gradually as the rapidly diminishing value of the domestic currency encouraged their use. If so, their gradual increase in circulation does not disrupt the continuity of the data but does make α higher than it would be if they could not be obtained. The quantities of chervontsi issued in Russia during 1922–23 are known. The magnitude of their effect on the real value of rubles is difficult to judge. In any event, real cash balances of rubles were not comparatively low in Russia during the last part of the hyperinflation, as the introduction of the chervonets may suggest. There are apparently no figures on the circulation of the tax pengö issued in Hungary in 1946, and so there is no way to determine the magnitude of its effect on the value of the regular pengö.

Other errors in the estimates arising from sources of an economic nature are probably minor. The main variable that was neglected in formulating the demand function for real cash balances is real income. Conceivably, changes in real income could have large effects on the balances. Such changes, however, are probably not a source of much error in the estimates. For some countries there are annual indexes of production and agricultural output to indicate the change in real income. These indexes have small movements compared with the fluctuations in real cash balances, at least until prices rise drastically in the last stages of hyperinflation. A semiannual index of output for Germany declines in the final stage of hyperinflation by as much as one-third in less than half a year.[12] Even so, an adjustment of the data to take account of real-income changes in Germany that assumes income elasticity of demand for the balances to be unity has little effect on the parameter estimates. This effect largely offsets that of the adjustment to include deposits. For the other countries, indexes of output suggest either no great change in real income during the months of hyperinflation or conflicting tendencies from which no conclusions can be drawn.

12. In so far as this drop in real income resulted from measures intentionally adopted to restrict the use of money, there need be no further effect on desired real cash balances.

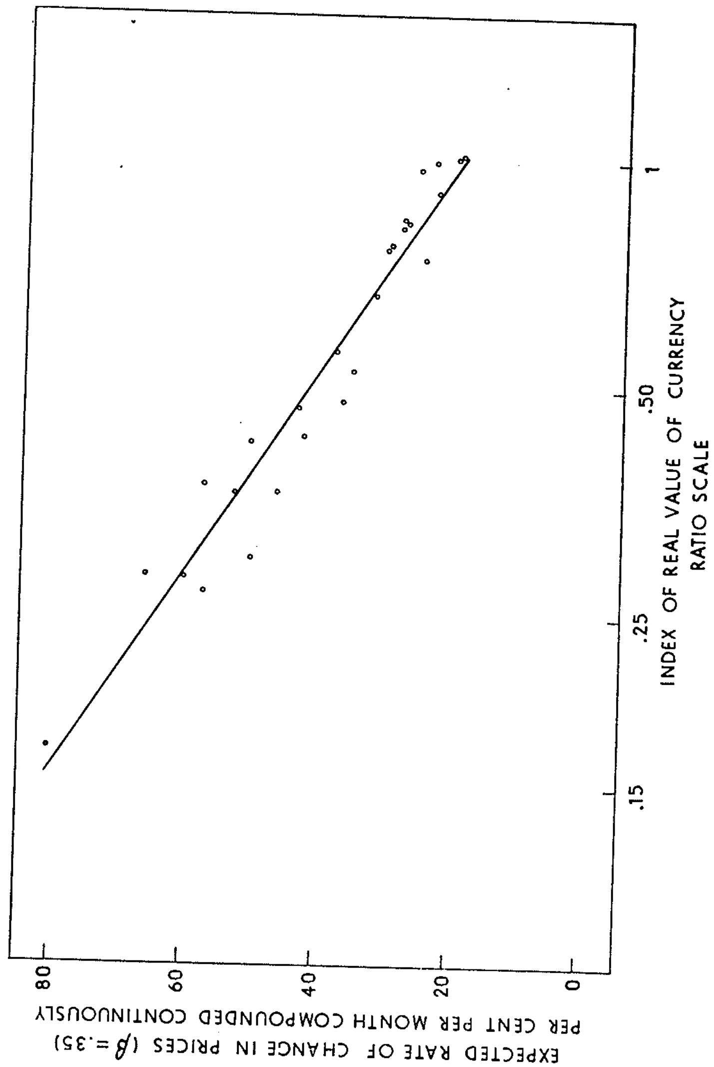

FIG. 14.—Russia—scatter diagram of end-of-month expected rates of change in prices and indexes of real value of hand-to-hand currency, and regression line (α = 3.06), December, 1921, to January, 1924.

3. OBSERVATIONS THAT DO NOT FIT THE REGRESSIONS

The periods covered by the statistical analysis exclude some of the observations near the end of the hyperinflations. The excluded observations are from the German, Greek, and second Hungarian hyperinflations, and they are dated in Figures 8–14. (Some of the points on the scatter diagram for the first Hungarian hyperinflation are also dated, but they were not excluded from the statistical analysis.) All the excluded observations lie considerably to the right of the regression lines, and their inclusion in the statistical analysis would improperly alter the estimates of α and β derived from the earlier observations of the hyperinflations.

One or both of two hypotheses could explain these observations with a much higher level of real cash balances than equation (2) implies. The first one is that in hyperinflation rumors of currency reform encourage the belief that prices will not continue to rise rapidly for more than a certain number of months. This leads individuals to hold higher real cash balances than they would ordinarily desire in view of the rate at which prices are expected to rise in the current month. As long as currency reform remains improbable for the near future, individuals adjust their real cash balances according to the rate at which they expect prices to change for some time. When they believe that the current rate at which they expect prices to rise will not last indefinitely, they are less willing to incur certain costs involved in keeping their balances low. For example, individuals would be unwilling to invest in risky equity stocks or secondhand durable goods whose principal attraction consists in the lack of better alternatives, unless there appeared to be no end to currency depreciation. Firms would not lease warehouses to maintain high inventory stocks as a substitute for cash assets, if the lease were expected to far outrun the rise in prices. Thus, when hyperinflation is expected to end in the near future, real cash balances will gradually rise, even though the expected rate of change in prices for the period preceding the end of hyperinflation remains constant or increases.

The second hypothesis is that the function that determines the demand for real cash balances does not conform to equation (2). This hypothesis suggests that the regression function shown on the scatter diagrams would fit the observations better if it curved upward on the left. To be consistent with the data, this hypothesis requires that all observations that lie to the right of the linear regressions shall fall in order along some curved regression function or, equivalently, that the value of α shall decline as the expected change in prices increases.[13]

13. It is conceivable, of course, that these high levels of real cash balances could be explained by more radical revisions of the model than are considered in the text. For

It would be difficult to test these two hypotheses, and I shall not attempt to do so. But it is not difficult, and is certainly worthwhile, to see to what extent they are consistent with the data plotted on the scatter diagrams. The evidence from these diagrams can be summarized briefly.

The Austrian data (see Fig. 8) do not support either of the two hypotheses. All the observations lie close to a linear regression line. The Russian data (see Fig. 14) only slightly support the two hypotheses. The four observations that lie substantially to the right of the upper half of the linear regression line refer to the last four months of hyperinflation. If these four observations were lowered to take into account the effects of expectations of currency reform, then all the observations might lie near a regression function that curved upward slightly. The Hungarian data after World War I (see Fig. 11) do not seem to support the two hypotheses. However, the four observations that lie to the right of the upper half of the regression line may show a high level of real cash balances because of expectations of currency reform. Figure 4 showed that the observations for August, September, and October, 1923, refer to a period in which the rate of change in prices fell sharply. It is possible that during these three months and the following month, November, there was an expectation that hyperinflation would soon end. A similar expectation may have affected the observations to the right of the lower half of the regression line. August through October, 1922, was a period of falling rates of change in prices. The difficulty of explaining the Hungarian data after World War I in this way is that some of the data from the other hyperinflations are inconsistent with the same explanation. In the Polish hyperinflation, for example (see Fig. 6), the rate of change in prices declined from February to March, 1923, but the data for this period indicate that the estimated level of real cash balances was *not* above the actual level for the entire period.

Observations near the end of the other hyperinflations lie at varying distances to the right of the regression lines, and regression functions that

example, new currency could be issued at such a high rate that people could not prevent the real value of their balances from rising even though expenditures rise at an increasing rate. However, a lag in the adjustment of actual real cash balances to their desired level, like that expressed by equation (1) in n. 2, fails to remedy the limitations of the model in the final months of hyperinflation. The lag suggests that the balances should decline when their actual level exceeds the desired level. In fact, they frequently rose still more in the ending months when the model indicates they already exceeded their desired level by large amounts. The failure of this lag to account for the facts reinforces the supposition that the first hypothesis in the text, while it cannot be directly supported, is the main explanation of these limitations.

curved upward would not fit them closely. However, expectations of currency reform may account for these observations. If they were adjusted for the effects of these expectations, it is possible that curvilinear regression functions would then fit the observations better than linear ones. The German hyperinflation provides a good example of this possibility (see Fig. 9). Prices had almost stopped rising in Germany by the middle of November, 1923. (This explains why real cash balances in the middle of that month were relatively so high.) The horizontal distance between the regression line and observations for the three preceding months is progressively greater for the months nearer to the end of hyperinflation. The configuration of these observations is thus consistent with an increasing effect on real cash balances of expectations of currency reform. If taking this effect on the balances into account shifted the points for August, September, and October, 1923, slightly to the left, there is some curvilinear regression function that would fit the data for these three months fairly well.

Similar remarks apply to the later observations for the Greek and Hungarian hyperinflations after World War II and the Polish hyperinflation after World War I (see Figs. 10, 12, and 13). The horizontal distance between the regression line and observations for the later months in these three hyperinflations is progressively greater for the months nearer to the end of the hyperinflations, which, according to the first hypothesis, suggests an increase in the effects of expectations of currency reform. Nevertheless, the observations from these hyperinflations, as well as those from the German one, are not inconsistent with the second hypothesis. Regression functions that curved upward slightly would fit the observations better than linear ones. The scatter diagrams indicate that the functions should begin to curve upward for an expected percentage rate of change in prices per month (compounded continuously) greater than 90–100, except for the Polish hyperinflation where the function should curve upward for a rate greater than 60–70.

The preceding discussion does not provide a direct test of the two hypotheses advanced above, but it does suggest possible revisions to make equation (2) more consistent with the data. Whether these revisions stand up in the light of all the relevant evidence has yet to be confirmed. The first is that expectations of currency reform acted to raise real cash balances near the end of hyperinflation. It is difficult to infer the extent to which this revision is required, because the second revision is also consistent with the data. What the second revision comes to is the possibility that the value of α falls at the higher levels reached by the expected change in prices.

4. INCREASES IN THE COEFFICIENT OF EXPECTATION

Figures 1–7 reveal that the residual errors by which estimated real cash balances differ from the actual levels have marked serial correlation. It is statistically significant at less than the .05 level in all hyperinflations except the second Hungarian.[14] And that one is probably not significant only because so few months were included in the regression. The regressions essentially involve fitting difference equations, in which uncorrelated random disturbances typically produce serial correlation, and its appearance in the results is not surprising. However, some, if not nearly all, of it can be attributed to shifts over time in the true value of the coefficient of expectation. Even if these shifts were purely random, the expected price change, being a sum of terms that contain past values of the coefficient (see eq. [6]), would include a component that involved the sum of successive values of a random variable. As is well known, such a sum will show positive serial correlation.

But random movements in the coefficient account for only part, probably a small part, of the serial correlation, because the evidence indicates that the coefficient does not shift in a purely random fashion but tends to increase over time. The index of real cash balances estimated from the regression functions tends to register in the early months a larger response to fluctuations in contemporaneous price changes than the actual index does, and in the later months a smaller response (see Figs. 1–7). This tendency implies that the estimated coefficients are too high for the early months and too low later on.[15] The tendency stands out clearly in five of the hyperinflations when they are divided into two parts: the Austrian, divided at June, 1921; the Greek, divided at November, 1943; the first Hungarian, divided at August, 1923; the Polish, divided at December, 1922; and the Russian, divided at April, 1923. Of the other two, the second Hungarian was too short for this tendency to show up, and the German (discussed in a moment) seems to show, if anything, the opposite tendency.

This tendency in five of the hyperinflations is confirmed by computed estimates of the coefficient, which have a higher value when the regression

14. For a description of the test for serial correlation used see J. von Neumann, "Distribution of the Ratio of the Mean Square Successive Difference to the Variance," *Annals of Mathematical Statistics*, XII (December, 1941), 367–95. For the significance tables see B. I. Hart and J. von Neumann, "Tabulation of the Probabilities for the Ratio of the Mean Square Successive Difference to the Variance," *Annals of Mathematical Statistics*, XIII (June, 1942), 207–14.

15. This cannot be due to a rising value of α, for then we should observe that points on the scatter diagrams, plotted for a given coefficient, curved downward to the left. Actually the points curve upward, if they curve at all.

fit includes only the later months than when the fit includes all the months. Table 6 gives estimates that were derived by fitting the regression to an arbitrary number of the later months. The three hyperinflations represented in the table were the only ones with an estimate for the later months that exceeded the estimate for all months by .05 or more. For these three, the last column in the table shows that the estimate for the later months as a percentage of the estimate for all months was 150 to over 200 per cent, that is, the former *exceeded* the latter by 50 to over 100 per cent. For Austria and Greece, computations not shown suggest that the estimate for the later months also exceeded the estimate for all months by as much as 50 per cent. However, for these two countries the

TABLE 6

ESTIMATES OF α AND β FOR THE LATER MONTHS OF THREE HYPERINFLATIONS*

Country	Period	Degrees of Freedom	Estimate of: α (Months)	Estimate of: β (Per Month)	Estimate of β for Later Months as Percentage of Estimate for All Months†
Hungary...	Feb., 1923–Feb., 1924	10	4.72	.15(±.05)	150
Poland.....	Aug., 1922–Dec., 1923	14	1.72	.45(±.05)	150
Russia.....	Apr., 1923–Jan., 1924	7	2.37	.80(±.10)	230

* Method of estimation and sources are the same as for Table 3.

† Preceding column in this table as a percentage of the corresponding estimates of the coefficient for all months shown in Table 3.

percentage difference between the two estimates could not be judged precisely without more exact estimates than have been computed. (The period covered by the regression for Hungary after World War II was too short to detect any change in the coefficient.) These estimates are of average values of the coefficient in the periods covered. If the coefficient was increasing within the periods, the results indicate that its percentage increase from the beginning to the end of these hyperinflations was at least greater than 50 per cent. Based on so few observations, these estimates are certainly subject to considerable error,[16] but they do suggest that increases in the coefficient over time may have been substantial.

16. It is likely that the percentages in the last column of Table 6 are somewhat too high. Errors of measurement in the index of prices, which is used in the index of real cash balances and of current change in prices, will produce a spurious correlation between these two and thus tend to bias estimates of the coefficient upward. This bias will be

For Germany, on the other hand, an estimate for the later months was about the same as the estimate for all months. The failure of the German data to indicate that the coefficient increased is puzzling in view of the other evidence. It is possible that the estimate for all months is too high for the early months even though the regression appears to fit equally well throughout. This result could occur if the extreme fluctuations in real cash balances for the later months dominated the regression, as appears probable only for Germany, and the relatively stable level of the balances from September, 1920, to July, 1921, contributed little. It is not clear, therefore, that the coefficient did not increase, though the amount could not have been very much.

That the coefficient should generally increase over time is certainly reasonable. It would be strange indeed if the public did not become conditioned by recent inflation to conclude that a fresh spurt in prices was not temporary but foretold more intense inflation to come. The assumption about expectations made in Section III pictured them as determined by a trial-and-error process, whereby expected price changes were adjusted to actual changes at a rate proportional to their difference. That assumption seems to provide an adequate approximation to the facts and sufficed for the model. But there is no reason to stop there. It is also reasonable to assume that with experience the public should make these adjustments more rapidly. We have found this assumption to be generally consistent with the data, though the effect of increases in the coefficient on the level of real cash balances is small compared with the major fluctuations that the balances undergo. Nevertheless, this assumption is important, for it ought to account for the level of the coefficients at the beginning of hyperinflation. The coefficient should be higher when past price rises have been more prolonged or larger.

Table 7 compares the level of the coefficient with the price changes preceding hyperinflation. The figures reveal some relation, though it is far from precise, between the level of the coefficient and the extent of past inflation. To what degree errors in the estimates of the coefficient account for the lack of precision in this relation is difficult to say. The very wide confidence intervals for the estimates (see Table 3) suggest that too

greater the shorter is the time period covered by the regression, because the variance of errors in the series, which is unrelated to the length of period covered, is then a larger fraction of the total variance, which is larger for longer periods. The amount of bias is difficult to judge. Indirect evidence that it might be fairly sizable is presented in the next section. But, since over-all fluctuations in the series in the periods covered are so large, it is unlikely to account entirely for all the increase in the coefficient over time shown by the estimates.

much reliance should not be placed on them. Yet it is possible to make sense of these figures "after the fact" and to reach some tentative conclusions. The low Austrian coefficient corresponds to relatively mild inflation in the preceding period, and the high Russian one to vigorous war and postwar inflation. For these two the relation seems clear cut. For the others it is less so. If we disregard Germany for the moment, we can attribute the high Polish coefficient to the large rise in prices over the entire

TABLE 7

COMPARISON OF EXPECTATION COEFFICIENT WITH PRICE CHANGES PRECEDING HYPERINFLATION*

Hyperinflation	Coefficient of Expectation	Price Change Preceding Beginning Month of Regressions		
		Since Prewar (1913–14 or 1937–39) (Per Cent)	One Year (Per Cent)	Two Years (Per Cent per Year)
Austrian	.05	6,500	94	82
Hungarian I	.10	17,300	314	N.a.
Hungarian II	.15	10,400	3,460†	520‡
Greek	.15	9,750	424	940§
German	.20	1,400	206	156
Polish	.30	75,000‖	241#	N.a.
Russian	.35	28,800,000	1,610	990

N.a. = Not available.
* For sources see Appendix B.
† 13 months at an annual rate.
‡ 25 months at an annual rate.
§ 22 months at an annual rate.
‖ Wholesale price index. A cost-of-living index gives 58,500 per cent.
\# Cost-of-living index. A wholesale-price index linked to an index of retail food prices gives 137 per cent.

period since 1913–14, and the lower coefficient for Greece and Hungary to a smaller rise over that period, in spite of their larger figures for the year immediately preceding hyperinflation. Apparently the rapid inflation preceding the second Hungarian hyperinflation was also too short to affect the coefficient much. Thus Hungary's experience with hyperinflation the first time does not seem to have influenced behavior appreciably the second time it occurred. This result suggests that memories of currency depreciation fade away in a quarter-century, at least the first time, and that speed of adjustment depends on how prolonged recent experience with inflation has been.

The medium-sized German coefficient at first sight seems out of line with the others in view of the mild inflation there during and immmediately after the war. The comparatively high level of this coefficient conceivably results from the advanced development, both financial and industrial, of the German economy at that time. Depreciation in the value of money may be more apparent, and its effects sooner felt, in an economy with proportionately less agriculture. Industrial firms and workers rely on the value of money in selling their products and services; in less urbanized economies, money is used much less. While these facts were also likely to make the coefficient for Austria higher relative to the others, its value was low primarily because hyperinflation there was milder and much shorter, ending just about the time the others, after World War I, were getting started.

By and large, it appears that the coefficient usually increased in response to continual inflation. This increase probably accounts for most of the serial correlation in the regression residuals. Whether the coefficient increased steadily or suddenly jumped to a higher level during the hyperinflations cannot be determined, since the data cover too short a period of time for gradual changes to show up. However, there is some evidence on this question in the behavior of real cash balances in the months preceding those included in the regressions. This evidence points to rather sudden increases in the coefficient from very low or even negative levels. A negative coefficient in the initial stages of inflation would indicate that the public expects prices to decline eventually. In Germany the balances rose throughout World War I, even though prices were continually going up. Shortly after the war the balances fell sharply by far more than the model can account for. Moreover, there is no indication that real income fell sharply at the same time. In Austria and Poland as well, the balances rose in the months preceding hyperinflation, despite continual inflation. There was also a rapid fall in real cash balances in these two countries shortly after the war. This fall seems to reflect a belated adjustment to inflation after a long period in which prices were expected to return to their prewar level.[17] (The figures on money do not go back far enough in time for Hungary during both world wars and for Greece during World War II to observe whether the same phenomenon occurred in these countries.)

17. In Russia, however, real cash balances fell continuously throughout World War I, even though the rate of change in prices fluctuated fairly closely about the same average level from the middle of 1917 to the end of 1921. The decline in real income and political uncertainties during the same period probably account for this steady fall in the balances.

The erratic changes in the balances preceding hyperinflation imply that the coefficient of expectation makes a sudden upward shift, presumably when the public first loses confidence in the prospect for stable or lower prices.[18] After that, if inflation persists and speeds up, the coefficient (as we have seen) appears to become higher, possibly again by sudden shifts rather than gradual increases. While the exact timing and pattern of these shifts remain unexplained, they bear at least some relation to both the rate and the duration of past inflation. Duration is important, for there is no one rate acting as a threshold that, if pierced, causes the coefficient to jump up. Thus even very mild inflation, if continued long enough, ought eventually to produce a fairly high coefficient. The further question about how high it might go cannot be given a definite answer. After it reaches a certain level this question might seem to lose significance. Russia's coefficient reached close to .8, which gives an average lag in expected behind actual price changes of only one and a quarter months. At that point expectations are extremely responsive to current price movements, and there is not much room for shortening the lag. Yet the exact value of the coefficient at high levels becomes very important in determining whether the price rise is stable or not, that is, whether the rise becomes self-generating. This question forms the subject matter of the next section. The evidence presented there suggests that the coefficients did not, and perhaps never would, reach levels that produce self-generating price rises.

An increasing coefficient of expectation produces an upward bias in estimates of α, because the constant value of the coefficient used in computing the expected rate of change in prices falls short of the true value by increasing amounts for successively later months of hyperinflation. The expected rate of change is then understated by a greater amount for successively later months, since the actual rate of change tends to increase. This understatement biases estimates of α upward.[19] Such bias would partly account for the fact that the estimates of α in Table 6 are lower than those in Table 3. The former would be the more accurate estimates if they were derived with the better estimates of the coefficient. However, the upward bias in estimates of the coefficient (described in n. 16), which bias is larger when the estimates are based on shorter

18. See A. J. Brown, *The Great Inflation, 1939–51* (London: Oxford University Press, 1955), pp. 190–92.

19. It should be pointed out that for this reason an increase in the coefficient is not responsible for pulling observations for the later months of hyperinflation above the regression lines. This could only result from a fall in the coefficient. These high observations can indeed be interpreted as a fall in the coefficient.

periods, also makes the estimates of α in Table 6 somewhat lower than those in Table 3. The true value, therefore, lies somewhere in between the estimates in these two tables, but which estimate is more accurate cannot be determined.

V. The Stability of Equilibrium in Hyperinflation

The coefficient of expectation determines the speed with which individuals revise their expectations of the rate of change in prices. The amount by which they change their real cash balances in accordance with their revised expectations depends on the elasticity of their demand for the balances, which is proportional to the value of α. But the total reaction of a given series of changes in the quantity of money on the price level, and therefore the condition for the stable moving equilibrium of prices, depends on the product of the two parameters.[20] This is proved by deriving the condition for stability from the model.

In a self-generating inflation, a small rise in prices causes such a "flight from money" that prices go up more than in proportion to the initial rise. This cannot occur if, for any change in the price level, its rate of change moves in the opposite direction. Then any increase in prices is dampened by a fall in their rate of rise. This condition for stable moving equilibrium is expressed by

$$\frac{\partial\left(\dfrac{d\log P}{dt}\right)}{\partial P}<0.$$

The condition can be derived for hyperinflations from equations (2) and (5). If the indexes of P and M are set at unity in the same month in which E is zero, γ in equation (2) vanishes, and the two equations can be written as follows:

$$\log\frac{M}{P}=-\alpha E, \tag{2a}$$

$$\frac{dE}{dt}=\beta\left(\frac{d\log P}{dt}-E\right). \tag{5a}$$

20. This dependence means that, for given data, the lower is the value of β, the higher is the value of α. As β becomes smaller, the individual weights for the average of past rates of change in prices become more equal, and the weighted average produces a smaller variation in the values of the expected changes. Consequently, a lower β implies a higher α, because the smaller variations in the expected rates of change in prices have to explain the same given variations in real cash balances.

The logarithms have the base e. To reduce the two equations to a single relation between observable variables, equation (2a) is first differentiated with respect to time, which gives

$$-\frac{1}{\alpha}\left(\frac{d\log M}{dt}-\frac{d\log P}{dt}\right)=\frac{dE}{dt}. \tag{2a'}$$

Substituting (2a) and (2a') into (5a) produces the following relation between P and M only:

$$\beta\,(\log P-\log M)=\frac{d\log M}{dt}-(1-\alpha\beta)\frac{d\log P}{dt}. \tag{12}$$

From this it follows that

$$\frac{\partial\left(\dfrac{d\log P}{dt}\right)}{\partial P}=\frac{-\beta}{1-\alpha\beta}\left(\frac{1}{P}\right), \tag{13}$$

since M is independent of P, and, therefore, partial derivatives of M and of its rate of change with respect to P are zero. Since P is never negative, it is apparent from (13) that prices are in stable equilibrium if $\alpha\beta$ is less than unity and that they are not so if $\alpha\beta$ is greater than unity.[21]

What the stability or instability of equilibrium implies about the course of hyperinflation can be inferred from the function that determines prices through time. The function can be derived by solving equation (12) for log P. This equation is a linear first-order differential equation in log P and t, where M is assumed to be a function of t, independent of P. The solution of log P in terms of t, assuming for the moment that $\alpha\beta$ is not unity, can be written as

$$(\log P)_t=(\log M)_t+H\,e^{-\beta t/(1-\alpha\beta)}+\frac{\alpha\beta}{1-\alpha\beta}\,\frac{\left(\displaystyle\int_{-T}^{t}\frac{d\log M}{dx}\,e^{\beta x/(1-\alpha\beta)}\,dx\right)}{e^{\beta t/(1-\alpha\beta)}}, \tag{14}$$

where $-T$ is an arbitrary number to be specified and H is the constant of integration. H is determined by specifying the values of the variables at a particular time. Without any loss of generality we can assume that, prior to the time when $t=0$, M and P were unity and that, at $t=0$, M began to increase at the rate $(d\log M/dt)_0=r_M$ and that prices began

21. The limiting case, when $\alpha\beta$ is unity and the partial derivative in (13) is zero, turns out to be equivalent to stable equilibrium (see eq. [16]). I am indebted to Professor Jacob Marschak for drawing my attention to this way of formulating the criterion of the stability of equilibrium.

to increase at the rate $(d \log P/dt)_0 = r_P$.[22] With these initial conditions, equation (14) takes on the following form:

$$(\log P)_t = (\log M)_t + \frac{\alpha\beta}{1-\alpha\beta} \frac{\left(\int_0^t \frac{d \log M}{dx} e^{\beta x/(1-\alpha\beta)} dx\right)}{e^{\beta t/(1-\alpha\beta)}} + \left(\frac{r_M}{\beta} - \frac{1-\alpha\beta}{\beta} r_P\right) e^{-\beta t/(1-\alpha\beta)} . \tag{15}$$

The course of inflation depends crucially on the value of $\alpha\beta$, the "reaction index."

When the reaction index is less than unity, the terms following the integral in equation (15) approach zero as t increases, so that prices eventually depend on two factors: the quantity of money and the integral, which is an exponentially weighted average of the past rates of change in the quantity of money. The logarithm of real cash balances thus becomes proportional to the negative of this integral.

When the reaction index equals unity, the last term in the original equation (12) drops out, and the level of prices can be derived immediately. We have

$$\log P = \log M + \frac{1}{\beta} \frac{d \log M}{dt},$$

or, since $\alpha\beta = 1$,

$$\log \frac{M}{P} = -\alpha \frac{d \log M}{dt}. \tag{16}$$

Real cash balances in this case are related to the current rate of change in the quantity of money. Adjustments in the balances are not influenced by the past costs of holding money.[23]

22. It is necessary to specify that $r_P > r_M$. In the identity

$$\frac{d \log \frac{M}{P}}{dt} \equiv \frac{d \log M}{dt} - \frac{d \log P}{dt},$$

the assumed initial rates can be substituted in the right-hand side, so that

$$\left(\frac{d \log \frac{M}{P}}{dt}\right)_0 \equiv r_M - r_P .$$

Since real cash balances fall when prices and the quantity of money are rising, it is necessary that each side of the preceding identity be negative.

23. The expected change in prices may still lag behind the actual change in prices unless β (or $1/\alpha$) is sufficiently large. In fact,

$$\left(\frac{d \log P}{dt} - E\right) = \frac{1}{\beta} \frac{d^2 \log M}{dt^2},$$

or, since $\alpha\beta = 1$, the difference also equals $\alpha(d^2 \log M/dt^2)$. This means that, when the quantity of money increases at an increasing rate, the expected rate of change in prices lags behind the actual rate. Actual real cash balances are then never as low as they would be if individuals could foresee the actual rates. This result is a consequence of the form of equation (5a) in the model, which shows how expectations are revised per period of time. Higher orders of revision of expectations are not taken into account. In fact, however, when prices actually rise at increasing rates for some time, individuals appear to revise their expectations with increasing rapidity. Equation (5a) nonetheless is adequate to account for most of the data.

When the reaction index is greater than unity, the terms following the integral in (15) are positive. The integral term is negative. The exponential weights in the integral are now turned around, so that the largest weight is given to the past rates of change in money. The integral term eventually stays relatively constant,[24] and the price level depends mainly on the logarithm of money and the two terms following the integral. These two terms following the integral rise at an exponential rate. Therefore, $\log P/M$ eventually rises at roughly an exponential rate.

The solution of equations (15) and (16) with the quantity of money rising at a constant percentage rate will illustrate the three cases of a reaction index less than, equal to, or greater than unity. Before $t = 0$, suppose that $\log P$ and $\log M$ are zero and, when $t \geq 0$, that $\log M = r_M t$. The equations for prices, the change in prices, and real cash balances are then as follows for $t \geq 0$:

$$(\log P)_t = r_M t + \alpha r_M - \frac{1-\alpha\beta}{\beta} (r_P - r_M) e^{-\beta t/(1-\alpha\beta)}, \tag{17a}$$

$$\left(\frac{d \log P}{dt}\right)_t = r_M + (r_P - r_M) e^{-\beta t/(1-\alpha\beta)}, \tag{17b}$$

$$\left(\log \frac{M}{P}\right)_t = -\alpha r_M + \frac{1-\alpha\beta}{\beta} (r_P - r_M) e^{-\beta t/(1-\alpha\beta)}. \tag{17c}$$

24. This statement is true as long as $\log M$ rises at less than an exponential rate. If $\log M$ were to start rising at an exponential rate greater than $-\beta/(1-\alpha\beta)$, the integral term takes on an increasing negative value. The price level would have to become infinite immediately. (This can be done by setting $r_P = \infty$). Otherwise, sooner or later $\log P$ would begin to fall even though $\log M$ were rising. This is inconsistent with our premises. There is no rising level of prices in this equation that does not immediately rise to infinity and yet is consistent with a $\log M$ that rises at an exponential rate greater than $-\beta/(1-\alpha\beta)$.

The situation in the intermediate cases, in which $\log M$ rises exponentially at a rate less than or equal to $-\beta/(1-\alpha\beta)$ can be found by referring to equation (15). The fact that, when $\log M$ rises at a great enough rate, $\log P$ must go to infinity, if it is to increase at all, arises more from the limitations of the model than from the realities of hyperinflation. The model is a first approximation to those realities only.

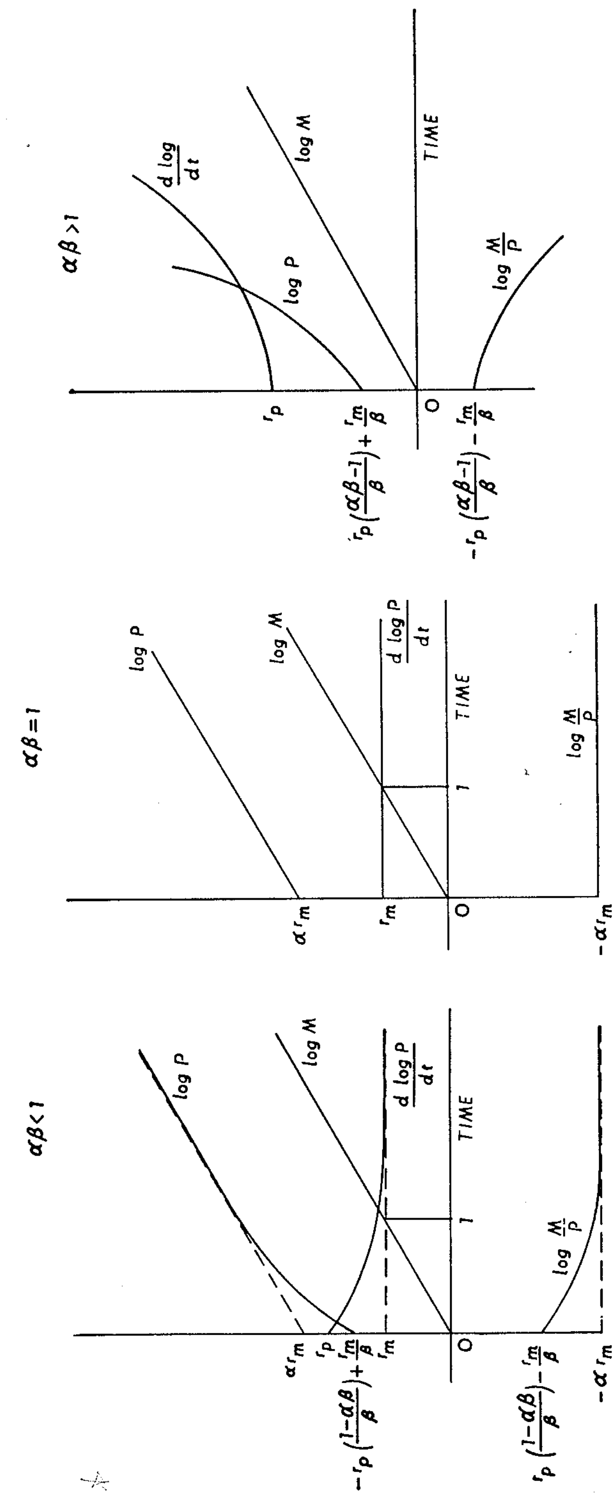

FIG. 15.—Three cases of model for constant rate of increase in quantity of money

These equations are plotted in Figure 15. When the index equals unity, the last term in (17*a*) and (17*c*) becomes zero. For this case, direct substitution of the assumed changes in the quantity of money in (16) yields the same result.

Table 8 gives estimates and confidence intervals of the reaction indexes. Only the German and Russian hyperinflations, of those examined, seem to have an index that is greater than unity. Except for the Austrian, Greek, and second Hungarian hyperinflations, however, the confidence intervals are too wide to conclude that the true value of the index is less than unity.

TABLE 8

REACTION INDEXES*

Country	Estimate of $\alpha\beta$	Confidence Intervals for $\alpha\beta$†
Austria	.43	.31 – .66
Germany	1.09	.92 –1.26
Greece	.61	.32‡– .85
Hungary (after World War I)	.87	.42‡–1.27
Hungary (after World War II)	.54	.47 – .76
Poland	.69	.39 –1.04
Russia	1.07	.94 –1.20

* For method of estimation see Appendix A.
† The confidence level for the intervals is .90. See Appendix A.
‡ End of confidence interval lies beyond the value shown. See n. § to Table 3.

Increases in the coefficient of expectation over time (for which supporting evidence was discussed under subsection 4 in the last section) do not necessarily produce larger reaction indexes for the later months than the indexes for all months listed in Table 8. As was pointed out, when a constant coefficient is used, these increases bias estimates of α upward and so those in Table 8. Taking these increases into account places only the Russian index above unity. Based on the estimates in Table 6 for the later months, the index for Russia becomes 1.90; but for Poland it becomes only .77, and for Hungary I it falls to .71. Computations not shown for the others indicate that the index does not come up to unity; estimates for the last half of the German hyperinflation reduce its value to .96. This evidence implies that only the Russian index clearly exceeds unity. For the later months the German index seems to stand slightly below; and the others, considerably below.

But this estimate of the Russian index may spuriously exceed unity, because there is another bias making estimates of the indexes too high that has to be considered, even though we cannot exactly calculate its

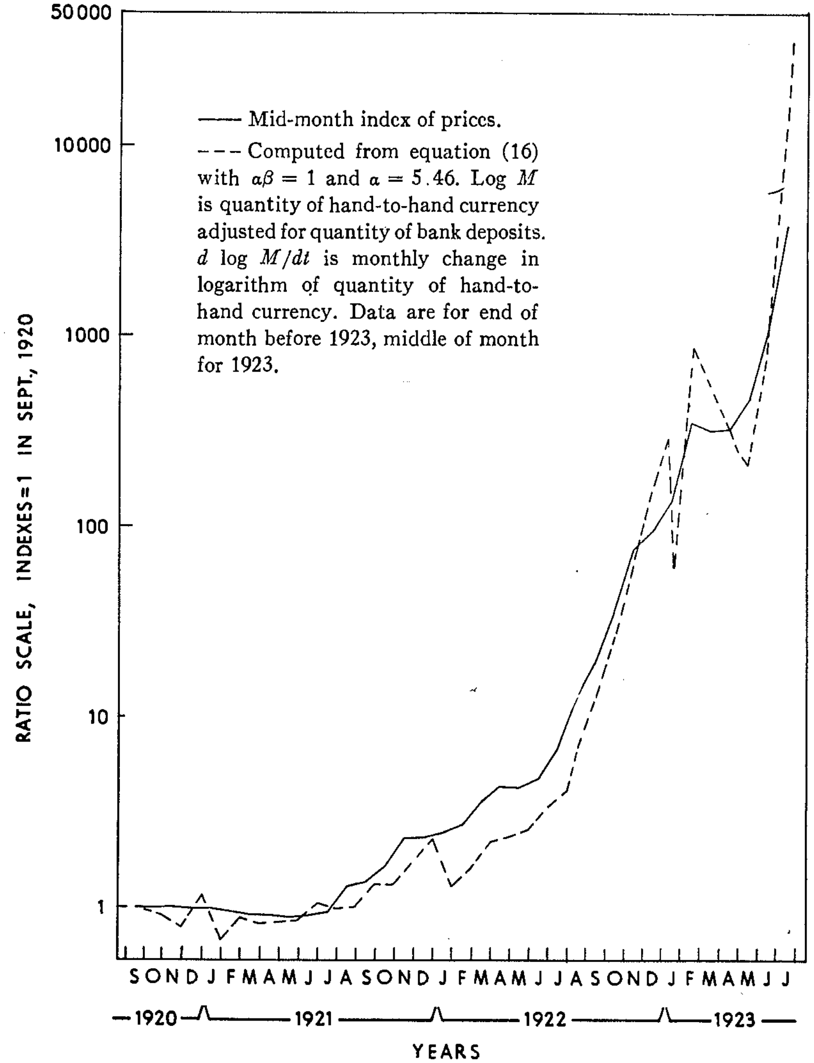

FIG. 16.—Index of prices generated by model and actual index of prices for Germany, September, 1920, to July, 1923.

effect. It is the upward bias in the estimated level of the coefficient of expectation (see n. 16). While this bias makes estimates of α too low, it does not in fact make them proportionately lower; correction for it would mean that the reaction indexes should be lowered somewhat. Its effect on estimates for the whole period of hyperinflation is probably small, but on those for a short period the effect could be substantial. Whether this bias is sufficient to account entirely for the fact that the Russian index exceeds unity is uncertain. There are other minor sources of bias, some operating in the other direction, that might be important when small amounts are crucial.[25] Thus while the true values of the others would all appear to lie below unity, the existence of many sources of possible error places even this conclusion in doubt. The estimate of the German index, being so close to unity, is especially doubtful.

Indirect evidence that the Russian index, as well as the German and others, are actually not greater than unity is provided by the estimated level of prices that can be computed from equation (15). Using the estimated value of the parameters for Germany and Russia from Table 8 in equation (15) to estimate the actual level of prices gives very poor results. Moreover, any other values of the index greater than unity are also unsatisfactory. On the other hand, values equal to, or slightly lower than, unity for Germany and Russia give more reasonable results, as indicated for the German hyperinflation by Figure 16, which compares the actual level of German prices with that generated by equation (16).

25. The two most important sources, which produce downward biases in the parameters, are described below. It is hard to believe that either of them could be very important, even though there is little basis on which to compute the exact amounts involved.

a) The exclusion of illegal currency issues from the series on money undoubtedly makes these data progressively too low by greater percentage amounts, because the incentives to issue without authorization were greatest in the peaks of hyperinflation coming at the end (see the discussion under subsection 1 in Sec. IV). By understating the series for real cash balances, this exclusion makes the estimates of α too low.

b) Least-squares estimates of the coefficient in a first-order difference equation are biased downward. This results from the failure of these estimates to take account of the serial correlation in the regression residuals, which is produced by difference equations even when the original random variable is not autocorrelated (see T. C. Koopmans [ed.], *Statistical Inference in Dynamic Economic Models* [New York: John Wiley & Sons, 1950], pp. 365–83). The regression function (11) is a difference equation of the Tth order. To what extent the estimates in Tables 3 and 8 are subject to the bias found for first-order equations is not known. Such bias, which has nothing to do with any of those mentioned heretofore, implies that estimates of the two parameters are too low. Conceivably it accounts for some of the actual serial correlation in the regression residuals, which was discussed under subsection 4 in Section IV and attributed to increases in the coefficient.

The reason why parameter values estimated from the model in one form are not always appropriate to it in another form is that one introduces biases not present in the other. While the two forms are mathematically identical, the regression functions derived from them involve different time series. For any point in time, the regression used relates the current money supply and prices to past changes in prices. The regression for the other form of the model relates current money and prices to past changes in money. While the former method of estimating the parameters does not sharply distinguish between values of the reaction index above or below unity, the latter does. Thus, even though the true index may lie below unity, the estimated value by the method used will sometimes turn out greater than unity because of random errors and bias.

This interpretation of the results seems inescapable, because a reaction index that exceeds unity is not consistent with the general characteristics of the hyperinflations. When the index exceeds unity, equation (15) implies that changes in the quantity of money, once the quantity initially rises, have very little to do with the course of inflation. The fall in real cash balances is so large that prices continue to rise under the impetus of falling balances *ad infinitum*.[26] This situation did not characterize the whole of the German and Russian hyperinflations, or the others either. Price changes did not always tend to increase at exponential rates. Yet if the reaction index were above unity, the *logarithm* of actual prices would have to rise at no less than an exponential rate before any solution of equation (16) would estimate prices with reasonable accuracy, unless perhaps there were continual changes in expectations. Even in the last part of the Russian hyperinflation, although the rate of change in prices climbed steadily, it was nowhere near exponential. In view of the actual price rises that occurred, therefore, none of the hyperinflations appears to have been self-generating.

Does it also follow that self-generating inflations are somehow impossible? There seems to be no reason why they could not occur; so far they have just not been observed. The world has seen inflations that go on for decades but do not develop self-generating characteristics. This evidence, combined with that from these seven relatively brief hyperinflations, suggests that price increases would become self-generating only after a prolonged period of hyperinflation. Even when the reaction index exceeded unity, if the percentage increase in the money supply were less than exponential, expectations would probably not sustain a percentage increase in prices at an exponential rate for very long. Prices would more likely exhibit a high degree of instability, with the percentage change in real cash balances alternately falling and rising at exponential rates depending on whether the increase in money in the immediate future were expected to accelerate or taper off. Under these circumstances it seems likely that reform of the currency would become a political and economic necessity. This could be the reason why self-generating price increases have never been observed—hyperinflation that otherwise proceeded to the stage where they began would not last long enough after to permit their detection.

26. In such a situation the inflation is self-generating in the sense that diminishing real cash balances create a continual rise in prices, which in turn leads to a continual fall in the balances. It is possible to prescribe values of the parameters so that, according to the model, even certain rates of *decrease* in the quantity of money will not stop the inflation once it gets under way. But we should not expect this solution of the model to apply. In the light of the sharp rise in the balances when a reform in the currency approaches, any diminution in the rate at which notes were issued would likely alter the prevailing expectation of a certain rate of future inflation to one of a less rapid rate, whether the reaction index were greater than unity or not. If so, the balances would rise at once if the policies of the note-issuing authorities justified more confidence in the future value of the currency. These sudden revisions in expectations cannot be accounted for in a model that predicts future prices on the basis of past changes in prices or money alone.

VI. A Note on Lags

We have seen that, if the lag in expectations in the model is too short, inflation becomes self-generating. When the coefficient of expectation is quite large, which means that expected price changes follow the actual changes without a perceptible lag, the reaction index exceeds unity. Then, once the money supply increases, the logarithm of prices begins to rise at no less than an exponential rate. Subsequent increases in the money supply only add to the rate of rise in prices;[27] a reduced rate of increase in the supply will not make the rate of price rise less than exponential, except perhaps through indirect effects on expectations. The condition for stable moving equilibrium given in Section V confirms this statement by showing that a reaction index greater than unity creates unstable conditions.

Actually, any function that relates real cash balances to the rate of change in prices without a lag violates the condition for stability. For, given such a function,

$$f\left(\frac{M}{P}\right)=\frac{d\log P}{dt}, \tag{18}$$

the condition for the stable moving equilibrium of prices requires that

$$\frac{\partial\left(\dfrac{d\log P}{dt}\right)}{\partial P}=\left(\frac{-M}{P^2}\right)f' \tag{19}$$

27. Subject to the conditions set forth in n. 24.

be less than zero, provided that M is independent of P. Since real cash balances generally fall in response to a rise in the rate of change in prices, f' is usually negative; hence this partial derivative is positive, implying instability. An increase in the rate of change in prices occurs whenever they rise. The inflation will thus continue under its own momentum, as it were.

Certainly no mild inflation exhibits the properties of instability, and the seven hyperinflations do not appear to have them either. Therefore, some lag (or other restraint on changes) affects the dependence of real cash balances on the rate of change in prices to a significant degree. The lag in expectations, which conforms to the data in most months of the seven hyperinflations exceptionally well, appears to be unusually long. Estimates of its average length (see Tables 2 and 3) range from three to twenty months. On the other hand, a lag associated with another variable in the model—the desired level of the balances—is assumed to be extremely short.[28] This section points out that these two lags cannot be empirically distinguished. The extreme length estimated for the first lag may thus result from the additional effects of the second, as well as from those of a third, also discussed below, which concerns the expected duration of any rate of rise in prices. These three seem to be the main lags that require consideration in a monetary analysis of hyperinflation.

To begin with, what can we say about a lag in the balances?[29] Suppose that the actual levels of real cash balances were not always equal to desired levels but were adjusted at a rate proportional to the difference in their logarithms:

$$\frac{d \log \frac{M}{P}}{dt} = \pi \left(\log \frac{M^d}{P} - \log \frac{M}{P}\right), \tag{20}$$

where the superscript d denotes desired levels, and π is a positive constant. This assumption is analogous to the one used to describe how the unobserved expected price changes adjust to the actual changes, except that here the actual level of real cash balances adjusts to the unobserved desired level. Suppose further that expectations involve no lag and that the desired balances depend on the actual change in prices as follows:

$$\log \frac{M^d}{P} = -\alpha \frac{d \log P}{dt}. \tag{21}$$

28. See above, n. 2.

29. The argument that follows has benefited from discussions I have had with Marc Nerlove.

Combining equations (20) and (21), we get

$$\frac{d \log \frac{M}{P}}{dt} + \pi \log \frac{M}{P} = -\alpha\pi \frac{d \log P}{dt}. \tag{12a}$$

This is identical to the reduced equation for the model derived in preceding sections, which used a lag in expectations instead. Compare equation (12*a*) with (12). The only difference is the substitution of π for β. When we treat (12*a*) as a differential equation and solve for log (M/P) in terms of $(d \log P)/(dt)$, the result is identical to the combination of equations (2) and (8), which underlies the regression function (11). The two lags thus imply the same relation between prices and money.

This equivalence of the two lags is perfectly reasonable. Individuals may well behave with only one or the other lag, but we cannot observe which one it is. Indeed, the behavior of two individuals who differed completely in this respect has identical effects on the data. One might adjust his expectations to current price changes rapidly but only slowly bring his cash balances in line with the amount desired. Another might tardily adjust his expectations but quickly change his cash balances. For both, we would observe a certain lag of changes in cash balances behind current price changes, and unless we assumed that the two lags affected the data differently,[30] we would not be able to distinguish between them.

The model can be viewed as an approximation, therefore, to a more general model involving both lags. By attaching the superscript d to real cash balances in (12*a*) and combining that equation with (20), we get the following second-order differential equation:

$$\frac{1}{(\beta+\pi)} \frac{d^2 \log \frac{M}{P}}{dt^2} + \frac{d \log \frac{M}{P}}{dt} + \left(\frac{\beta\pi}{\beta+\pi}\right) \log \frac{M}{P} = -\alpha \left(\frac{\beta\pi}{\beta+\pi}\right) \frac{d \log P}{dt}. \tag{22}$$

This, too, confounds the two lags. Interchanging β and π does not alter the equation in any way.

The model that was used is equivalent to this equation except for the first term, and what I have previously called the "coefficient of expectation" here approximates $(\beta\pi)/(\beta + \pi)$, an amalgam of two lags.[31] If

30. Suppose eq. (1) in n. 2 had $-d \log P/dt$ for its left-hand side. It would then be similar to (20) but would give a lag in the balances with implications that are not all equivalent to a lag in expectations.

31. Under the original assumption made in Sec. II that π is very large, this expression is only slightly less than β. For, as π increases, $(\beta\pi)/(\beta + \pi)$ approaches the value of β from below.

$\beta + \pi$ is fairly large or the second derivative fairly low, the first term contributes little to the relation. That the contribution of this term was fairly small in the seven hyperinflations is suggested by the good results obtained with the model used. The model describes most months quite well,[32] even though it ignores the first term. Of course, the above equation, incorporating both lags, would probably fit the data better than the model, though whether much better is questionable. Because of the many statistical difficulties as well as extensive computations involved, however, a fit using this equation has not been attempted.

Even though the coefficient of expectation in the model must be interpreted as a mixture of two lags that cannot be distinguished, it hardly seems possible that the lag in the balances could be more than a fraction as long as the lag in expectations. Once a person decides on the desired level of his balances, he can easily adjust his actual balances by spending them or by selling other assets for cash. The time required is negligible, because he adjusts his balances not so much by changing the level of his consumption over a period of time as by altering the form in which he holds his wealth. In forming his expectations, however, he may very well look far back in time in order to assess the current trend of prices. Nevertheless, there is no direct evidence on the relative importance of the two lags, and it should be understood that to some extent the estimates of the coefficient of expectation also reflect a lag in the balances and are to that extent too low as a measure of the lag in expectations alone. For simplicity of exposition, I have referred throughout only to the lag in expectations.

The estimates of α, on the other hand, are not subject to any such ambiguity, and the elasticity of demand defined by equation (4) does represent, as was intended, the percentage change in desired real cash balances with respect to changes in the expected rate of increase in prices.

Another kind of expectation that may be important, although it was not explicitly taken into account, concerns the expected duration of a rate of price change. It affects the desired level of the balances. A rate of price rise of 50 per cent per month may be entirely ignored if the rate is expected to last only one day; but balances will be considerably reduced if the rate is thought likely to continue for six months. Expected duration might follow rumors and conjectures about the future level of prices instantly without regard to the past. Then it would tend to produce erratic and unpredictable changes in desired real cash balances. But, if it depended primarily on the extent and duration of inflation in the recent past, it would behave much like the expected price change. Since the model describes most months of hyperinflation fairly well even though the effects of expected duration on the balances were not explicitly taken into account, quite likely the model implicitly incorporates these effects. Thus the exponentially weighted average of the rates of change in prices probably represents not the expected rate for the immediate future, as was assumed, but the average rate that is expected to last long enough to justify the trouble and expense of reducing cash balances. This rate could fall far short of the rate that might be expected to prevail in the coming weeks or months and thus could explain why the estimated values of the coefficient of expectation are so low. An average lag as long as twenty months may reflect not so much a failure to revise expected rates (or to adjust cash balances) rapidly as an unwillingness to incur the costs of low cash balances before the continued tendency for prices to rise is viewed as relatively permanent. Throughout the hyperinflations the expected duration of the price rise seems to have exercised considerable restraint on reductions in desired real cash balances.

32. Moreover, neither lag accounts for the high levels of the balances in the ending months of hyperinflation (see n. 13).

Other factors than the past performance of prices undoubtedly influenced the expected duration at certain times. The balances rose near the end of all but two of the seven hyperinflations, while the model predicts they should have fallen (see Sec. IV). Prospects for the probable end of currency depreciation apparently made the public willing to hold larger balances immediately. Before the beginning months, too, the expected duration of inflation apparently underwent a sudden rise (see Sec. IV). The sharp decline in the balances following their long rise in the months preceding hyperinflation seems to reflect a sudden realization by the public that greater price increases lay ahead. The precise timing of such shifts in expectations appears incapable of prediction by economic variables, even though we may be certain such shifts will eventually occur under the circumstances. But, when the shifts are absent, expectations of price changes depend closely on past events.

VII. The Tax on Cash Balances

The analysis so far has linked the large price increases during hyperinflation to large increases in the quantity of money. But there is still the question, "Why did the quantity of money increase and by so large an amount?" The answer is twofold: (1) printing money was a convenient way to provide the government with real resources, though it tended to preclude other methods; and (2) the effectiveness of this method declined over time and so required ever larger issues.

In the unsettled conditions following the two world wars, governments were too weak to enact adequate tax programs and to administer them effectively. Issuing money was a method of raising revenue by a special kind of tax—a tax on cash balances. This tax is often appealing because it does not require detailed legislation and can be administered very simply. All that is required is to spend newly printed notes. The resulting inflation automatically imposes a tax on cash balances by depreciating the value of money.

The base of the tax is the level of real cash balances; the rate of the tax is the rate of depreciation in the real value of money, which is equal to the rate of rise in prices. Revenue (in real terms) from the tax is the product of the base and the rate,

$$\frac{M}{P}\left(\frac{dP}{dt}\frac{1}{P}\right).$$

The note-issuing authorities "collect" all the revenue; however, when prices rise in greater proportion than the quantity of money, that is, when real cash balances decline, part of the revenue goes to reduce the real value of the outstanding money supply.[33] Thus total revenue per period of time is the sum of two parts: first, the real value of new money issued per period of time,

$$\frac{dM}{dt}\frac{1}{P};$$

and, second, the reduction in outstanding monetary liabilities, equal to the decline per period of time in the real value of cash balances,

$$\frac{d\left(\frac{M}{P}\right)}{dt}.$$

This is demonstrated by the following identity:

$$\frac{dM}{dt}\frac{1}{P}-\frac{d\left(\frac{M}{P}\right)}{dt}\equiv\frac{M}{P}\left(\frac{dP}{dt}\frac{1}{P}\right). \tag{23}$$

The note-issuing authorities do not set the tax rate directly. They set the rate at which they increase the money supply, and this rate determines the tax rate through the process described in preceding sections.

Typically, institutions other than the government also have the authority to issue money. In so far as they exercise this authority, they

33. Inflation also reduces the real value of the principal and interest charges of debt fixed in nominal terms. However, amortization of debt cannot have required a very large fraction of government expenditures once hyperinflation began.

receive some of the revenue from the tax, even though the initiating factor is government creation of money. In the hyperinflations, as government issues swelled the reserves of private commercial banks, an expansion of bank credit could and did take place. Banks largely dissipated the revenue from their share of the tax by making loans at nominal rates of interest that did not take full account of the subsequent rise in prices, so that the real rate of interest received was on the average below the real return that could be obtained on capital. The revenue dissipated by the banks went to their borrowers.[34]

The revenue received by the government thus depends on the tax rate and on the fraction of the revenue that goes to banks and their borrowers. In addition, it depends on the tax base, that is, the level of real cash balances. A higher tax rate will not yield a proportionately higher revenue, because the balances will decline in response to the higher rate. Indeed, the balances may ultimately decline more than in proportion to the rise in the rate, so that a higher rate may yield less revenue. This will not be so immediately, however, because of the lag in expectations.[35] It takes time for expected price changes to adjust to the actual changes. Consequently, a rise in the tax rate, that is, in the rate of change in prices, will at least initially increase the revenue from the tax.

This fact helps to explain why a similar time pattern of revenue emerged in all the seven hyperinflations. The revenue was high at the start, when the expected rate of price increase was still low; tended to decline in the middle, as the expected rate started to rise considerably; and rose near the end, when the rate of new issues skyrocketed.[36] The

34. The government's share of total revenue depends on the proportion of money issued by governmental agencies. If this proportion is not the same for the quantity of money outstanding as for the amount currently issued, the government's share of the two parts of total revenue will differ. To compute the government's share of the total, we must multiply each part in (23) by separate fractions. For the first part, the appropriate fraction is the ratio of money currently issued by the government to total issues in a period of time; for the second, the fraction is the ratio of outstanding government issues to the total money supply.

35. This follows from the relation

$$\frac{d\log\frac{M}{P}}{dt}=-\alpha\beta\,(C-E)\,,$$

derived from equations (2) and (5), and from the condition that $\alpha\beta<1$, found to hold for all the hyperinflations (see Sec. V).

36. Part of this rise in revenue resulted from the failure of real cash balances to make further declines in the final months, apparently because the end of hyperinflation appeared imminent (see Sec. IV).

note-issuing authorities in all the hyperinflations evidently responded to the belated decline in the balances in much the same manner. In the beginning months, when the quantity of money rose at rates much higher than those previously attained, the revenue was also much higher than before. Because of the lag in expectations, many months passed before real cash balances declined very much. When the balances finally began to decline by substantial amounts, the revenue decreased from the high level of the beginning months. The revenue could be enlarged, as for a short time it was, only by inflating at successively higher rates. Rates were quickly reached, however, that completely disrupted the economy, and they could not be long continued. The attempt to enlarge the revenue in the closing months thus produced the characteristic pattern of the hyperinflations: price increases did not peter out; they exploded.

The time pattern aside, the productivity of the tax can be analyzed most simply by comparing the revenue that was actually raised in the seven hyperinflations with the revenue that could have been raised if the quantity of money had risen at a constant rate. Under the latter condition, the actual and expected rate of rise in prices would eventually become practically equal to this constant rate,[37] and C can replace E in equation (3), the demand function for real cash balances. This substitution gives

$$\frac{M}{P} = e^{-\alpha C-\gamma} . \tag{24}$$

Since total revenue, R, equals the rate of rise in prices times real cash balances, we have, substituting (24),

$$R = C\, e^{-\alpha C-\gamma} . \tag{25}$$

Total revenue is a maximum when the derivative of this expression with respect to C is zero and the second derivative is negative, namely:

$$\begin{aligned} \frac{dR}{dC} &= (1-\alpha C)\ e^{-\alpha C-\gamma} = 0 , \\ \frac{d^2R}{dC^2} &= (\alpha^2 C - 2\alpha)\ e^{-\alpha C-\gamma} < 0 . \end{aligned} \tag{26}$$

These conditions are satisfied when $(1-\alpha C)$ equals zero. Therefore, the rate of tax on cash balances, that is, the rate of inflation, that yields the maximum revenue in (25) is $1/\alpha$. At this rate of tax, the demand for real

37. We can then refer to these three rates simply as "the rate of inflation." Their equality follows from the constancy of real cash balances, which in turn follows from equation (15), given the constant rate of increase in money.

cash balances has an elasticity of -1. Table 9 shows this rate for the seven hyperinflations. It must be emphasized that the rates in columns 1 and 2 yield the maximum revenue only in the special sense of the revenue that can be maintained indefinitely. When the tax is first imposed, that is, at the beginning of inflation, there is no maximum yield; the higher the rate, the higher the yield, thanks to the delayed adjustment in the balances produced by the lag in expectations.

TABLE 9

RATE OF INFLATION THAT MAXIMIZES THE ULTIMATE REVENUE FROM A TAX ON CASH BALANCES*

Country	Rise in Prices and Quantity of Money That Maximizes Ultimate Revenue: Rate per Month Compounded Continuously ($1/\alpha$) (1)	Rise in Prices and Quantity of Money That Maximizes Ultimate Revenue: Per Cent per Month $[(e^{1/\alpha}-1)100]$ (2)	Average Actual Rate of Rise in Prices (Per Cent per Month) (3)
Austria	.117	12	47
Germany	.183	20	322
Greece	.244	28	365
Hungary (after World War I)	.115	12	46
Hungary (after World War II)	.236	32	19,800
Poland	.435	54	81
Russia	.327	39	57

* Rate of change in quantity of money is assumed constant. The values of α used are the estimated values shown in Table 3. Column 3 is reproduced from Table 1, row 7.

The table also lists for comparison the average rates of rise in prices that actually prevailed. The actual rates were well above the constant rates that would have maximized the ultimate revenue. With Table 10 we can compare the yield of the actual rates with the ultimate yield that could have been attained with a constant rate.

Table 10 presents various measures of the average revenue. The closing months of some of the hyperinflations were omitted (see note * to Table 10). To make comparisons among the countries, it is necessary to express the revenue relative to some standard of reference. Two alternative standards were used for Table 10. The first is the level of real cash balances in the beginning month of hyperinflation. Its use allows for differences in the base of the tax. The second is national income in a "normal" year. Its use allows for differences in economic resources.

TABLE 10

Revenue from the Tax on Cash Balances

Country and Period*	Ratio of Government Money to Total Quantity†	Average Monthly Revenue as a Percentage of‡				
		Real Cash Balances in Beginning Month of Hyperinflation			National Income in a Base Year	
		Ultimate Maximum for a Constant Rate of Increase in Quantity of Money	Total $\left(\frac{M}{P}\frac{dP}{dt}\frac{1}{P}\right)$	From Issues of New Money $\left(\frac{dM}{dt}/P\right)$	From Issues of New Money $\left(\frac{dM}{dt}/P\right)$	From Issues of Government Money (Cols. [1] × [5])
	(1)	(2)	(3)	(4)	(5)	(6)
Austria Oct., 1921—Aug., 1922	.30	9	24	18	26	8
Germany Aug., 1922—July, 1923	.46–.59	30	30	25	12	6–7
Greece Nov., 1943—Aug., 1944	(.50)	22	30	22	(11)§	(6)§
Hungary Mar., 1923—Feb., 1924	.66–.95	19	25	21	20	13–19
Hungary Aug., 1945—Feb., 1946	.83–.93	18	32	21	7	7‖
Poland Jan.—Nov., 1923	.65–.75	36	36	31	4	3
Russia Dec., 1921—Jan., 1924	1.00	41	41	41	0.5	0.5

* The period of the averages in cols. 3–6 is from the beginning month of hyperinflation to the last month included in the calculation of the regression lines (see Tables 1 and 3). For two reasons this period was not carried to the end of all the hyperinflations, shown in Table 1. First, the closing months frequently had extreme fluctuations in real cash balances. Using the end-of-month level of the balances would provide a poor approximation to their average level during the months. Second, in the closing months the level of the balances was sometimes unaccountably high in view of the large expected change in prices (see Sec. IV). These high levels very likely resulted from an expectation that hyperinflation would soon end. They temporarily made the revenue from the tax much higher than in the preceding months.

† "Government money" comprises notes issued by governmental agencies and deposits held by the public in the national bank and postal savings accounts. The total quantity of money comprises all notes and bank deposits held by the public. With one exception the ratios are rough approximations, since they were built up from annual figures for deposits that lack complete coverage. The exception refers to Hungary after World War II, for which fairly complete monthly data were used. Owing to the unavailability of Greek deposit figures during the war, the figure for Greece refers to 1941 and is apt to be a poor estimate for the hyperinflationary period, but it is the only one obtainable. For sources see Appendix B.

‡ Column 2: Product of the rate of change in prices shown in col. 1 of Table 9 and the index of real cash balances (with appropriate change of base) corresponding to this rate on the regression lines in Figs. 8–15.

Column 3: Average product for the period covered of the rate of change in prices ($\log_e P_t - \log_e P_{t-1}$) from the beginning to the end of the month and an end-of-month index of the real value of notes. Since the index tended to fall over most months, the use of its value at the end of the month makes this average an understatement of an average based on daily figures.

Column 4: The total revenue less the average change in real cash balances per month (see eq. [23]). This average change is equivalent to the difference between an index of the real value of notes at the beginning and at the end of the period, divided by the number of months covered.

Column 5:

$$(\text{Col. 4}) \times \left(\frac{\text{Index of real cash balances in beginning month of hyperinflation}}{\text{Index in earlier months}}\right) \times \left(\frac{\text{Quantity of money in the normal year}}{\text{Average monthly national income in the normal year}}\right)$$

Though desirable, using months from the normal year in the third factor for the earlier months in the second factor would involve difficulties and was not attempted. The earlier months used, except for Hungary I and Poland, were prewar months to which the index of real cash balances could be readily extended. The normal years were later years for which reasonably reliable data on national income were available. The above product assumes that the third factor holds for the earlier months used in the second factor. This assumption is likely to be approximately correct, because the earlier months and the normal year had relatively stable prices. The normal year and earlier months used, respectively, were for Austria, 1929 and all of 1914; Germany, 1927 and all of 1913; Greece, 1949–52 and June, 1941; Hungary I, 1936 and July, 1921; Hungary II, 1936 and August–December, 1939; Poland, 1929 and January, 1921; and Russia, 1926–29 and December, 1913. The ratios of money to income in the normal years were calculated by Martin J. Bailey (see his "The Welfare Cost of Inflationary Finance," *Journal of Political Economy*, Vol. LXIV [April, 1956], Table 2

Column 3 gives the actual total revenue. It is the average product of the monthly rate of change in prices and an end-of-month index of real cash balances. Column 4 gives the amount of total revenue collected by banks and the government from spending new money. The total revenue in column 3 exceeds this amount by the average reduction in real cash balances per month. Column 2 shows the maximum ultimate revenue under the restriction that the rate of inflation is held constant, which is equivalent to the condition that the balances eventually remain constant. A comparison of columns 4 and 2 indicates that, in all countries except Austria, a constant rate of inflation could have yielded a revenue as large, on the average, as the amount actually collected.

This outcome seems surprising in view of the high and rising tax rates that were imposed. No purpose was served by such high rates unless the authorities intended to take advantage of the lag in expectations to collect more revenue than could ultimately be obtained with a constant rate. They could succeed in this intention by a policy of inflating at increasing rates and so repeatedly take advantage of this lag. The authorities successfully pursued this policy in the beginning and ending months and in much of the period preceding hyperinflation. The policy was temporarily abandoned in the middle months of all the hyperinflations, when the rate of price increase stopped rising and even fell somewhat. The average revenue exceeds the maximum amount in column 2 if most of the middle months are excluded. It is not clear why the authorities allowed the revenue to decline during these months. Perhaps they hesitated to continue a policy that promised to destroy the monetary system, and they hoped to restore some of the public's confidence in the currency. Apparently the need for revenue soon overrode whatever considerations prompted them to let the revenue decline, for the cumulative process of inflating at increasing rates was resumed in the closing months.

On this interpretation, the actual rate of rise in prices depended pri-

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col. 5). These ratios do not apply to years as close to the periods of hyperinflation as the data sometimes permit, but they seem adequate for use in the above estimates.

Columns 3, 4, and 5 were derived from changes in figures for notes (except for Hungary II) that were used to approximate changes in notes and deposits combined. This derivation involves the assumption that notes and deposits increased at equal rates. The assumption is obviously not precisely true, since the ratio of notes to deposits varied (see Appendix B). Moreover, relatively small variations in this ratio imply large differences in their rates of increase. Where notes rose faster than deposits (Germany and Hungary I), the percentages in cols. 3 and 4 are somewhat overstated; and, where notes rose less rapidly (Poland), the percentages are understated.

Column 6: This gives the revenue from new issues collected by the government; private banks received the remaining revenue from new issues. These figures use changes in notes to approximate changes in government money, which was composed almost entirely of all the notes in circulation. Consequently, the figures are almost wholly free from the error to which cols. 3–5 are subject, discussed above.

§ The figures for Greece are inclosed in parentheses to emphasize their unknown reliability. Their derivation involves the assumption that the ratio of notes to total money given for 1941 holds for the later hyperinflationary period without alteration.

‖ Monthly data on deposits were used for the later Hungarian hyperinflation, and its revenue was multiplied by the ratio of government money to the total supply for each month before taking the average. Column 1 indicates the range of these monthly ratios for this hyperinflation.

marily on the revenue needs of the governments. We cannot measure these needs directly. But it will facilitate judging the outcome in terms of need to express the actual revenue as a percentage of national income rather than of initial real cash balances.

Rough estimates of this percentage are given in columns 5 and 6 of Table 10. Because data on national income are not available for the periods of hyperinflation, it was necessary to assume that the ratio of the quantity of money to national income in a later "normal" year, for which data on income are available, was the same as in earlier months, to which the index of real cash balances can be readily extended (see note to Table 10 for column 5). This assumption allowed a conversion of column 4 into percentages of national income. These percentages are shown in column 5. In so far as output during the hyperinflations fell below its level in the normal year, revenue as a percentage of contemporaneous national income would be somewhat higher. The government's share of this revenue was found by multiplying column 5 by the ratio of government money to the total supply. Unfortunately, this ratio cannot be estimated very closely, except for Hungary after World War II. Monthly data on deposits are unavailable for the other hyperinflations. Ratios based on annual data only, shown in column 1, were used. Column 6, which is the product of columns 1 and 5, shows the revenue going to the governments as percentages of normal-year national income. These percentages are not subject to one main error affecting those in columns 3–5 (see next to last paragraph of note ‡ to Table 10) and, while rough, are probably reasonably accurate.

In printing money to provide revenue, the governments of these countries collected on the average less than 10 per cent of normal levels of income. Hungary after World War I was the only exception.[38] Even a figure of 10 per cent seems low for ordinary governmental needs, which, as indicated by the little evidence available, probably ranged at the lowest from 10 to 20 per cent of national income. Germany, for example, collected 12 per cent of national income from all taxes in 1925, the second year after the hyperinflation. These results are consistent with the assumption made above that the authorities imposed increasing tax rates in the attempt

38. The range given for its revenue in that period may be too high, because the calculation of this range involved a comparison with the postwar year 1921, when real cash balances were probably somewhat below normal levels.

This qualification applies to the figures for Poland also, but any tendency for them to be too high was counteracted by the unusually low level to which real cash balances had fallen in the beginning month relative to their level in the year 1921 used for comparison. This fall reflects the high rate of inflation there during and after the war (see Table 7).

to collect somewhat more revenue than could be obtained with a constant rate. Other taxes yielded some revenue, of course, though presumably an amount insufficient for desired total expenditures. As the rate of price increase rose, however, the real value of whatever funds were raised by other taxes undoubtedly diminished, and during the later stages of hyperinflation these funds must have become nearly worthless owing to delays in collecting them.[39] Then the only recourse of the government for procuring all the funds needed immediately was to increase the tax on cash balances.

The results of this tax as recorded by the percentages in column 6 are remarkably similar in outcome, except for Russia,[40] despite the greater possible differences due to errors in the data. Certainly the differences are very much smaller than the corresponding differences in the behavior of prices. If, as these results suggest, the different countries sought and, by and large, collected from this tax a percentage of national income roughly of the same order of magnitude, the differences among the hyperinflations in the rate of increase in the quantity of money can be explained by corresponding differences in the factors affecting the amount of revenue. The rate of new issues required for a given revenue is larger when a higher fraction of the revenue goes to banks and when the tax base is smaller; and conversely. Variations in the tax base, namely, real cash balances, are

39. In 1945 Hungary instituted a scheme, which was only temporarily successful, for collecting taxes in a money of constant purchasing power (see Bertrand Nogaro, "Hungary's Recent Monetary Crisis and Its Theoretical Meaning," *American Economic Review*, XXXVIII [September, 1948], 530 ff.).

40. Its low percentage reflects the unusually low level of real cash balances already reached in 1921 before hyperinflation began, and the low ratio of the balances to national income relative to that of the other countries in the later years used for comparison. The low level of the balances in 1921 was undoubtedly due to the social upheaval in that country during 1914–21, which along with war reduced output to about one-half of prewar levels and shook the public's confidence in the currency. In the other countries the balances declined over this period but not to the same extent. The low ratio for Russia in the later years may possibly reflect special circumstances and may be too low as an estimate of prewar levels. The implication is that Russia's revenue was low because the yield of the tax for any feasible rates was exceptionally low. To a great extent this was undoubtedly true. But very likely the revenue the government wished to collect with this tax was also fairly low. The Russian currency reform, unlike those bringing the other hyperinflations to a close, was accomplished gradually. Between the end of 1922 and March 10, 1924, when the depreciating rubles were completely abandoned, the Russian government issued a stable-valued currency, the chervonets, which gradually replaced the old rubles. In so far as the government collected its regular taxes in chervontsi, the revenue did not lose value in the process of collection, and these taxes retained their productivity.

determined in turn by the reaction index and past rates of change in the quantity of money.

The model implies that there is no limit to the revenue that can be collected if the rate of increase in money can be raised to any level. Whether this implication holds for *any* rate, no matter how high, is an interesting but largely irrelevant question, for the rate cannot in fact be raised to any level, except perhaps momentarily. The disruption of the economy caused by extreme rates would arouse political pressures on a scale that would quickly force the government to curtail its issues. The most spectacular of all recorded hyperinflations, that in Hungary after World War II, did not last long. Price increases quickly built up to rates that must have made it impossible for the economy to function effectively. All rates of price increase in the other hyperinflations much above even 150 per cent per month came in the final months preceding a currency reform. In varying degrees all the hyperinflations showed that rising rates imposed in an attempt to collect a larger revenue than can be obtained with a constant rate very soon reach such tremendous heights that the monetary system verges on chaos, and a return to orthodox taxing methods becomes an economic and political necessity.

VIII. Summary of Findings: The Theory of Hyperinflation

This study set out to explain the monetary characteristics of hyperinflation as displayed by seven such episodes following the two world wars. These characteristics are summarized by the pattern of time series for money and prices: (1) the ratio of the quantity of money to the price level—real cash balances—tended to fall during hyperinflation as a whole but fluctuated drastically from month to month and (2) the rates at which money and prices rose tended to increase and in the final months preceding currency reform reached tremendous heights. This second pattern supplies the identifying characteristic of hyperinflation, but the explanation of the first holds the key to an explanation of the second and logically comes first in order of presentation.

1. Fluctuations in Real Cash Balances

The evidence given in the preceding sections verifies the hypothesis that these fluctuations result from changes in the variables that determine the demand for real cash balances. With a change in demand, individuals cannot alter the nominal amount of money in circulation, but they can alter the real value of their collective cash balances by spending or hoarding money, and so bid prices up or down, respectively. Only one of the variables that determine this demand has an amplitude of fluctuation during hyperinflation as large as that of the balances and could possibly account for large changes in the demand. That variable is the cost of holding money, which during hyperinflation is for all practical purposes the rate of depreciation in the real value of money or, equivalently, the rate of rise in prices.

To relate the rate of price rise to the demand for the balances, it is necessary to allow for lags. There are two lags that could delay the effect of a change in this rate on the demand. First, there will be a lag between the expected and the actual rate of price rise; it may take some time after a change in the actual rate before individuals expect the new rate to continue long enough to make adjustments in their balances worthwhile. Second, there will be a lag between the desired and the actual level of the balances; it may take some time after individuals decide to change the actual level before they achieve the desired level. The method used to take account of these lags relates actual real cash balances to an average of past rates of price change, weighted by an exponential curve, so that price changes more recent in time are given greater importance. The weights never fall to zero, but past price changes sufficiently distant in time receive too small a weight to have any influence on the weighted average. The steepness of the weighting pattern indicates the length of period over which most of the weight is distributed. This method of allowing for the two lags does not distinguish between them. However, the period of time required for adjusting the balances to desired levels seems negligible compared with the past period of time normally reviewed in forming expectations. For this reason I have assumed that the actual level of real cash balances always equals desired levels and that the weighted average of past rates of price change measures only the "expected rate of price change." But there is no direct evidence on the relative importance of the two lags, and the name given to the weighted average may be lacking somewhat in descriptive accuracy.

The specific form of the hypothesis, restated to allow for lags, asserts that variations in the expected rate of price change account for variations in real cash balances during hyperinflation, where the expected rate is an exponentially weighted average of past rates. The hypothesis was tested by fitting a least-squares regression to time series for the balances and the expected rate. The regression fits the data for most months of the seven hyperinflations with a high degree of accuracy, and thus the statistical results strongly support the hypothesis.

The regression functions derived from these fits provide good approximations to the demand function for the balances and so reveal certain characteristics of this demand during hyperinflation. The elasticity of demand with respect to the expected rate of price change increases in absolute value as this expected rate rises. This contradicts the often stated view that the degree to which individuals can reduce their holdings of a depreciating currency has a limit. The demand elasticity indicates that they reduce their holdings by an increasing proportion of each successive rise in the expected rate. Indeed, the reason why issuing money on a grand scale does not almost immediately lead to extreme flight from the currency is not due to inelasticity in the demand for it but to individuals' lingering confidence in its future value. Their confidence maintains the lag in expectations, whereby the expected rates of price change do not at first keep pace with the rapidly rising actual rates. However, the weighting pattern for the lag appears to become much steeper in the later months, indicating that the lag in the expected behind the actual rates tends to shorten in response to continual inflation.

Thus the large changes in the balances during hyperinflation correspond to large changes in the rate of price change with some delay, not simultaneously. The demand function that expresses this correspondence can be interpreted to represent a dynamic process in which the course of prices through time is determined by the current quantity of money and an exponentially weighted average of past rates of change in this quantity. The process implies that past and current changes in the quantity of money cause the hyperinflation of prices. This link between changes in prices and money is only broken when the absolute value of the slope of the demand function is especially high or the lag in expectations is especially short. In that event price increases become self-generating. What this means is that the rise in prices immediately produces a proportionately greater decline in real cash balances. Then the effect of percentage changes in prices and the balances on each other does not diminish, as a stable moving equilibrium of prices requires, but grows. Such a process sends up the *percentage* change in prices at no less than an exponential rate, even if the quantity of money remains constant. Apparently the demand slope and the lag never reached the critical level in the seven hyperinflations, for none had self-generating price increases. Instead of running away on their own, price increases remained closely linked to past and current changes in the quantity of money and could have been stopped at any time, as they finally were, by tapering off the issue of new money.

2. THE TREMENDOUS INCREASE IN MONEY AND PRICES

If in fact price increases were not self-generating, what accounted for their tremendous size? The above explanation of their behavior in terms of large increases in the quantity of money only raises the further question, "Why did this quantity increase so much?" Clearly, issuing money on a large scale serves as a major source of funds for government expenditures. The inflation resulting from new issues places a tax on cash balances by depreciating the value of money. The revenue in real terms raised by this tax is the product of the rate of rise in prices (the tax rate) and real cash balances (the tax base). By setting the rate of increase in the quantity of money, the note-issuing authorities indirectly determine the rate of tax through the process implied by the demand function. The simplicity of administering this tax undoubtedly explains why governments resorted to continual issues of money in the difficult periods after the two world wars. An explanation of why those issues became so large, however, is found in the response of the tax base to the tax rate.

If the tax rate remains constant, the tax base and, therefore, the revenue ultimately become constant. Among all constant rates, there is one that yields a maximum ultimate revenue. With a tax rate that increases rapidly enough, however, the revenue forever exceeds this maximum amount for a constant rate because of delays in adjusting the tax base produced by the lag in expectations. In the beginning and closing months of the seven hyperinflations, the authorities successfully pursued a policy of inflating at increasing tax rates to take advantage of this lag and collected more revenue thereby than they could have obtained with any constant rate. This policy led to actual rates far above the constant rate that would have maximized ultimate revenue and produced the tremendous increases in money and prices characteristic of hyperinflation.

In the middle months the rate of increase in the money supply tapered off, for what reason it is not entirely clear, and the revenue temporarily decreased. As a result, the revenue collected with the actual tax rates was not greater on the average than the amount that could have been obtained with a constant rate. The resumption of increasing rates in the closing months restored the revenue to amounts at least as large as those in the beginning months. In order to compensate for the low level to which the tax base fell after many months of hyperinflation, the tax rates rose to astronomical heights. This explosion of the rates in the final months completely disrupted the economy and forced the government to substitute a traditional tax program for a policy of printing money.

In Section I it was suggested that the seven hyperinflations represent the same economic process because of the similarity in the ratios of the average change in prices to the average change in the quantity of money (row 9 in Table 1). The model of hyperinflation described above depicts the nature of this process. But these ratios of averages cover up an extraordinary dissimilarity. Rows 13 and 15 also present ratios involving prices and the quantity of money but not averages, and they differ widely. The model shows that these differences originate, not in the differing responses of the public to a depreciating currency, but in the varying rates at which money was issued. The average share of national income that these new issues procured for the different governments was 3 to about 15 per cent, except in Russia, which had the unusually low percentage of 0.5. The differences in these percentages are not large when compared with the very much larger differences between the hyperinflations in the rates at which money rose. To some extent the governments may have collected less revenue than was planned. But, in so far as the actual collections met budgetary plans, the rates required to procure the intended amount in any month roughly equal the actual rates. The differences between the hyperinflations in the required rates that can be derived from the model thus account to a great extent for the corresponding differences in the actual rates at which money rose.

The model used has definite limitations: it only applies accurately to large price increases, and it fails to describe the closing months in four of the hyperinflations. In the closing months real cash balances sometimes rose when the model indicates they should have fallen. This limitation likely results from expectations that current price increases would not last very long. Such expectations are not related in any direct or obvious way to past changes in prices. To take account of this limitation of the model does not seem to require revisions that would contradict the premise of this study that domestic monetary factors alone explain hyperinflations.

Many prevailing theories of economic disturbances emphasize external monetary factors like the foreign-exchange rate, as well as real factors like the level of employment and real income, the structure of trade unionism, the rate and extent of capital formation, and so on. These factors are prominent primarily in discussions of depression. Yet they also enter into discussions of inflation. The theory of the cost-price spiral, which borrows its concepts and framework from theories of income and employment common in discussions of depression, has been applied to inflation with the suggestion, sometimes explicit, that it applies to hyperinflation as well. Closely related and often identical to the theory of the cost-price spiral is the explanation of hyperinflation in terms of the depreciation of the foreign-exchange rate.[41]

41. References to discussions of the cost-price spiral are too extensive to give even a partial list here. The most explicit application of this theory to hyperinflation that I

These theories postulate that a rise in prices results from increases in wages or prices of imported goods and precedes increases in the quantity of money. This study points to the opposite sequence and indicates that an extreme rise in prices depends almost entirely on changes in the quantity of money. By implication, the rise in wages and the depreciation of the foreign-exchange rate in hyperinflations are effects of the rise in prices. Extreme changes in a short period of time in exchange rates will primarily reflect variations in the real value of the currency. It is quite true that the public might well expect depreciation of the currency to show up more accurately in depreciation of exchange rates than in any set of readily available commodity prices and so follow these rates in adjusting their balances. Circumstances are easy to imagine in which *for a short time* the exchanges might depreciate faster than prices rise and so appear to move in advance of prices. But this result would not mean that the rise in prices had become the effect rather than the cause of exchange depreciation. Real cash balances would be related to this depreciation only so long as it remained a good indicator of price changes.

The model suggests in addition that the spiral theory places emphasis on the wrong factors. *Hyper*inflation at least can be explained almost entirely in terms of the demand for money. This explanation places crucial importance on the supply of money. While the monetary authorities might capitulate to pressures for sustaining wage increases, as the spiral theory presumes, they will typically attend to many other considerations. The most important of these in hyperinflation is the revenue raised by issuing money, which was analyzed above. More precise analysis than this of the determinants of the money supply goes beyond a mechanistic account of the inflationary process and involves the motives of governments, with whom the authority to open and close the spigot of note issues ultimately lies.

have found is by Mrs. Joan Robinson in "Review of Bresciani-Turroni's *The Economics of Inflation*," *Economic Journal*, XLVIII (September, 1938), 507.

To my knowledge no one has argued that depreciation of the foreign-exchange rate alone is sufficient to explain hyperinflations, but it is often considered to be a causal factor. The attempts to find statistical confirmation of this view are inadequate and unconvincing. Probably the best attempt is that by James Harvey Rogers (see his *The Process of Inflation in France* [New York: Columbia University Press, 1929], chap. vii), on which Frank Graham based his interpretation of the German episode (see his *Exchange, Prices, and Production in Hyper-inflation: Germany, 1920–23* [Princeton, N.J.: Princeton University Press, 1930], esp. p. 172).

APPENDIX A
STATISTICAL METHODS

Equations (2) and (9) were combined to give the regression function (11), which has the form,

$$Y_t + \alpha E_t + \gamma = \epsilon_t \, , \tag{27}$$

where Y_t stands for $\log_e (M/P)_t$ and ϵ_t is a random variable. The method of computing E_t was given in note 6. The variance of the random variable is estimated by

$$V(\epsilon) = \frac{\Sigma (Y + \alpha E + \gamma)^2}{N} . \tag{28}$$

For simplicity the subscripts have been dropped with the understanding that the summation is over the values of Y and E that refer to the same period of time. N is the number of observations in the summation.

The parameters α and β can be estimated by the method of least squares. The estimates of α and β are those values that make (28) a minimum for all values of γ. The estimates in Table 3 were computed by maximizing the total correlation coefficient after making a substitution for α and γ, a procedure that is equivalent to minimizing (28). The total correlation coefficient, R, is defined as

$$R^2 = 1 - \frac{\Sigma (Y + \alpha E + \gamma)^2}{\Sigma Y^2 - N \bar{Y}^2} , \tag{29}$$

where the bar indicates the average value of the variable.

The values of γ and α that make R^2 a maximum for given values of β are $\hat{\gamma}$ and $\hat{\alpha}$ and are found as follows:

$$\begin{aligned} \frac{\partial R^2}{\partial \gamma} &= \frac{-2\gamma N - 2N\bar{Y} - 2\alpha N\bar{E}}{\Sigma Y^2 - N\bar{Y}^2} = 0 \, , \\ \hat{\gamma} &= -\bar{Y} - \alpha \bar{E} \, , \\ \frac{\partial R^2}{\partial \alpha} &= \frac{-2\alpha\Sigma E^2 - 2\Sigma YE - 2\gamma N\bar{E}}{\Sigma Y^2 - N\bar{Y}^2} = 0 \, , \end{aligned} \tag{30}$$

and, if $\hat{\gamma}$ is substituted for γ,

$$-\hat{\alpha} = \frac{\Sigma YE - N\bar{E}\bar{Y}}{\Sigma E^2 - N\bar{E}^2} . \tag{31}$$

R^2 is derived as a function of β by inserting (30) and (31) in (29).

$$R^2(\beta) = \frac{(\Sigma YE - N\bar{Y}\bar{E})^2}{(\Sigma E^2 - \bar{E}^2)\,(\Sigma Y^2 - N\bar{Y}^2)} . \tag{32}$$

The value of β, $\hat{\beta}$, that makes $R^2(\beta)$ a maximum can be found by trying successive values of β that differ by .05 in (32). Given $\hat{\beta}$, $\hat{\alpha}$ is computed from (31). This was the procedure used to calculate the estimates in Tables 3 and 5. It gives the same estimates as would minimizing (28), because the substitutions for α and γ, $\hat{\gamma}$ in (30) and $\hat{\alpha}$ in (31), give, for every β, the maximum of R^2 for all γ and α. The maximum of $R^2(\beta)$ for all β gives the maximum with respect to all the variables, which is equivalent to the minimum of (28).

The reason for choosing a β that gives the maximum value of $R^2(\beta)$, instead of directly solving for the values that set to zero the partial derivatives of (28) with respect to each of the three variables, was one of computational efficiency. The length of the weighted average for computing E is determined by the value of β and can be fixed only by first assuming a value for β, which was the procedure that was followed. But, in the method of solving the partial derivatives simultaneously, the value of β remains to be determined, and therefore the minimum number of items that the accumulated product (9) should include to satisfy (10) (shown in Sec. III) cannot be fixed beforehand. This procedure would in this respect result in more computations than would be strictly necessary. Also, the method of finding the value of β that maximizes $R^2(\beta)$ gives as a byproduct the values of $R^2(\beta)$ for different values of β, which are necessary for calculating confidence intervals for the estimates. One further advantage to the method used is that, by knowing many values of $R^2(\beta)$ over a wide range, we are sure that there is a unique maximum for all positive values of β. As β becomes large, E approaches C, and $R^2(\beta)$ quickly approaches a limit.

A method of finding confidence intervals for the estimates of α and β that utilizes information gained in estimating the two parameters is afforded by the likelihood ratio. If the residuals from the regression function (27) are assumed to be independent and normally distributed with mean zero and variance σ^2, the likelihood function, L, is defined as

$$L = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^N \exp\left\{\frac{-1}{2\sigma^2}\sum_1^N (Y + \alpha E + \gamma)^2\right\} . \tag{33}$$

The likelihood ratio is then defined by

$$\lambda = \frac{L(\hat{w})}{L(\hat{\Omega})} , \tag{34}$$

where $L(\hat{w})$ is the maximum of L over the region w of the null hypothesis, and $L(\hat{\Omega})$ is the maximum of L over the region Ω of all alternative hypotheses. As N becomes large, the distribution of $-2 \log_e \lambda$ approaches $\chi^2(r)$, where r is the number of restrictions on the values of the parameters specified in the null hypothesis.[42]

To find confidence intervals for the estimates of β, the values of σ, γ, and α that maximize L are inserted in (33). These values are found by partial differentiation of log L.

$$\frac{\partial \log L}{\partial \sigma} = \frac{-N}{\sigma} + \frac{1}{\sigma^3} \Sigma (Y + \alpha E + \gamma)^2 = 0 .$$

The maximum likelihood estimate of σ is

$$\hat{\sigma}^2 = \frac{\Sigma (Y + \alpha E + \gamma)^2}{N},$$

which, by (29), can be written as

$$\hat{\sigma}^2 = (1 - R^2) \Sigma (Y - \bar{Y})^2 . \tag{35}$$

The values of γ and α that maximize L are $\hat{\gamma}$ and $\hat{\alpha}$, as given by (30) and (31). The value of β that maximizes L is the same as the estimated value, $\hat{\beta}$, which was found to an accuracy of $\pm .05$. The components of the likelihood ratio are therefore expressed as follows:

$$\log L (\hat{w}) = \log L (\hat{\sigma}_0^2, \hat{\gamma}, \hat{\alpha}, \beta_0) =$$
$$-\frac{N}{2} \log 2\pi - \frac{N}{2} \log \hat{\sigma}_0^2 - \frac{1}{2\hat{\sigma}_0^2} \Sigma [Y + \hat{\alpha} E (\beta_0) + \hat{\gamma}]^2 , \tag{36}$$

where $\hat{\sigma}_0^2$ is computed using $\hat{\gamma}$, $\hat{\alpha}$, and β_0; and $E(\beta_0)$ is computed using β_0.

$$\log L (\hat{\Omega}) = \log L (\hat{\sigma}^2, \hat{\gamma}, \hat{\alpha}, \hat{\beta}) =$$
$$-\frac{N}{2} \log 2\pi - \frac{N}{2} \log \hat{\sigma}^2 - \frac{1}{2\hat{\sigma}^2} \Sigma [Y + \hat{\alpha} E (\hat{\beta}) + \hat{\gamma}]^2 , \tag{37}$$

where $\hat{\sigma}^2$ is computed using $\hat{\gamma}$, $\hat{\alpha}$, and $\hat{\beta}$; and $E(\hat{\beta})$ is computed using $\hat{\beta}$. Consequently,

$$-2 \log \lambda = N \log \hat{\sigma}_0^2 - N \log \hat{\sigma}^2, \quad \text{or}$$
$$-2 \log \lambda = N \log [1 - R^2 (\beta_0)] - N \log [1 - R^2 (\hat{\beta})] , \tag{38}$$

which is distributed as $\chi^2(1)$. Only the right-hand tail of the chi-square distribution should be used in this test.

Confidence intervals for β in Table 3 were computed by inserting numbers that differed by .05 for β_0 in (38). The lowest and highest values,

42. S. S. Wilks, *Mathematical Statistics* (Princeton, N.J.: Princeton University Press, 1950), p. 151.

β_L and β_U, that just make (38) significant at the .10 level are the bounds of the confidence interval. This gives a confidence coefficient of .90.

Confidence intervals for estimates of α were found in a similar manner from the likelihood ratio. $L(\hat{w})$ is the maximum of L for given values of α. The lowest and highest values, α_L and α_U, that make the likelihood ratio significant at the .10 level are the bounds of the confidence interval. Since the value of β that maximizes L for given α cannot be found directly, a trial-and-error procedure was followed using values of β that differed by .05.

The confidence intervals for the estimates of $\alpha\beta$ in Table 8, $(\alpha\beta)_L$ and $(\alpha\beta)_U$, are based on the likelihood ratio. $L(\hat{w})$ is the maximum of L under the one restriction that $\alpha\beta = (\alpha\beta)_0$. $(\alpha\beta)_L$ and $(\alpha\beta)_U$ are the extreme values of $\alpha\beta$ that make the likelihood ratio significant at the .10 level. It turns out that $(\alpha\beta)_L = \alpha_U\beta_L$ and $(\alpha\beta)_U = \alpha_L\beta_U$. Computations by trial and error for each hyperinflation verified that they are the extreme values at the .10 level of significance for all β that differ by .05.

The significance tests summarized in Table 4 and described in note 9 are based on the likelihood ratio and its asymptotic properties. The likelihood function of this ratio for the tests is defined as

$$L = \sum_{i=1}^{7} \left(\frac{1}{\sqrt{2\pi}\,\sigma_i}\right)^{n_i} \exp \left\{\frac{-1}{2\sigma_i^2} \sum_{j=1}^{n_i} (Y + \alpha_i E (\beta_i) + \gamma_i)_j^2 \right\}. \tag{39}$$

The major summation is over the seven hyperinflations. The number of observations from each hyperinflation is given by n_i. The total number of observations from all seven hyperinflations is

$$\sum_{1}^{7} n_i = 149 .$$

The likelihood ratio, (34), for the tests is defined as follows: The denominator, $L(\hat{\Omega})$, is the same in all the tests. It is the unrestricted maximum of the likelihood function. The values of α_i, β_i, and γ_i, which determine σ_i from (35), are those estimated from the observations of each hyperinflation separately. In the test of significance used for Table 4, the numerator of the likelihood ratio, $L(\hat{w})$, is the maximum of the likelihood function under the twelve restrictions that for all seven hyperinflations α_i is the same and β_i is the same. Here the appropriate number of degrees of freedom for chi square is twelve. In the tests of significance for the two fits described in note 9, $L(\hat{w})$ is the maximum of the likelihood function under six restrictions, which makes six the appropriate number of degrees of freedom to use in finding the significance level of the chi-square distribution. For the first fit the restrictions are that β_i is the same for all seven hyperinflations. For the second fit the six restrictions are that a_i is the same for all seven hyperinflations.

Finding the values of α, β, and γ that maximize the likelihood function under these various restrictions is equivalent to finding the values that minimize the following sum of squares under the given restrictions:

$$\sum_{i=1}^{7}\left\{\sum_{j=1}^{n_i}(Y+a_iE(\beta_i)+\gamma_i)_j^2\right\}. \tag{40}$$

There is no direct way to minimize the above relation, and the procedure followed was to approximate the minimum for values of β that differ by .05. The minimum is particularly difficult to compute under the six restrictions that all a_i are the same. Consequently, for these six restrictions the minimum of (40) was only shown to lie above a certain number. Greater accuracy was not necessary to prove that the appropriate likelihood ratio based on these restrictions is significant at the .005 level.

APPENDIX B
DATA AND SOURCES

This appendix contains the following three monthly time series for each hyperinflation:

1. *Logarithm of real cash balances.*—In the tables this series is shown as $\log_{10}(P/M)$, where P and M are indexes of prices and the quantity of authorized hand-to-hand currency.[43] This logarithm, when multiplied by $-1/\log_{10}e$, becomes $\log_e(M/P)$, which appears in equation (11). This series, as well as those described below, was computed more easily in its present form, and it could be used for all the statistical work of this study without conversion into natural logarithmic form. The values of the parameters α and β are independent of the base of the logarithms in the equations.

2. *Rate of change in prices per month.*—In the tables this series is shown as $\log_{10}(P_i/P_{i-1})$, where P_{i-1} and P_i are successive values of an index of the price level, the latter on the date opposite the item of the series and the former on the preceding date.[44] Multiplication of this series by $1/\log_{10}e$

43. Counterfeiting was widespread in all the countries during hyperinflation (see J. van Walré de Bordes, *The Austrian Crown* [London: P. S. King & Son, Ltd., 1924], Annex III), but it does not seem unreasonable to assume that the amount of counterfeit notes was negligible compared with the tremendous quantities of legal notes issued.

44. When this period between the two dates differs appreciably from thirty days, the logarithm is multiplied, unless otherwise noted, by the appropriate factor to make the figure a rate per thirty days. Where no earlier date is shown after the last item in the column, the period between the values of the price index used for this item is one month.

gives $\log_e(P_i/P_{i-1})$, which is the form of the series used in the equations. This series measures the average rate of change in prices during each month. When the rate is fairly steady, the series is a good approximation to the daily rate throughout the month; but, when the rate rises considerably, this series represents the daily rate at the end of the month more closely than it does the rate at the middle of the month.

3. *Expected rate of change in prices per month, $E(\beta)$.*—The derivation of this series is presented in Section III, and methods for its computation are given in note 6. The value of β used for each hyperinflation is the estimated value shown in Table 3. This series is shown in logarithms to the base 10. To convert to natural logarithms, multiply by $1/\log_{10} e$.

The tables of these series start with the final month of hyperinflation and carry back in time to include all the observations used in the regressions. The series of the rate of change in prices are carried back even further to include all the rates used to compute all the values of E that are included in the regressions.

Notes on sources and comments on the comprehensiveness and reliability of the data accompany each table. Series on deposits and real output are included along with supporting notes.

NOTES ON AUSTRIAN DATA

For 1919 and 1920 P is an index of the cost of living. Rents are excluded, since they were controlled and would bias the index downward compared with an index of the general price level. For 1921 and 1922 P is a different index of the cost of living that excludes rents but includes many prices not included in the earlier index.[45] The earlier index is linked to the later one by a factor that expresses the ratio of the averages of the indexes over the six-month period in which they overlap.

The rate of change in prices for the period preceding January 15, 1919, was found by assuming that the price index was unity on December 30, 1914. If, however, it was unity on June 30, 1914, the last rate in the second series should be changed to .0024, a difference that would have a negligible effect on the series for E. It is not clear whether the base for P in 1914 should be June or December of that year.

M is an index of the quantity of bank notes in circulation.[46] For the base year 1914 the quantity was taken as five hundred million crowns, a figure that Walré de Bordes considers slightly too low.[47] The bank notes were those of the Austro-Hungarian National Bank and were stamped by

45. Walré de Bordes, *op. cit.*, pp. 88 ff.

46. *Ibid.*, pp. 48 ff.

47. *Ibid.*, pp. 38, 64.

the Austrian government. Unstamped bank notes also circulated, but they disappeared gradually, probably completely by June, 1920.[48]

Table B2 gives figures on deposits less bank reserves (since reserves are included in M above) and the ratio of an index of bank notes to an

TABLE B1

TIME SERIES FOR AUSTRIA

Date	$\log_{10}(P/M)$ (1914=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.05)$ (Per Month) (3)
1922			
Aug. 31	1.5988	.3699	.0885
July 31	1.4654	.2842	.0740
June 30	1.3365	.1247	.0633
May 31	1.3522	.1492	.0601
Apr. 30	1.2625	.0648	.0555
Mar. 31	1.2553	.0117	.0550
Feb. 28	1.3118	.0557	.0573
Jan. 31	1.3139	.1550	.0573
1921			
Dec. 31	1.2742	.1630	.0523
Nov. 30	1.2718	.2511	.0467
Oct. 31	1.1430	.2154	.0362
Sept. 30	1.0414	.1230	.0270
Aug. 31	.9956	.1193	.0221
July 31	.9085	−.0467	.0171
June 30	.9956	.0604	.0204
May 31	.9731	.0025	.0183
Apr. 30	.9731	.0187	.0191
Mar. 31	.9956	.0296	.0191
Feb. 28	.9956	.0611	.0186
Jan. 31	.9777	.0261	.0166
1920			
Dec. 31	....	.0427	....
Oct. 15	....	.0127	....
July 15	....	.0126	....
Apr. 15	....	.0248	....
Jan. 15	....	.0334	....
1919			
July 15	....	.0065	....
Jan. 15	....	.0027	....
1914			
Dec. 30	....	....	....

index of bank notes plus deposits in circulation for Austria. The deposit figures include postal savings, deposits in savings banks, sight liabilities of the national bank, and an estimate of deposits in commercial banks. Cash reserves are those in commercial banks.

The figures for the post office include savings but not current accounts

48. *Ibid.*, p. 45.

(i.e., sight deposits),[49] the amounts of which are unavailable. In 1923 the total debits of current accounts in the post office were 25 per cent of the debits of current accounts in the national bank.[50] If the amount of these accounts in the post office were also roughly 25 per cent of their amount in the national bank in the preceding years, the deposits in the post office that are excluded were a negligible proportion of total bank deposits, because all deposits in the national bank were a small proportion of the total.

The deposits of savings banks included above comprise all but about 1 per cent of the total deposits in such banks.[51]

Sight liabilities of the national bank[52] were held by the public and are properly included.[53]

TABLE B2

DEPOSIT DATA FOR AUSTRIA

End of Year	Bank Deposits *less* Cash Reserves (Million Kronen)	Ratio of Index of Bank Notes to Index of Bank Notes *plus* Deposits in Circulation
1913	7,423.1	1.00
1920	71,371.0	4.75
1921	494,914	4.12
1922	9,475,665	4.77

Figures for deposits held by the public less cash reserves in commercial banks for the years 1920–22 are based on the deposits and cash-reserve ratios of seven large Vienna banks.[54] To estimate deposits in commercial banks, the deposits in these seven banks were increased by 78.85 per cent, the percentage by which they fell short of the deposits (current, savings, and *Giro* accounts) of twenty-seven Vienna banks in 1920.[55] These twenty-

49. League of Nations, *Memorandum on Currency and Central Banks, 1913–1925* (Geneva: Publications Department of the League of Nations, 1926), II, 61.

50. B. H. Beckhart and H. Parker Willis, *Foreign Banking Systems* (New York: Henry Holt & Co., 1929), chap. ii.

51. League of Nations, *op. cit.*, p. 61.

52. Walré de Bordes, *op. cit.*, p. 53.

53. See Beckhart and Willis, *op. cit.*, pp. 133–34, and League of Nations, *Memorandum on Central Banks, 1913 and 1918–1923* (Geneva: Publications Department of the League of Nations, 1924), pp. 83, 91.

54. League of Nations, *Memorandum on Commercial Banks, 1913–1929* (Geneva: Publications Department of the League of Nations, 1931), pp. 62 ff.

55. Walré de Bordes, *op. cit.*, p. 56.

seven Vienna banks constituted almost the whole body of commercial banking proper within Austrian territory at the time. A great number of new banks established during the hyperinflation remained small.[56]

The above figures of bank deposits exclude deposits in the following institutions:[57]

1. Wiener Giro- und Cassen-Verein and Soldierungs-Verein, which were engaged in clearing transactions. The former's business was large, but most of its activities involved stock-exchange transactions. Before 1924 the latter's deposits can probably be considered negligible, since its debits were only 10 per cent of deposit debits for the national bank in 1923.

2. Provincial mortgage institutions, credit associations, and agricultural credit associations, which held savings deposits and greatly declined in importance during hyperinflation.[58]

Even though the coverage of bank deposits in the above figures is fairly comprehensive, the inadequacy of the data for commercial banks means that the figures in Table B2 must be taken as rough approximations.

Changes in real income in Austria during the hyperinflation were apparently not great.[59] Unemployment figures available show a rise in 1922, but there is no evidence that the relative amount of unemployment became important.[60]

NOTES ON GERMAN DATA

P is an index of wholesale prices compiled by the Statistisches Reichsamt.[61] For the months before 1923 P is the average monthly level of prices and is used to approximate the level of prices at the middle of the month. For the first half of 1923 P is the level of prices on a particular day and is available for ten-day intervals. For the second half of 1923 it refers to a particular day at weekly intervals. Where the period covered by the rate of change in prices in 1923 differs from thirty days, the figures were adjusted to give the rate per thirty days. In computing E, each rate of change in prices was treated as though it pertained to the fifteenth of the month. Adjusting the exponential weights to take account of observations not falling on the fifteenth would have involved an unnecessary complication.

56. League of Nations, *Memorandum on Commercial Banks, 1913–1929*, p. 66.

57. Beckhart and Willis, *op. cit.*, chap. ii.

58. *Ibid.*, p. 169.

59. Walré de Bordes, *op. cit.*, p. 158.

60. Statistisches Reichsamt, *Statistisches Jahrbuch für das Deutsche Reich* (Berlin: Reimar Hobbing, 1925), Vol. XLIV: *1924–25*, International Section, Table 12.

61. Sonderhefte zur Wirtschaft und Statistik, *Zahlen zur Geldentwertung in Deutschland 1914 bis 1923* (Berlin: R. Hobbing, 1925), pp. 33 ff.

M is an index of the quantity of authorized bank notes in circulation.[62] For the months before 1923 the quantity of these bank notes is available for the end of the month, and linear interpolations of these figures were used to estimate M for the middle of the month. For 1923 the gold value of the quantity of bank notes is available weekly. These figures were converted to marks by multiplying by the dollar-exchange rate and by 4.198 (the number of gold marks that equaled one dollar before November, 1923). Weekly figures for the quantity of bank notes were interpolated to estimate M for the same day in each month in 1923 to which P refers. By that time the quantity of bank notes was increasing so rapidly that ordinary arithmetic interpolation would have been subject to large errors. Therefore, interpolations for 1923 were linear between the logarithms of the quantity of bank notes up to June and were linear between second logarithms thereafter. These interpolations are probably in error by negligible amounts for the months preceding September.

Illegal currencies issued in Germany in 1923 were estimated not to have exceeded 192 quintillion marks.[63] These currencies comprised over two-thirds of the outstanding legal bank notes on November 15, 1923, though a rapidly diminishing fraction thereafter. There is some reason to believe that the circulation of the illegal currencies was somewhat localized and that most of them were issued when the so-called needs for currency became acute, which occurred at the peaks of hyperinflation and would imply that they were issued at a rate roughly in proportion to that of the authorized bank notes. In so far as failure to take account of illegal currencies has made the figures for real cash balances for Germany in 1923 too low, the estimate of the parameter α is too high. While similar sources of upward bias in the estimates of α may exist for the other countries, there seems to be no indication that substantial amounts of illegal currencies were issued. Apparently, only in Germany were unauthorized currencies issued by local governments and private organizations.

Table B4 gives figures on deposits less bank reserves (M above includes these reserves) and the ratio of an index of bank notes to an index of bank notes plus deposits in circulation for Germany. The figures include deposits in the post office, savings banks, the Reichsbank, and a rough estimate of deposits in all commercial banks. Cash reserves are those of commercial banks.

The amount of deposits with the post office[64] are available for the years

62. *Ibid.*, pp. 45 ff.

63. German Government, *Germany's Economy, Currency and Finance* (Berlin: Zentral-Verlag G.m.b.H., 1924), p. 67.

64. Statistisches Reichsamt, *op. cit.*, p. 314.

TABLE B3

TIME SERIES FOR GERMANY

Date (Middle of Month unless Otherwise Noted)	$\log_{10}(P/M)$ (1913=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.20)$ (Per Month) (3)
1923			
Nov. 13*	1.9445	2.5560	1.1155
Oct. 16	2.6263	1.5881	.7965
Sept. 18	2.6415	1.6259	.6212
Aug. 14	2.5717	1.1385	.3987
July 17	2.1303	.4842	.2349
June 15	1.9868	.3789	.1797
May 15	1.7782	.1542	.1356
Apr. 14	1.7076	.0155	.1315
Mar. 15	1.8261	−.0587	.1571
Feb. 15	2.0755	.3899	.2049
Jan. 15	1.9445	.1546	.1640
1922			
Dec.	1.9381	.1066	.1661
Nov.	2.0481	.3094	.1792
Oct.	1.9247	.2949	.1503
Sept.	1.7747	.1746	.1183
Aug.	1.7084	.2807	.1058
July	1.5024	.1556	.0671
June	1.3955	.0369	.0476
May	1.3983	.0070	.0501
Apr.	1.4231	.0681	.0597
Mar.	1.3886	.1219	.0579
Feb.	1.2934	.0490	.0438
Jan.	1.2550	.0218	.0427
1921			
Dec.	1.2608	.0088	.0473
Nov.	1.2975	.1426	.0559
Oct.	1.1855	.0756	.0367
Sept.	1.1351	.0327	.0281
Aug.	1.1225	.1279	.0271
July	1.0043	.0192	.0048
June	.9978	.0189	.0017
May	.9886	−.0059	−.0021
Apr.	.9983	−.0040	−.0013
Mar.	1.0056	−.0121	−.0007
Feb.	1.0220	−.0195	.0018
Jan.	1.0370	−.0003	.0065
1920			
Dec.	1.0410	−.0203	.0081
Nov.	1.0730	.0126	.0144
Oct.	1.0652	−.0094	.0148
Sept.	1.0884	.0141	.0202

* Prices still rose sharply but at diminishing rates until November 20, 1923, when the mark exchange rate with the dollar was officially fixed and prices became relatively stable.

TABLE B3—*Continued*

Date (Middle of Month unless Otherwise Noted)	$\log_{10}(P/M)$ (1913=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.20)$ (Per Month) (3)
1920—*Continued*			
Aug.	....	.0256	....
July	....	−.0047	....
June	....	−.0379	....
May	....	−.0167	....
Apr.	....	−.0376	....
Mar.	....	.0061	....
Feb.	....	.1276	....
Jan.	....	.1943	....
1919			
Dec.	....	.0735	....
Nov.	....	.0815	....
Oct.	....	.0569	....
Sept.	....	.0675	....
Aug.	....	.0951	....
July	....	.0416	....
June	....	.0158	....
May	....	.0164	....
Apr.	....	.0186	....
Mar.	....	.0064	....
Feb.	....	.0131	....
Jan.	....	.0291	....
1918			
Dec.	....	.0200	....
Nov.	....	.0000	....
Oct.	....	.0075	....
Sept.	....	−.0094	....
Aug.	....	.0530	....
July	....	.0020	....
June	....	.0126	....
May	....	−.0021	....
Apr.	....	.0129	....
Mar.	....	.0000	....
Feb.	....	−.0129	....
Jan.	....	.0021	....
1917			
Dec.	....	.0000	....
Nov.	....	.0043	....
Oct.	....	.0043	....
Sept.	....	−.0086	....
Aug.	....	.0720	....
July	....	.0180	....
June	....	.0053	....

1913–22 but not for 1923. The magnitude of these deposits was relatively small before 1923, and their slow growth up to 1923 suggests that their magnitude in that year was insignificant.

The figures for savings banks include all deposits of almost all such banks.[65]

The figures for deposits with the Reichsbank cover private (non-government) current accounts.[66] They are not separated in the figures from government current accounts for the years before 1921, however. For 1913 and 1919 this item is omitted from the estimate of deposits. This understates the estimate by probably no more than 5 per cent for

TABLE B4

DEPOSIT DATA FOR GERMANY

End of Year	Bank Deposits *less* Cash Reserves (Million Marks)	Ratio of Index of Bank Notes to Index of Bank Notes *plus* Deposits in Circulation
1913	29,640	1.00
1919	93,340	2.09
1920	138,261	2.22
1921	197,800	2.29
1922	272,220	1.93
1923	$1{,}959 \times 10^{12}$	3.33

1919 and by even less for 1913, since the total of all Reichsbank deposits was less than 15 per cent of all bank deposits at the end of 1919. Furthermore, cash reserves of commercial banks, included in the figures for these two years, were mostly balances in the Reichsbank and account for over one-half of its private accounts.

Private accounts in the Reichsbank for 1920 were estimated by the figure for January, 1921, since the private accounts seem to have been stable during this period. Even if this January figure is not applicable to the preceding month, the greatest amount by which it could differ from the actual figure for the end of 1920 would not greatly bias the estimate of bank deposits.

Commercial bank deposits held by the public less cash reserves were

65. League of Nations, *Memorandum on Commercial Banks, 1913–1929*, p. 135.

66. League of Nations, *Memorandum on Central Banks, 1913 and 1918–1923*, p. 199.

extrapolated from figures for almost all commercial banks.[67] Benchmarks for the estimates were provided by complete figures for all commercial banks for the end of 1913 and 1923.[68] The figures for the two years 1921 and 1922 were estimated by a linear interpolation of the second logarithms of the figures for December 30, 1920, and January 1, 1924, including an adjustment for deviations from the same kind of trend in the deposits of the eight major Berlin banks. These estimates could have a substantial error but probably not enough to create the relative fall in the ratio of bank notes to deposits in 1922.

Data on deposits in mortgage, public, and co-operative banks are not available for the relevant years, but their position in 1913 and 1924 indicates that they declined greatly during the hyperinflation.[69] Their deposits in 1923 were probably less than 10 per cent of those of commercial banks alone.

TABLE B5

INDEX OF OUTPUT FOR GERMANY

Year	Index of Output (1913=100)	Year	Index of Output (1913=100)
1914	82	1920	66
1915	74	1921	73
1916	69	1922	80
1917	67	1923	61
1918	66	1923 (Sept.–Dec.)	42
1919	55		

Table B5 gives an index of output for Germany that is a simple average of indexes of industrial production, agricultural output, and commercial transportation figures. The index portrays in a general way changes in real income.[70]

The index of output for the last part of 1923 is the midpoint of an interval estimate of the aggregate real income of German industrial workers. It is likely to be an understatement of total real income. The

67. League of Nations, *Memorandum on Commercial Banks, 1913–1929*, pp. 129 ff., and P. Barret Whale, *Joint Stock Banking in Germany* (London: Macmillan & Co., Ltd., 1930), p. 191.

68. League of Nations, *Memorandum on Commercial Banks, 1913–1929*, pp. 129 ff.

69. *Ibid.*

70. Graham, *op. cit.*, p. 316.

figures in Table B5 take into account territorial changes resulting from the loss of Upper Silesia to Poland on October 12, 1921. However, the German marks in that area continued to circulate in fixed number along with the Polish marks, which were introduced in March, 1923. By the time the Polish mark was made sole legal tender in November of that year, the value of the German marks had depreciated to nothing.[71]

NOTES ON GREEK DATA

P is an index of the cost of food in Athens.[72] The rate of change in prices for the period before 1941 is based on the fact that prices rose no more than threefold over the entire period 1938–41.[73] In computing *E*, however, the rate for the period June, 1938, to December, 1940, was taken to be zero. If the rate was actually as high as .0145 for this entire period, the error in *E* for any of the months would be no greater than .0005.

M is an index of the quantity of bank notes issued by the Bank of Greece.[74] The circulation of these bank notes was largely limited to the area near Athens,[75] which provides some justification for having confidence in a price index of food in Athens only. Nevertheless, little stock can be placed in figures of such limited coverage. Furthermore, data on deposits and changes in real income are apparently nonexistent. Bank deposits should not be dismissed as entirely insignificant, though their effects in the other hyperinflations were minor, because deposits in Greece were as large in value as the quantity of bank notes in circulation during the hyperinflation.[76]

The series are not shown for dates later than November 10, 1944. On this date the country made a second attempt at currency stabilization, which was not entirely successful. Prices continued to rise afterward, though at rates far below those that prevailed before November, 1944.

71. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924* (Geneva: Publications Department of the League of Nations, 1925), II, 128.

72. William C. Cleveland and Dimitrios Delivanis, *Greek Monetary Developments, 1939–1948* ("Indiana University Publications, Social Science Series," No. 6 [Bloomington: Indiana University, 1949]), Appendix.

73. Vera Lutz, "The Record of Inflation: European Experience since 1939," chap. iii of preliminary draft for "The American Assembly" (Graduate School of Business, Columbia University, n.d.), p. 86. (Mimeographed.)

74. Cleveland and Delivanis, *op. cit.*

75. *Ibid.*, p. 99.

76. Lutz, *op. cit.*, p. 96.

TABLE B6

TIME SERIES FOR GREECE

Date	$\log_{10}(P/M)$ (June, 1941=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.15)$ (Per Month) (3)
1944			
Nov. 10	3.5580	5.9320	.9095
Oct. 31	2.5369	1.9540	.6694
Sept. 30	2.5310	1.3030	.4615
Aug. 31	2.3491	.6524	.3253
July 31	2.3213	.6077	.2724
June 30	2.0752	.2040	.2181
May 31	2.1626	.4120	.2204
Apr. 30	2.0192	.3022	.1894
Mar. 31	2.0554	.2800	.1712
Feb. 29	1.9499	.1832	.1536
Jan. 31	1.8791	.3884	.1488
1943			
Dec. 31	1.5867	.1294	.1101
Nov. 30	1.5999	.2790	.1069
Oct. 31	1.4442	.1687	.0791
Sept. 30	1.4002	.1331	.0646
Aug. 31	1.3553	.0590	.0535
July 31	1.3835	.1278	.0526
June 30	1.3419	.0465	.0405
May 31	1.3538	.0268	.0395
Apr. 30	1.3732	.1224	.0415
Mar. 31	1.3310	.0446	.0284
Feb. 28	1.3505	−.0514	.0258
Jan. 31	1.4402	−.0704	.0383
1942			
Dec. 31	....	−.1268	....
Nov. 30	....	−.0590	....
Oct. 31	....	.1730	....
Sept. 30	....	.0887	....
Aug. 31	....	.1030	....
July 31	....	.1065	....
June 30	....	.1414	....
May 31	....	.0577	....
Apr. 30	....	.1190	....
Mar. 31	....	.1231	....
Feb. 28	....	.0630	....
Jan. 31	....	.0328	....
1941			
Dec. 31	....	.1696	....
Nov. 30	....	.1472	....
Oct. 31	....	.1569	....
Sept. 30	....	.1200	....
Aug. 31	....	.1012	....
July 31	....	.1776	....
June 30	....	.1396	....
May 31	....	.0969	....
Apr. 30	....	.0000	....
Mar. 31	....	.0215	....
1940			
Dec. 31	....	.0145	....
1938			
June 30	....	....	....

NOTES ON HUNGARIAN DATA FOR AFTER WORLD WAR I

For the years before December, 1923, P is an index of retail prices based on the year 1913. Thereafter, P is an index of wholesale prices based on the year 1914. The two indexes were linked together on the assumption that prices changed very little between the two base years.[77]

TABLE B7

TIME SERIES FOR HUNGARY FOLLOWING WORLD WAR I

Date (End of Month)	$\log_{10}(P/M)$ (July, 1921=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.10)$ (Per Month) (3)
1924			
Feb.	1.7332	.2536	.1086
Jan.	1.5514	.1124	.0934
1923			
Dec.	1.5051	.0958	.0914
Nov.	1.4472	.0340	.0909
Oct.	1.4728	.0253	.0969
Sept.	1.5490	.0784	.1044
Aug.	1.6385	.2086	.1072
July	1.6776	.2966	.0965
June	1.5453	.1867	.0755
May	1.4713	.0514	.0638
Apr.	1.4969	.1022	.0651
Mar.	1.4800	.1983	.0611
Feb.	1.3201	.0357	.0463
Jan.	1.2923	.0618	.0479
1922			
Dec.	1.2201	.0105	.0464
Nov.	1.2304	−.0040	.0502
Oct.	1.2480	.0923	.0559
Sept.	1.2330	.0945	.0520
Aug.	1.2405	.0898	.0476
July	1.2330	.1300	.0432
June	....	.0691	....
May	....	.0099	....
Apr.	....	.0359	....
Mar.	....	.0662	....
Feb.	....	.0209	....
Jan.	....	−.0078	....
1921			
Dec.	....	−.0026	....
Nov.	....	.0898	....
Oct.	....	.0334	....
Sept.	....	.0634	....
Aug.	....	.1092	....
July	....	.0193	....
1914			
July	....	....	....

77. John Parke Young, *European Currency and Finance* (Commission of Gold and Silver Inquiry, U.S. Senate, Serial 9 [Washington, D.C.: Government Printing Office, 1925]), II, 322.

M is an index of the quantity of notes issued by the State Note Institute.[78] These figures neglect issues of *bons de caisse* in 1923, which apparently circulated as a media of exchange. However, they remained below about 3 per cent of the number of notes in circulation.[79]

Table B8 gives figures on deposits less bank reserves (M above includes reserves) and the ratio of an index of notes to an index of notes plus deposits in circulation for Hungary for the years after World War I. The figures include deposits of all the important commercial banks, current accounts (excluding checking and savings deposits) of the post office, and deposits in savings banks.[80] Cash reserves are those of commercial banks.

TABLE B8

DEPOSIT DATA FOR HUNGARY FOLLOWING WORLD WAR I

End of Year	Bank Deposits *less* Cash Reserves (Million Kronen)	Ratio of Index of Notes to Index of Notes *plus* Deposits in Circulation
1920	18,398	1.00
1921	28,496	1.07
1922	53,038	1.35
1923	159,958	1.95
1924	5,201,805	1.06

Current accounts in the State Note Institute are excluded, because the Institute served mainly as a bankers' bank.[81] Even if these accounts are included in the totals, the pattern of the ratios in Table B8 still retains the peak in 1923. The same kinds of deposits as are included in the figures in the table accounted for 83 per cent of total bank deposits held by the public in 1925.[82] Besides figures on checking and savings deposits in the post office and deposits in small commercial banks, such figures on deposits in municipal savings banks and co-operative credit societies are also excluded, because they are unavailable. These last two institutions probably lost business during the hyperinflation, in which case the coverage of deposits in Table B8 would be nearly complete.

78. *Ibid.*, p. 321.

79. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, I, 123.

80. League of Nations, *Memorandum on Currency and Central Banks, 1913–1925*, II, 65, 85.

81. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, II, 92.

82. League of Nations, *Memorandum on Commercial Banks, 1913–1929*, p. 170.

An unusual aspect of deposits for this period in Hungary was the phenomenal growth of current accounts in the post office. They increased over a thousand times from 1920 to 1924. Deposits in commercial banks, which seem to account for most of the growth in deposits during other hyperinflations, increased only a little over two hundred times.

Data on output for Hungary after World War I are apparently unavailable.

NOTES ON HUNGARIAN DATA FOR AFTER WORLD WAR II

For the months after July, 1945, P is an index of prices compiled by Professor Varga.[83] For the earlier period P is an index of the cost of living in Budapest.[84] The rates of change in prices based on the cost-of-living index differ in level from those given in the second column of Table B9

TABLE B9

TIME SERIES FOR HUNGARY FOLLOWING WORLD WAR II

Date (End of Month)	$\log_{10}(P/M)$ (Dec., 1939=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.15)$ (Per Month) (3)
1946			
July	4.5879	14.6226	3.2211
June	2.9782	4.9264	1.3758
May	3.1740	2.4992	.8011
Apr.	3.0255	1.2821	.5263
Mar.	2.9761	.6323	.4040
Feb.	3.2040	.7804	.3670
Jan.	2.9718	.2411	.3001
1945			
Dec.	3.0655	.5041	.3097
Nov.	2.8954	.7283	.2783
Oct.	2.6866	.8070	.2055
Sept.	2.2630	.3456	.1081
Aug.	2.1359	.2118	.0696
July	2.0825	.1352	.0466
June	....	.0003	....
May	....	.0004	....
Apr.	....	.0520	....
1944			
June	....	.0077	....
1943			
June	....	.0056	....
1942			
June	....	....	....

83. Stefan Varga, "Zerfall und Stabilisierung der ungarischen Währung," *Neue Zürcher Zeitung*, January 7, 1947, p. 4.

84. Statistical Office of the United Nations, *Monthly Bulletin of Statistics*, June, 1947, No. 6, p. 120.

for the final months of the hyperinflation; however, the former rates do not show a materially different pattern from the rates in the table.

M is an index of the quantity of bank notes issued by the National Bank of Hungary and the quantity of deposits in the thirty major commercial and savings banks.[85] The ratio of deposits to bank notes rose astronomically during the hyperinflation. In view of the fact that this ratio ordinarily declined in the other hyperinflations, a breakdown of deposits into various kinds would be desirable, because deposits in a new unit of currency, which was stable in real value when it was first introduced, appeared in January, 1946.[86] Deposits of this new currency should be excluded from the figures as long as the currency had a stable value. Presumably they are excluded from the published figures,[87] but, since the data on deposits are not broken down, this presumption cannot be verified.

Output in Hungary seems to have been stable during the hyperinflation following World War II, though the evidence is spotty. Production figures on basic raw materials do not decline, and unemployment figures only increase after the date of the reform in the currency.[88] Undoubtedly real income was much below prewar levels.

NOTES ON POLISH DATA

For the months before September, 1921, P is an index of the retail prices of foods only. Thereafter it is a geometric average of fifty-seven commodity prices at wholesale.[89] The two indexes were linked together.

M is the quantity of bank notes issued by the Bank of Poland.[90] The Polish mark was introduced into Upper Silesia on March 1, 1923, and was made legal tender in November of that year. It gradually replaced the German mark during this period.[91] One result of transferring this territory to Poland from Germany was that the former's currency was given a larger area within which to circulate, and more Polish marks could be issued without affecting the Polish price level. The ratio of M to P could

85. *Ibid.*, pp. 54, 106.

86. Nogaro, *op. cit.* (The tables in Nogaro's article contain some errors. Original sources were used.)

87. L'Office Central Hongrois de Statistique, *Revue hongroise de statistique*, October–December, 1946, Nos. 10–12, p. 154.

88. Statistical Office of the United Nations, *op. cit.*, esp. p. 20.

89. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, II, 298, and Young, *op. cit.*, p. 349.

90. Young, *op. cit.*, p. 347.

91. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, II, 128.

thus rise, even if real cash balances within the original borders were constant. This no doubt partly explains the slight rise that occurred in real cash balances in October and November, 1923, above the level at which the expected rate of change in prices suggests they should have been.

TABLE B10

TIME SERIES FOR POLAND

Date (End of Month)	$\log_{10}(P/M)$ (Jan., 1921=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.30)$ (Per Month) (3)
1924			
Jan.	2.2279	.2309	.2860
1923			
Dec.	2.3962	.3211	.3053
Nov.	2.4472	.3947	.2997
Oct.	2.4150	.5740	.2665
Sept.	2.1553	.1396	.1590
Aug.	2.2279	.2367	.1657
July	2.1761	.2127	.1409
June	2.0645	.2232	.1158
May	1.9542	.0264	.0782
Apr.	1.9956	.0299	.0963
Mar.	2.0719	.0609	.1196
Feb.	2.2041	.1979	.1401
Jan.	2.1173	.1966	.1199
1922			
Dec.	1.9823	.0992	.0930
Nov.	1.9590	.1364	.0908
Oct.	1.8808	.1210	.0749
Sept.	1.8573	.0501	.0588
Aug.	1.8865	.1260	.0618
July	1.8195	.0639	.0394
June	1.8062	.0473	.0308
May	1.7924	.0199	.0250
Apr.	1.7993	.0097	.0268
Mar.	....	.0636	....
Feb.	....	.0299	....
Jan.	....	.0164	....
1921			
Dec.	....	.0115	....
Nov.	....	−.0488	....
Oct.	....	.0368	....
Sept.	....	.0526	....
Aug.	....	.0676	....
July	....	.1105	....
June	....	.0353	....
May	....	.0126	....
Apr.	....	−.0159	....
Mar.	....	.0142	....
Feb.	....	.1024	....
Jan.	....	.0304	....
1914			
June	....	....	....

Table B11 gives figures on deposits less bank reserves (M above includes reserves) and the ratio of an index of bank notes to an index of bank notes plus deposits in circulation for Poland. The figures include deposits held by the public in the Bank of Poland,[92] in the post office (savings deposits only),[93] and in almost all commercial banks.[94] Cash reserves are those of commercial banks.

Non-government deposits in the Bank of Poland were held by banks, businesses, and individuals.[95] The part held by banks was largely excluded by deducting that part of commercial banks' cash reserves that includes balances with the national bank. However, the portion of total deposits in the Bank of Poland not held by the government in 1920 and 1923 had to be estimated, because it is not differentiated from government deposits for those two years. The estimates for 1920 and 1923 are based on the proportion of non-government to government deposits in the Bank of Poland at the end of 1921 and in September, 1923, respectively. Since deposits in the Bank of Poland not held by the government were large compared with total bank deposits in Poland at the time, the figure for total bank deposits in the table depends heavily on the accuracy of these estimates. Their accuracy will be only fair at best, because they are based on ratios that were not necessarily constant.

TABLE B11

DEPOSIT DATA FOR POLAND

End of Year	Bank Deposits *less* Cash Reserves (Million Marks)	Ratio of Index of Bank Notes to Index of Bank Notes *plus* Deposits in Circulation
1920	12,094	1.00
1921	47,857	1.03
1922	224,290	0.97
1923	64,229,000	0.94

Furthermore, figures on deposits in commercial banks for 1923 are unavailable and had to be estimated from deposits in the sixteen major

92. League of Nations, *Memorandum on Central Banks, 1913 and 1918–1923*, p. 285.

93. League of Nations, *Memorandum on Currency and Central Banks, 1913–1925*, II, 86.

94. League of Nations, *Memorandum on Commercial Banks, 1913–1925* (Geneva: Publications Department of the League of Nations, 1931), p. 230.

95. League of Nations, *Memorandum on Central Banks, 1913 and 1918–1923*, p. 292.

commercial banks.[96] The deposits in these sixteen banks at the end of 1923 were increased by 91 per cent to serve as an estimate of total commercial-bank deposits in that year. Ninety-one is the percentage by which deposits in the sixteen banks at the end of January, 1923, fell short of deposits in all commercial banks at the end of 1922. The 91 is a slightly low percentage to use, because it would have been desirable to use a figure for deposits in the sixteen banks for the first of January, 1923, rather than for the end of that month. Deposits at the first of that month were undoubtedly somewhat smaller than deposits at the end, but a figure for the ending date is not available. The cash-reserve ratio of commercial banks at the end of 1923 is taken to be 40 per cent, the approximate ratio that it had been at the end of the previous four years.

The figures in Table B11 exclude deposits in the National Economic Bank and the State Land Bank. At the end of 1925 the kinds of deposits included in the table were 65 per cent of the total deposits in all forms not held by the government in Poland.[97] Even under the assumption that the banks whose deposits are included in the above table gained relatively to the smaller non-commercial banks whose deposits were excluded, the estimates of total deposits cannot be considered very comprehensive. Yet, since the figures cover a large proportion of total deposits, the ratios in the table reflect to some extent the major movements in total deposits relative to bank notes.

The only evidence available related to real income is figures on unemployment. These show a decline for the period from 1922 to August, 1923, except for a short rise in the early part of 1923. Unemployment seems not to have risen until after the first of 1924.[98]

NOTES ON RUSSIAN DATA

P is an index of retail prices for all of Russia published by the Central Bureau of Labor Statistics.[99] Two other available indexes, one of retail prices and the other of wholesale prices, agree substantially with the index used.[100]

M is an index of the quantity of paper rubles in circulation.[101] The total circulation of paper money and coins at the beginning of 1914 is estimated

96. League of Nations, *Memorandum on Currency and Central Banks, 1913–1925*, II, 86.

97. League of Nations, *Memorandum on Commercial Banks, 1913–1929*, pp. 230 ff.

98. Statistisches Reichsamt, *op. cit.*, Table 13.

99. Young, *op. cit.*, Table 81, p. 360.

100. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, I, 199, and Young, *op. cit.*, p. 360.

101. Young, *op. cit.*, p. 359.

at 2,512 million rubles.[102] All coins had disappeared from circulation by 1916. The figures for 1923 are given in either chervontsi or chervonets gold rubles. The procedure for converting these units into those that apply to the rubles in circulation before 1923 is as follows: Convert ten chervonets gold rubles into one chervonets. Then convert chervontsi into

TABLE B12

TIME SERIES FOR RUSSIA

Date (First of Month)	$\log_{10}(P/M)$ (1913=1) (1)	$\log_{10}(P_i/P_{i-1})$ (Per Month) (2)	$E(.35)$ (Per Month) (3)
1924			
Feb.	3.1106	.4958	.3478
Jan.	2.8854	.3728	.2858
1923			
Dec.	2.7694	.3226	.2493
Nov.	2.7126	.2224	.2186
Oct.	2.8633	.2938	.2170
Sept.	2.7033	.2360	.1848
Aug.	2.5944	.2192	.1633
July	2.5145	.1677	.1399
June	2.4548	.1644	.1283
May	2.3541	.1443	.1131
Apr.	2.3424	.1012	.1000
Mar.	2.3820	.0503	.0996
Feb.	2.4183	.1153	.1202
Jan.	2.4281	.0975	.1223
1922			
Dec.	2.4594	.1667	.1327
Nov.	2.4232	.1972	.1185
Oct.	2.3365	.0880	.0855
Sept.	2.3345	.0305	.0844
Aug.	2.4713	−.0158	.1070
July	2.6571	.0566	.1585
June	2.7767	.0872	.2011
May	2.9079	.2172	.2489
Apr.	2.8949	.3403	.2622
Mar.	2.7767	.3254	.2295
Feb.	2.6656	.2770	.1894
Jan.	2.6160	.3195	.1527

old paper rubles according to the official daily rate of exchange.[103] (One new ruble, issued in 1923, equaled one hundred 1922 rubles and one million pre-1922 rubles.) The official daily exchange rates were based on the free-market rates. The difference between the two quotations was rarely more than $3\frac{1}{2}$ per cent in Moscow, though it sometimes reached as high as

102. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, II, 140.

103. S. S. Katzenellenbaum, *Russian Currency and Banking, 1914–1924* (London: P. S. King & Son, Ltd., 1925), p. 111.

20 per cent in the provinces.[104] Gold treasury bonds (certificates of the *centrocassa* of the Commissioner of Finance) were not included in M, because they circulated mainly among state enterprises and institutions.[105]

In December, 1922, the government began to issue a separate currency called the chervonets ruble, which was not increased in such quantities as to depreciate much in value. The two currencies circulated together and exchanged according to free-market rates, as noted above. An index of wholesale prices in chervontsi is available.[106] The chervontsi were perfect substitutes for the depreciating rubles. The amount outstanding of the former is not included in M, which comprises rubles only, because only prices quoted in rubles were undergoing hyperinflation. The fact that such a perfect substitute for rubles existed undoubtedly contributed to the speed at which the rubles declined in value.

TABLE B13

TIME SERIES FOR RUSSIA

Date (First of Month)	$\log_{10}(P_i/P_{i-1})$ (Per Month)
1921	
Dec.	.1599
Nov.	.0667
Oct.	.0302
Sept.	−.0216
Aug.	−.0022
July	.1145
June	.1620
May	.0777
Apr.	.1118
Mar.	.1064
Feb.	.1092
Jan.	.1461
1920	
Dec.	.0580
Nov.	.0380
Oct.	.0180
Sept.	.0076
Aug.	.0470
July	.0930
June	.0557
May	.0834
Apr.	.0976
Mar.	.0909
Feb.	.1062
Jan.	.1309
1919	
Dec.	.1194
Nov.	.1683
Oct.	.0546

All forms of private bank credit soon disappeared after the nationalization of Russian banking in December, 1917. At the beginning of 1914 deposits of all commercial banks totaled 2,545 million rubles; by 1920 deposits of the public were virtually extinct.[107] Credit was extended by the new State Bank established in 1922, but very few of its deposits were held by the public.[108] With the establishment of the State Bank, private banking was also allowed, but it did not develop to any degree until 1923. By 1923 all banking was conducted in relatively stable chervontsi rather than in regular rubles.[109] Consequently, if state enterprises are considered to be a part of the government, deposits held by the public were small enough to neglect.

104. *Ibid.*, pp. 120–21.

105. League of Nations, *Memorandum on Currency and Central Banks, 1913–1924*, II, 143.

106. Statistisches Reichsamt, *op. cit.*, Table 14.

107. Katzenellenbaum, *op. cit.*, pp. 150, 152.

The currency reform that finally resulted in the complete abandonment of the ruble was begun in February, 1924. It was completed on March 10, 1924, when 1 chervonets officially exchanged for 500,000 1923 rubles or 500 billion old (pre-1922) rubles. Russian data during these years are based on the Julian calendar. Add thirteen days to the above dates to convert to the Gregorian calendar.

TABLE B14

INDEXES OF OUTPUT FOR RUSSIA

Year	Index of (1913=100)	
	Industrial Production	Agricultural Output
1920	....	65
1921	13	55
1922	24	69
1923	35	76
1924	49	69

Table B14 gives an index of industrial production and an index of agricultural output.[110] On the supposition that agricultural output dominated the total product of the economy, the movement in its index suggests that total real income rose until sometime in 1923 or 1924 and then fell. Another index of real income for the fiscal years 1922–23 and 1924–25 is 58 and 77 per cent, respectively, of the 1913 level.[111]

108. *Ibid.*, p. 159.

109. *Ibid.*, pp. 183 ff.

110. Jean Dessirier, "Indices comparés de la production industrielle et de la production agricole en divers pays de 1870 à 1928," *Bulletin de la statistique générale de la France*, XVIII, Sec. 1 (October–December, 1928), 104.

111. Serge N. Prokopovicz, *Histoire économique de l'U.R.S.S.* (Paris: Chez Flammarion, 1952), p. 567.