## Boston University Economics 702 Fall 2008

## Introductory Study Problem #3: A Wealth Difference Equation

Consider an agent who can accumulate wealth  $(a_t)$  according to

$$a_{t+1} = (1+r_t)[a_t + z_t]$$

where  $z_t$  is his saving and  $r_t$  is the rate of return.

(a) Assuming that the rate of return is constant over time, solve this difference equation from an initial condition  $a_0$  given an exogenous sequence of savings,  $\{z_t\}_{t=0}^T$ , to determine wealth at date T + 1.

(b) Supposing that wealth must be positive at date T + 1, show that the condition

$$a_{T+1} \ge 0$$

is equivalent to an economic requirement on the present value of saving

$$a_0 + \sum_{t=0}^T (\frac{1}{1+r})^t z_t \ge 0$$

(c) Now suppose that the rate of return is not constant through time. Solve this difference equation from an initial condition  $a_0$  given an exogenous sequence of savings,  $\{z_t\}_{t=0}^T$ , to determine wealth at date T + 1.

(d) Show that the requirement that wealth must be positive at date T+1,  $a_{T+1} \ge 0$ , is equivalent to an economic requirement on the present value of saving

$$a_0 + \sum_{t=0}^T p_t z_t \ge 0$$

and determine how the  $p_t$  relate to the  $r_t$ .