Rational Expectations: An Introduction

Muth: “Rational Expectations and the Theory of Price Movements”

Lucas: “Econometric Policy Evaluation: A Critique”

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A. Rational Expectations and the Theory of Price Movements

- Muth’s question: How should prices vary in a marketplace where beliefs about the future are important?
- Background: Assumption that beliefs are important in certain agricultural markets and a prevailing approach to modeling these markets as involving various ad hoc forms of beliefs.

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Muth's work

- Initially largely ignored, later at center of macroeconomic research
- Provided a careful definition of Rational Expectations
- Provided two very clean examples of how expectation formation be important for price formation within a model – production which takes time and inventory speculation--and showed the consequences of RE for price dynamics in each case.
- Developed a benchmark method of solving linear rational expectations models: the method of undetermined coefficients.

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Muth's Model 1

- Designed to be simplest possible vehicle for displaying
  - How price dynamics work under ad hoc expectations
  - How price dynamics work under Muth's alternative, rational expectations.
- Simple linear model with only "one step ahead" expectation formation

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Market

- Supply: based on prior expectation of price (production takes time) and a shock (eg weather)
- Demand: based on current price
- Equilibrium: price determined so supply equals demand (active markets in grains, etc.)
- Linear model; abstracts from constant terms for simplicity.

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Equations of model

- Supply depends positively on expected price (p^e) and on a supply shock (u)

$$S : y_t = \gamma p_t^e + u_t$$

- Demand (consumption) depends negatively on price

$$D \ : \ d_t = -\eta p_t$$

- Supply equals demand

$$E \ : \ d_t = y_t$$

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Market Clearing Price

- Equating supply and demand, determine how the price depends on expectations and on shocks.

$$p_t = -\frac{\gamma}{\eta} p_t^e - \frac{1}{\eta} u_t$$

- The negative dependence on each variable reflects the fact that both are supply shifters – if people thought, at planting time, that that there was going to be a higher price today then they would produce more output and the price would be lower as a result.
- Muth: price and price expectations will be negatively related statistically, which seems to be a strong form of market irrationality.

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The Rational Expectations alternative

- Price expectation is the same as the prediction of the model,

$$p_t^e = Ep_t \mid I_{t-1}$$

- Using the solution for the market-clearing price above, this implies that

$$p_t = -\frac{1}{\eta}\{\gamma Ep_t \mid I_{t-1} + u_t\} \Rightarrow Ep_t \mid I_{t-1} = -\frac{1}{\eta}\{\gamma Ep_t \mid I_{t-1} + Eu_t \mid I_{t-1}\}$$

$$\Rightarrow Ep_t \mid I_{t-1} = -\frac{1}{\gamma+\eta} Eu_t \mid I_{t-1} \Rightarrow p_t = -\frac{1}{\eta}\{-\frac{\gamma}{\gamma+\eta} Eu_t \mid I_{t-1} + u_t\}$$

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Expected versus unexpected supply shifts

- An unexpected increase in supply has same effect as described before: it depresses price.
- An expected increase in supply has a smaller magnitude (less negative) effect on price because producers cut back on their production, knowing that price will be low.

$$p_t = -\frac{1}{\eta}\{-\frac{\gamma}{\gamma+\eta} Eu_t \mid I_{t-1} + u_t\}$$

$$= -\frac{1}{\eta}\{(u_t - Eu_t \mid I_{t-1}) + \frac{\eta}{\gamma+\eta} Eu_t \mid I_{t-1}\}$$

- Get the "stabilizing effect on price" of having an upward sloping (as opposed to vertical) supply schedule, but only to extent expected (verify this by looking at market in which expected price is replaced by actual price)

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Solving the model: the method of undetermined coefficients

- Muth pioneered an approach to solving RE models, by (i) assuming a particular driving process for u; (ii) hypothesizing an "undetermined coefficients" form of the solution; and then (iii) determining the values of coefficients which are consistent with rational expectations

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A variant of Muth's method

- Assumed process for u

$$u_t = \rho u_{t-1} + e_t$$

- Conjectured form of price solution

$$p_t = \theta_u u_{t-1} + \theta_e e_t$$

- Variant in sense that "guess" that lagged u rather than the history of e's is relevant—if wrong reach contradiction (as in state vector in HW problem)

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Variant (cont'd)

- Implications for expectation

$$E_{t-1}u_t = \rho u_{t-1} \quad and \quad E_{t-1}p_t = \theta_u u_{t-1}$$

- Restrictions on price solution

$$-\eta p_t = \gamma E_{t-1}p_t + u_t \Rightarrow -\eta[\theta_u u_{t-1} + \theta_e e_t] = \gamma\theta_u u_{t-1} + [\rho u_{t-1} + e_t]$$

- Deliver implications for r coefficients matching our prior discussion of "expected versus unexpected supply shifts"

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Working out the solutions

$$-\eta p_t = \gamma E_{t-1} p_t + u_t$$

$$\Rightarrow -\eta[\theta_u u_{t-1} + \theta_e e_t] = \gamma[\theta_u u_{t-1}] + \rho u_{t-1} + e_t$$

$$\Rightarrow -\eta\theta_e = 1 \;\; and \; -\eta\theta_u = \gamma\theta_u + \rho$$

$$\Rightarrow \theta_e = -\frac{1}{\eta} \;\; and \;\; \theta_u = -\frac{1}{\gamma + \eta}\rho$$

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Interpreting the UDC solution

- Effect of lagged u is

$$\theta_u = -\frac{1}{\gamma + \eta}\rho$$

- A product
  - The effect of expected u on price
  - The effect of lagged u on expected u.

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A second model in the style of Muth

- Add inventory speculation, subtract “production takes time”.
- By contrast, Muth’s second model has both.
- New element

$$k_{t+1} - k_t = i_t$$
$$k_{t+1} = \phi[\beta E_t p_{t+1} - p_t]$$
$$d_t + i_t = y_t = \gamma p_t + u_t$$

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Implications of market equilibrium

- Supply = Demand
- Inventory flow demand alters market demand

$$\boxed{\begin{array}{l} System \\ k_{t+1} = \phi[\beta E_t p_{t+1} - p_t] \\ -\eta p_t + (k_{t+1} - k_t) = \gamma p_t + u_t \end{array}}$$

$$\boxed{\begin{array}{l} Alternate\ system \\ p_t = \dfrac{1}{\gamma + \eta}[(k_{t+1} - k_t) - u_t] \\ (\gamma + \eta)k_{t+1} = \phi[\beta E_t[(k_{t+2} - k_{t+1}) - u_{t+1}] \\ \qquad\qquad - \phi[(k_{t+1} - k_t) - u_t]] \end{array}}$$

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Analyzing this system

- Muth sets up an undetermined coefficients approach to (a generalization of this) system and then solves it.
  - If we proceed down this route with the second system above, one issue that we encounter is that there are two roots of the inventory stock (k) difference equation, one stable and one unstable. Muth's approach is to suppress the unstable root.
  - An alternative is to view the dynamic system in the first form, which is a first order vector system

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A linear difference system

- The equations may be written as a first order *expectational* difference equation system

$$AE_tY_{t+1} = BY_t + Cu_t$$

$$\begin{bmatrix} -\phi\beta & 1 \\ 0 & 1 \end{bmatrix} E_t \begin{bmatrix} p_{t+1} \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} -\phi & 0 \\ (\gamma+\eta) & 1 \end{bmatrix} \begin{bmatrix} p_t \\ k_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t$$

$$E_tY_{t+1} = WY_t + \Psi u_t$$
$$(with W = A^{-1}B \ and \ \Psi = B^{-1}C)$$

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Muth's second example

- A characteristic form for RE models studied by Blanchard-Kahn (next lecture)
- Contains a predetermined variable (past inventory stock)
- We will study this system as we move forward in lectures and homeworks, but not comment further on it now.
- Eigenvalues play a key role: these are roots to |A*z-B|=0 or |I*z=W|=0

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B. Lucas on Econometric Policy Evaluation

- An important paper that was also timely
- Backdrop:
  - the breakdown of the "Phillips curve"
    - High unemployment (low output) associated with low inflation and vice-versa
  - US experience: a period of "stagflation"
    - high unemployment (low output) and high inflation

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Cruel tradeoff

- What average rate of inflation should the government set, given that high inflation meant low unemployment and vice versa.
- Dynamic Phillips Curve model (Solow-Gordon)

$$\pi_t = \alpha\pi_{t-1} + \lambda u_t + \ldots + e_t$$

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Controlling inflation

- Cut unemployment by amount Δu permanently starting at t. The effect on inflation is shown at right
- Values of α that are large mean much bigger long run multipliers.

$$\Delta\pi_t = \lambda\Delta u$$
$$\Delta\pi_{t+1} = \lambda\Delta u + \alpha\lambda\Delta u$$
$$\Delta\pi_{t+2} = \lambda\Delta u + \alpha\lambda\Delta u + \alpha^2\lambda\Delta u$$
....
$$\Delta\pi_{t+n} = \lambda\Delta u + \alpha\lambda\Delta u + \alpha^n\lambda\Delta u$$
....
$$\Delta\pi_{\infty} = \frac{\lambda}{1-\alpha}\Delta u$$

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An alternative model

- Friedman/Lucas/Sargent
- Only surprise changes in inflation affect unemployment: there is no long-run tradeoff.
- People have rational expectations about inflation.

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A Model Producing an apparent, but not real, LR tradeoff

$$u_t = \theta(\pi_t - E_{t-1}\pi_t)$$

$$\pi_t = \rho\pi_{t-1} + v_t \Rightarrow E_{t-1}\pi_t = \rho\pi_{t-1}$$

$$\pi_t = \rho\pi_{t-1} + \frac{1}{\theta}u_t$$

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FSL prediction

- An attempt to exploit LR Phillips curve (permanent increases in inflation) will produce no benefits
- Interpretation as increase in ρ toward one
- Prediction that model coefficients will "shift", seemed consistent with problems encountered by then-current macro models.

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Econometric Policy Evaluation: A Critique

- Uses a series of forceful examples to make the case that there is a general presumption that econometric model equations are not "policy invariant"
  - Phillips curve
  - Investment
  - Consumption
- Other key example added later
  - Term Structure (Poole)

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Lucas Critique

- Stimulated economists to think about
  - Rational expectations models
  - Optimizing macro models
  - New directions in policy design
    - Time (in) consistency of optimal plans
    - Dynamically optimal policy under commitment
    - Dynamically optimal policy that is credible under discretion

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