Solving international RBC models

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March 22, 2006

1 A basic international RBC model

Consider te following functions

$$\begin{aligned} u(c_t, n_t) &= \frac{1}{1-\sigma} c_t^{1-\sigma} - \frac{\chi}{1+\gamma} n_t^{1+\gamma} \\ y_t &= a_t k_t^{1-\alpha} n_t^{\alpha} \end{aligned}$$

In each country, the maximization involves the selection of $[c_t, n_t, i_t, k_{t+1}]$ and the restrictions are

$$\begin{aligned} (1-\delta) k_t + i - k_{t+1} &= 0 \\ (1-\delta) k_t^* + i^* - k_{t+1}^* &= 0 \\ \pi \left(a k_t^{1-\alpha} n_t^{\alpha} - c_t - i_t \right) + (1-\pi) \left(a^* k_t^{*1-\alpha} n_t^{*\alpha} - c_t^* - i_t^* \right) &= 0 \end{aligned}$$

The maximization problem is:

$$\max L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \pi u(c_t, n_t) + (1-\pi) u(c_t^*, n_t^*) \\ +\pi \eta_t \left[(1-\delta) k_t + i - k_{t+1} \right] \\ +(1-\pi) \eta_t \left[(1-\delta) k_t^* + i^* - k_{t+1}^* \right] \\ +\lambda_t \left[\pi \left(a k_t^{1-\alpha} n_t^{\alpha} - c_t - i_t \right) + (1-\pi) \left(a^* k_t^{*1-\alpha} n_t^{*\alpha} - c_t^* - i_t^* \right) \right] \end{array} \right\}$$

The related FOCs are

$$
\begin{aligned}
c_t &: \frac{1}{c_t} - \lambda_t = 0 \\
c_t^* &: \frac{1}{c_t^*} - \lambda_t = 0 \\
n_t &: -\chi n^{\gamma} - \lambda_t \alpha \frac{y_t}{n_t} = 0 \\
n_t^* &: -\chi n^{*\gamma} - \lambda_t \alpha \frac{y_t^*}{n_t^*} = 0 \\
i_t &: \eta_t - \lambda_t = 0 \\
i_t^* &: \eta_t^* - \lambda_t = 0 \\
k_{t+1} &: -\eta_t + \beta \eta_{t+1} (1-\delta) + \beta \lambda_{t+1} (1-\alpha) \frac{y_{t+1}}{k_{t+1}} = 0 \\
k_{t+1}^* &: -\eta_t^* + \beta \eta_{t+1}^* (1-\delta) + \beta \lambda_{t+1} (1-\alpha) \frac{y_{t+1}^*}{k_{t+1}^*} = 0 \\
\eta_t &: (1-\delta) k_t + i - k_{t+1} = 0 \\
\eta_t^* &: (1-\delta) k_t^* + i^* - k_{t+1}^* = 0 \\
\lambda_t &: \pi [y_t - c_t - i_t] + (1-\pi) [y_t^* - c_t^* - i_t^*] = 0
\end{aligned}
$$

The steady-state involves solving the set of FOCs.

Loglinearized system:

1.

$$
\begin{aligned}
c \cdot \frac{-1}{c^2} \cdot d \ln c_t &= \lambda \cdot d \ln \lambda_t \\
d \ln c_t &= d \ln \lambda_t \\
d \ln c_t^* &= d \ln \lambda_t
\end{aligned}
$$

2.

$$
\begin{aligned}
n \cdot -\chi \gamma n^{\gamma-1} \cdot d \ln n_t &= \lambda \cdot \alpha \frac{y}{n} \cdot d \ln \lambda_t \\
&\quad + y \cdot \lambda \alpha \frac{1}{n} \cdot d \ln y_t \\
&\quad + n \cdot \lambda \alpha \frac{-y}{n^2} \cdot d \ln n_t
\end{aligned}
$$

$$
\begin{aligned}
(1+\gamma)\cdot d\ln n_t &= d\ln \lambda_t + d\ln y_t \\
(1+\gamma)\cdot d\ln n_t^* &= d\ln \lambda_t + d\ln y_t^*
\end{aligned}
$$

3.

$$
\begin{aligned}
d\ln \lambda_t &= d\ln \eta_t \\
d\ln \lambda_t &= d\ln \eta_t^*
\end{aligned}
$$

4.

$$
\begin{aligned}
d\ln \eta_t &= \beta(1-\delta)\cdot d\ln \eta_{t+1} \\
&+\beta\left(1-\alpha\right)\frac{y}{k}\cdot d\ln \lambda_{t+1} \\
&+\beta\left(1-\alpha\right)\frac{y}{k}\cdot d\ln y_{t+1} \\
&-\beta\left(1-\alpha\right)\frac{y}{k}\cdot d\ln k_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
d\ln \eta_t^* &= \beta(1-\delta)\cdot d\ln \eta_{t+1}^* \\
&+\beta\left(1-\alpha\right)\frac{y^*}{k^*}\cdot d\ln \lambda_{t+1} \\
&+\beta\left(1-\alpha\right)\frac{y^*}{k^*}\cdot d\ln y_{t+1}^* \\
&-\beta\left(1-\alpha\right)\frac{y^*}{k^*}\cdot d\ln k_{t+1}^*
\end{aligned}
$$

5.

$$
\begin{aligned}
d\ln k_{t+1} &= \frac{i}{k}d\ln i_t + (1-\delta)\,d\ln k_t \\
d\ln k_{t+1}^* &= \frac{i^*}{k^*}d\ln i_t^* + (1-\delta)\,d\ln k_t^*
\end{aligned}
$$

6.

$$
\begin{aligned}
d\ln y_t &= \frac{c}{y}d\ln c_t + \frac{i}{y}d\ln i_t \\
d\ln y_t^* &= \frac{c^*}{y^*}d\ln c_t^* + \frac{i^*}{y^*}d\ln i_t^*
\end{aligned}
$$

7.

$$
\begin{aligned}
d\ln y_t &= d\ln a_t + (1-\alpha)\, d\ln k_t + \alpha d\ln n_t \\
d\ln y_t^* &= d\ln a_t^* + (1-\alpha)\, d\ln k_t^* + \alpha d\ln n_t^*
\end{aligned}
$$

2 An int'l RBC model with capital adjustment costs

For the international RBC model with capital adjustment costs, consider te following functions:

$$
\begin{aligned}
u(c_t, n_t) &= \frac{1}{1-\sigma} c_t^{1-\sigma} - \frac{\chi}{1+\gamma} n_t^{1+\gamma} \\
y_t &= a_t k_t^{1-\alpha} n_t^{\alpha}
\end{aligned}
$$

In each country, the maximization involves the selection of $[c_t, n_t, i_t, k_{t+1}]$ and the restrictions are

$$
\begin{aligned}
(1-\delta)\, k_t + \phi\left(\frac{i_t}{k_t}\right) k_t - k_{t+1} &= 0 \\
(1-\delta)\, k_t^* + \phi\left(\frac{i_t^*}{k_t^*}\right) k_t^* - k_{t+1}^* &= 0 \\
\pi\left(a k_t^{1-\alpha} n_t^{\alpha} - c_t - i_t\right) + (1-\pi)\left(a^* k_t^{*1-\alpha} n_t^{*\alpha} - c_t^* - i_t^*\right) &= 0
\end{aligned}
$$

The maximization problem is:

$$
\max L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{
\begin{array}{c}
\pi u(c_t, n_t) + (1-\pi)\, u(c_t^*, n_t^*) \\
+\pi \eta_t \left[(1-\delta)\, k_t + \phi\left(\frac{i_t}{k_t}\right) k_t - k_{t+1}\right] \\
+(1-\pi)\, \eta_t \left[(1-\delta)\, k_t^* + \phi\left(\frac{i_t^*}{k_t^*}\right) k_t^* - k_{t+1}^*\right] \\
+\lambda_t \left[\pi\left(a k_t^{1-\alpha} n_t^{\alpha} - c_t - i_t\right) + (1-\pi)\left(a^* k_t^{*1-\alpha} n_t^{*\alpha} - c_t^* - i_t^*\right)\right]
\end{array}
\right\}
$$

The related FOCs are

$$
\begin{aligned}
c_t &: \frac{1}{c_t} - \lambda_t = 0 \\
c_t^* &: \frac{1}{c_t^*} - \lambda_t = 0 \\
n_t &: -\chi n^{\gamma} - \lambda_t \alpha \frac{y_t}{n_t} = 0 \\
n_t^* &: -\chi n^{*\gamma} - \lambda_t \alpha \frac{y_t^*}{n_t^*} = 0 \\
i_t &: \eta_t \cdot \phi' - \lambda_t = 0 \\
i_t^* &: \eta_t^* \cdot \phi' - \lambda_t = 0 \\
k_{t+1} &: -\eta_t + \beta \eta_{t+1} \left((1-\delta) - \phi' \left(\frac{i_{t+1}}{k_{t+1}} \right) \cdot \left(\frac{i_{t+1}}{k_{t+1}} \right) + \phi \left(\frac{i_{t+1}}{k_{t+1}} \right) \right) + \beta \lambda_{t+1} (1-\alpha) \frac{y_{t+1}}{k_{t+1}} = 0 \\
k_{t+1}^* &: -\eta_t^* + \beta \eta_{t+1}^* \left((1-\delta) - \phi' \left(\frac{i_{t+1}^*}{k_{t+1}^*} \right) \cdot \left(\frac{i_{t+1}^*}{k_{t+1}^*} \right) + \phi \left(\frac{i_{t+1}^*}{k_{t+1}^*} \right) \right) + \beta \lambda_{t+1} (1-\alpha) \frac{y_{t+1}^*}{k_{t+1}^*} = 0 \\
\eta_t &: (1-\delta) k_t + \phi \left(\frac{i_t}{k_t} \right) k_t - k_{t+1} = 0 \\
\eta_t^* &: (1-\delta) k_t^* + \phi \left(\frac{i_t^*}{k_t^*} \right) k_t^* - k_{t+1}^* = 0 \\
\lambda_t &: \pi \left[y_t - c_t - i_t \right] + (1-\pi) \left[y_t^* - c_t^* - i_t^* \right] = 0
\end{aligned}
$$

The steady-state involves solving the set of FOCs.

Loglinearized system:

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\begin{aligned}
c \cdot \frac{-1}{c^2} \cdot d \ln c_t &= \lambda \cdot d \ln \lambda_t \\
d \ln c_t &= d \ln \lambda_t \\
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\begin{aligned}
n \cdot -\chi \gamma n^{\gamma-1} \cdot d \ln n_t &= \lambda \cdot \alpha \frac{y}{n} \cdot d \ln \lambda_t \\
& \quad + y \cdot \lambda \alpha \frac{1}{n} \cdot d \ln y_t \\
& \quad + n \cdot \lambda \alpha \frac{-y}{n^2} \cdot d \ln n_t
\end{aligned}
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\begin{aligned}
(1+\gamma)\cdot d\ln n_t &= d\ln\lambda_t + d\ln y_t \\
(1+\gamma)\cdot d\ln n_t^* &= d\ln\lambda_t + d\ln y_t^*
\end{aligned}
$$

3.

$$
\begin{aligned}
d\ln\lambda_t &= d\ln\eta_t - \frac{1}{\theta}\cdot d\ln i_t + \frac{1}{\theta}\cdot d\ln k_t \\
d\ln\lambda_t &= d\ln\eta_t^* - \frac{1}{\theta}\cdot d\ln i_t^* + \frac{1}{\theta}\cdot d\ln k_t^*
\end{aligned}
$$

4.

$$
\begin{aligned}
d\ln\eta_t &= \beta(1-\frac{i}{k})\cdot d\ln\eta_{t+1} \\
&\quad +\beta\frac{1}{\theta}\frac{i}{k}\cdot d\ln i_{t+1} \\
&\quad \left(-\beta\frac{1}{\theta}\frac{i}{k} - \beta\left(1-\alpha\right)\frac{y}{k}\right)\cdot d\ln k_{t+1} \\
&\quad +\beta\left(1-\alpha\right)\frac{y}{k}\cdot d\ln\lambda_{t+1} \\
&\quad +\beta\left(1-\alpha\right)\frac{y}{k}\cdot d\ln y_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
d\ln\eta_t^* &= \beta(1-\frac{i^*}{k^*})\cdot d\ln\eta_{t+1}^* \\
&\quad +\beta\frac{1}{\theta}\frac{i^*}{k^*}\cdot d\ln i_{t+1}^* \\
&\quad \left(-\beta\frac{1}{\theta}\frac{i^*}{k^*} - \beta\left(1-\alpha\right)\frac{y^*}{k^*}\right)\cdot d\ln k_{t+1}^* \\
&\quad +\beta\left(1-\alpha\right)\frac{y^*}{k^*}\cdot d\ln\lambda_{t+1} \\
&\quad +\beta\left(1-\alpha\right)\frac{y^*}{k^*}\cdot d\ln y_{t+1}^*
\end{aligned}
$$

5.

$$
\begin{aligned}
d\ln k_{t+1} &= \frac{i}{k}d\ln i_t + \left(1-\frac{i}{k}\right)d\ln k_t \\
d\ln k_{t+1}^* &= \frac{i^*}{k^*}d\ln i_t^* + \left(1-\frac{i^*}{k^*}\right)d\ln k_t^*
\end{aligned}
$$

6.

$$
\begin{aligned}
d\ln y_t &= \frac{c}{y} d\ln c_t + \frac{i}{y} d\ln i_t \\
d\ln y_t^* &= \frac{c^*}{y^*} d\ln c_t^* + \frac{i^*}{y^*} d\ln i_t^*
\end{aligned}
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$$
\begin{aligned}
d\ln y_t &= d\ln a_t + (1-\alpha)\, d\ln k_t + \alpha d\ln n_t \\
d\ln y_t^* &= d\ln a_t^* + (1-\alpha)\, d\ln k_t^* + \alpha d\ln n_t^*
\end{aligned}
$$